**In this lesson, we will**:

- Add **circulation** to our circular cylinder potential flow by superposing a **vortex**, and explain how this generates lift on the cylinder
- Define **aerodynamic circulation**
- Discuss the **physical significance:** Spinning cylinders and spheres (the **Magnus effect**)

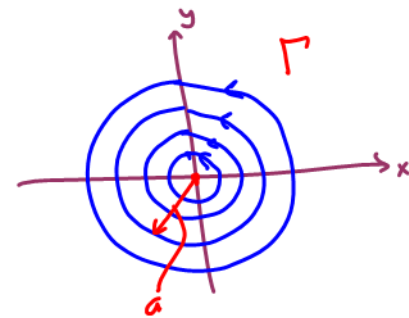## Potential Flow Around a Circular Cylinder: Superposition of a Vortex

- Recall Superpose a uniform Stream & a doublet ⇒ FLOW OVER A CIRCULAR CYLINDER
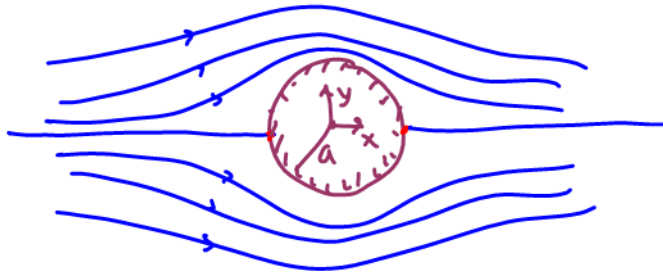
DRAG = 0
LIFT = 0

- LINE VORTEX → HAS CIRCULAR STREAMLINES

LET $\boxed{\Gamma_a = -\Gamma}$ → Γ_a = aerodynamic circulation

- SUPERPOSE   UNIFORM STREAM + DOUBLET + LINE VORTEX
                          OF STRENGTH $\Gamma = -\Gamma_a$
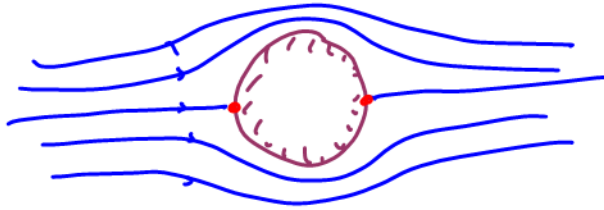
Γ = ⊕ counterclockwise ↻

Γ_a = ⊕ clockwise ↺

U

Effect of Increasing Aerodynamic Circulation (Γ_a)
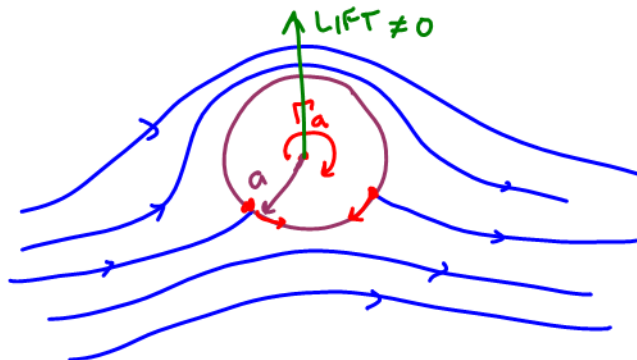
- $\Gamma_a = 0$

DRAG = 0

LIFT = 0

DRAG & LIFT here have dimensions of $\{force/length\}$

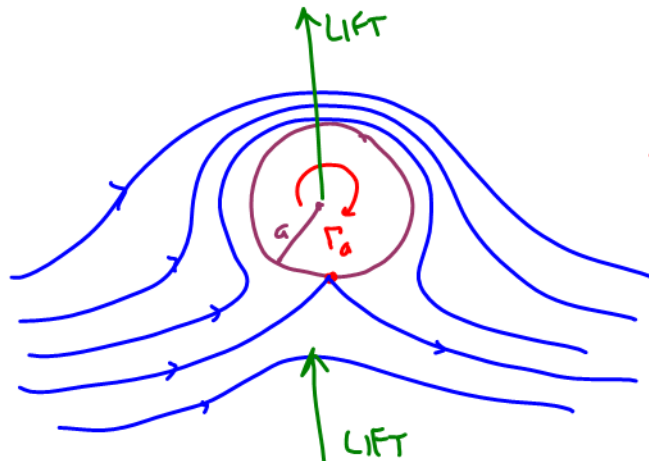(drag per unit depth into the page)

(lift per unit depth into the page)

- $0 < \Gamma_a < 4\pi a U$

LIFT $\neq 0$

DRAG = 0

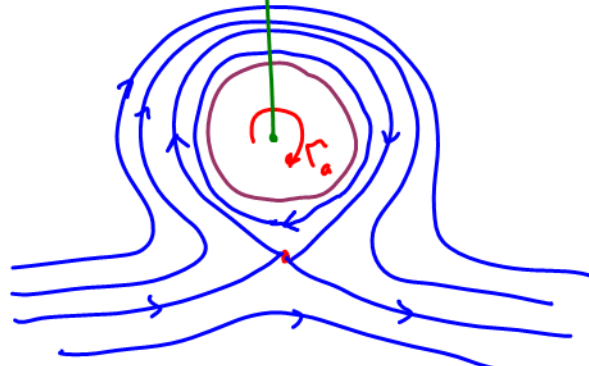LIFT = $\rho U \Gamma_a$

BELOVED BERNOULLI → THIS DOES HAVE LIFT

- $\Gamma_a = 4\pi a U$

LIFT

DRAG = 0

LIFT = $\rho U \Gamma_a$

- $\Gamma_a > 4\pi a U$

LIFT

DRAG = 0

LIFT = $\rho U \Gamma_a$

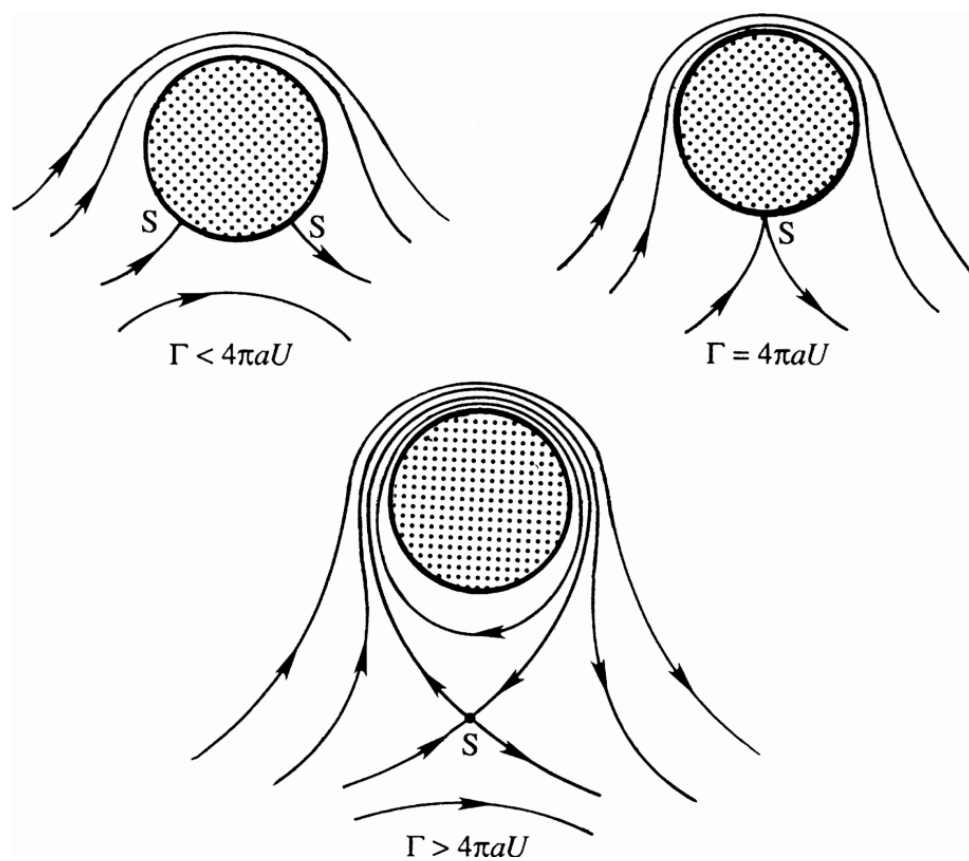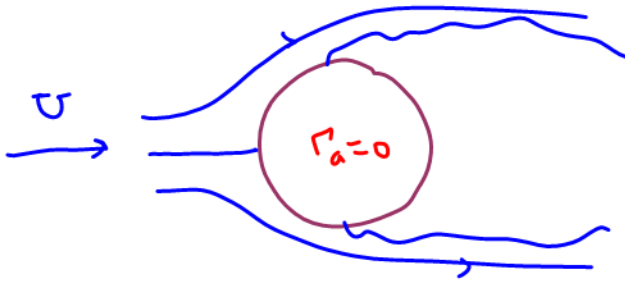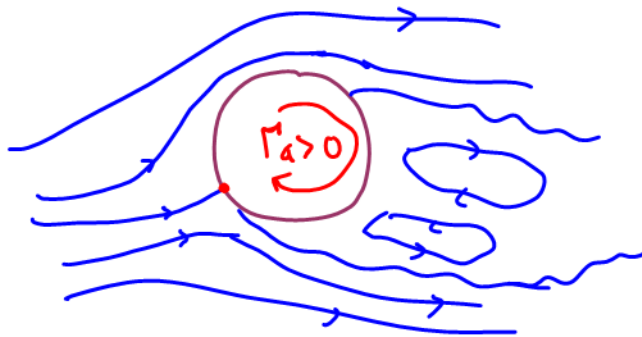$\Gamma < 4\pi a U$

$\Gamma = 4\pi a U$

$\Gamma > 4\pi a U$

Figure from P. K. Kundu, I. M. Cohen & D. R. Dowling, *Fluid Mechanics* 6e, Elsevier Inc., 2016.

Physical Significance of Cylinders With Circulation
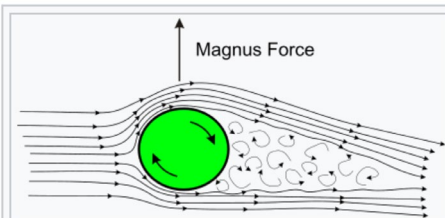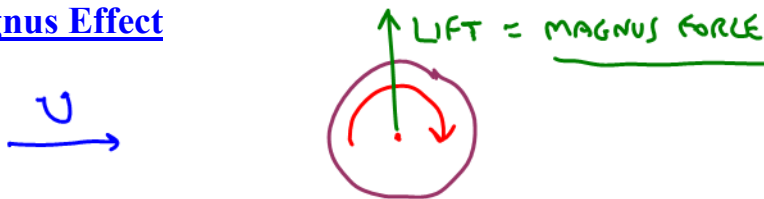
· WE MODEL A ROTATING CYLINDER

$\Gamma_a = 0$

DRAG ≠ 0
LIFT = 0

$\Gamma_a > 0$

DRAG ≠ 0
LIFT ≠ 0

The Magnus Effect

↑ LIFT = MAGNUS FORCE
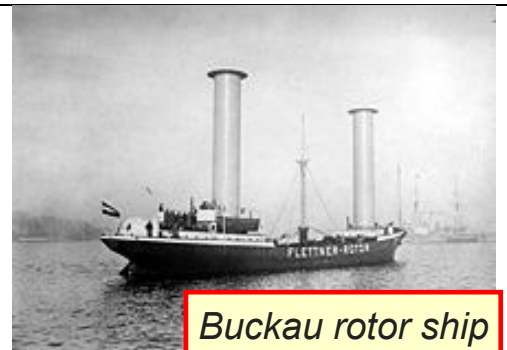

Magnus Force

The Magnus effect, depicted with a backspinning cylinder in an airstream. The arrow represents the resulting sideways force that can be used to help propel a ship. The curly flow lines represent a turbulent wake. The airflow is deflected in the direction of spin.
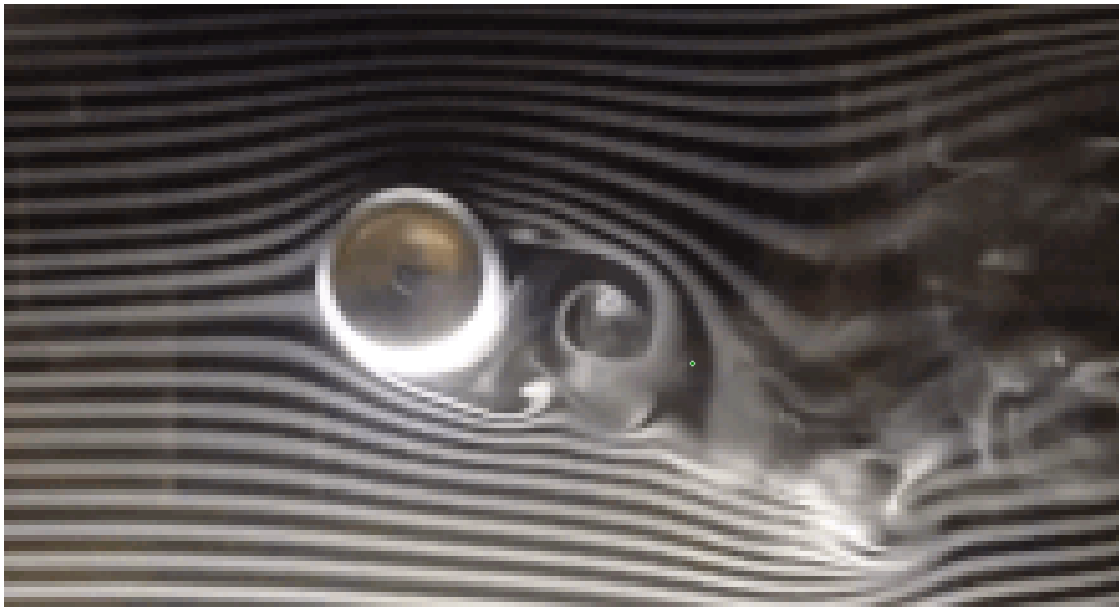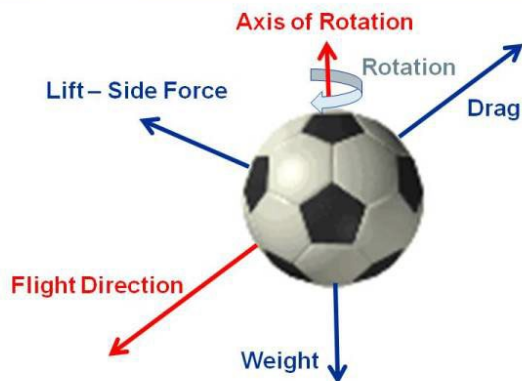
E-Ship 1 rotor ship

Buckau rotor ship

Maersk Pelican

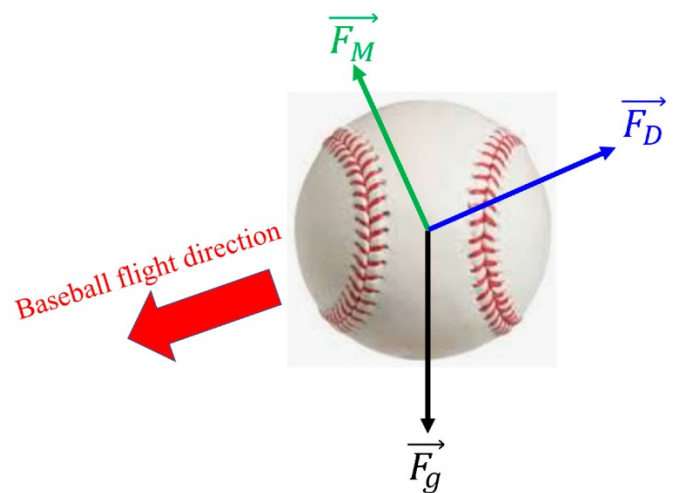Figures from https://en.wikipedia.org/wiki/Rotor_ship.

Video from https://en.wikipedia.org/wiki/Magnus_effect.

Figures from:
https://ffden-2.phys.uaf.edu/webproj/211_fall_2018/Adam_Liland/Adam_Liland/spin.html (left)
and https://www.mdpi.com/2076-3417/12/11/5540 (right).