Falkner-Skan Wedge Flows

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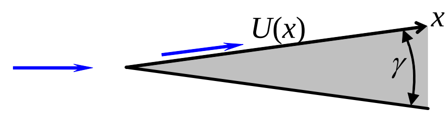
Latest revision: 11 December 2007

1. Introduction

- In class we found a similarity solution for the laminar flat plate boundary layer, where the potential outer flow was simply $U(x) = U$ = constant. It turns out that a similarity solution can be found for some much more *general* boundary layer problems, even when $U(x)$ is not constant. Consider steady, incompressible, two-dimensional, laminar boundary layer flow over some body where the potential outer flow $U(x)$ is *known*.



- In particular, consider 2-D wedge flows, where $U(x)$ follows a power law: $U(x) = Bx^m$ where B and m are constants. The relationship between exponent m and the wedge angle γ was found previously from potential flow theory.

2. Similarity Solution

- We expect a similarity solution since there is *no length scale in the problem*; how big or small the observer is does not matter.
- Choose a similarity variable, $\eta = \frac{y}{\delta_c(x)}$, and let $u(x,y) = U(x) f'(\eta)$, just as was done in the Blasius case.
- Assume that $\delta_c(x)$ also follows a power law, $\delta_c(x) = Cx^b$ where C and b are constants.
- Substitution of these similarity assumptions into the boundary layer equations yields $f''' + \alpha f f'' + \beta[1-(f')^2] = 0$, where β and α are parameters in the problem.
- Falkner and Skan chose $\alpha = 1$ (arbitrarily). β is called the "***Falkner-Skan parameter***," and is defined as $\beta \equiv \frac{\delta_c^2}{\nu}\frac{dU}{dx}$. Thus, their final similarity equation reduces to $f''' + f f'' + \beta[1-(f')^2] = 0$.
- The constants and exponents B, m, C, b, and β are all inter-related as follows (these relationships come from the similarity analysis, after some algebra):
  $b = \frac{1-m}{2}$, $C = \sqrt{\frac{\nu(2-\beta)}{B}}$, $\beta = \frac{2m}{1+m}$ or $m = \frac{\beta}{2-\beta}$.
- The above equation yields a whole *family* of solutions (representing different kinds of flows) depending on the choice of constant β.

3. Special Cases

- $\underline{\beta = 0}$. The Blasius equation results (with the arbitrary Blasius constant c equal to 1). Verification to be completed in class.
- $\underline{\beta = 1}$. To be done in class.
- Other β values and their physical significance to be discussed in class.