Let \textit{unprimed} coordinates \(x_1, x_2,\) and \(x_3\) denote the \textit{original} axes.
Let \textit{primed} coordinates \(x'_1, x'_2,\) and \(x'_3\) denote the \textit{rotated} axes, in this case rotated to the principal axes.

\textbf{Note:} In this diagram and discussion, superscript 2 refers to eigenvalue #2, and is \textit{not} an exponent. E.g., \(b^2\) does \textit{not} mean “\(b\) squared”.

When the rotated coordinate system aligns with the \textbf{principal axes}, the normal (diagonal) components of the tensor are the \textbf{eigenvalues}, and the off-diagonal components are all zero. The unit vectors pointing in the directions of the principal axes are called the \textbf{eigenvectors}. Each eigenvalue has a corresponding eigenvector. Shown above is the eigenvector corresponding to the second eigenvalue, i.e., the eigenvalue associated with the rotated axis direction \(x'_2\) (the rectangular box is drawn to aid in visualizing this in three dimensions). This eigenvector is given the notation \(b^2\), corresponding to eigenvalue \(\lambda^2\). Vector \(b^2\) has three components \((b^2_1, b^2_2, b^2_3)\) relative to the original (unprimed) set of axes, and is one unit in length.

The three \textbf{invariants} of a \textit{symmetric} second-order tensor, \(A_{ij}\), are:

- \(I_1 = A_{11} + A_{22} + A_{33}\)
- \(I_2 = A_{11}A_{22} + A_{22}A_{33} + A_{33}A_{11} - A_{12}^2 - A_{23}^2 - A_{31}^2\)
- \(I_3 = \det (A_{ij})\) \text{ [determinant of the matrix \(A_{ij}\)]}

\textit{By definition, these invariants do not change, no matter how we rotate the axes.}