The Heat Flux-Vorticity Analogy

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Note: For simplicity, consider two-dimensional incompressible, Newtonian flow in the x-y plane.

1. Heat Flux
- Consider a wall that is parallel to the x-axis, and let \( q_y \) be the y-component of the heat flux vector (rate of heat transfer per unit area to the fluid). From Fourier’s law of conduction, \( q_y = -k \frac{\partial T}{\partial y} \). Right next to the wall, \( q_{wall} = -k \left( \frac{\partial T}{\partial y} \right)_{wall} \).
- In words, Fourier’s law states that the heat flux at a wall is directly proportional to the negative of the temperature gradient at the wall. The actual value of \( T \) is not important, just its slope.
- There are three cases of interest:
  1. \( \frac{\partial T}{\partial y} \) wall < 0 Here, \( q_y \) at the wall is positive, and heat flows from the wall to the fluid. Since energy is being added to the fluid, we think of this as a source of internal energy (temperature) at the wall, or production of internal energy at the wall.
  2. \( \frac{\partial T}{\partial y} \) wall > 0 Here, \( q_y \) at the wall is negative, and heat flows from the fluid to the wall. Since energy is being removed from the fluid, we think of this as a loss of internal energy (temperature) at the wall, or production of negative internal energy at the wall.
  3. \( \frac{\partial T}{\partial y} \) wall = 0 (insulated wall) Here, \( q_y \) at the wall is zero, and no heat flows from the wall to the fluid or from the fluid to the wall. In this case, no energy is being added to or removed from the fluid. There is no production of internal energy, but simply a redistribution of internal energy that already exists in the fluid. (Note, however, that internal energy is still produced within the fluid because of viscous dissipation.)

2. Vorticity Flux - an analogy
- Again consider a wall that is parallel to the x-axis, and let \( \omega_z \) be the z-component of the vorticity vector.
- The analogy: Vorticity is analogous to temperature; vorticity flux is analogous to heat flux.
- In words, just as heat flux at a wall is directly proportional to the negative of the temperature gradient at the wall, the vorticity flux at a wall is directly proportional to the negative of the vorticity gradient at the wall. The actual value of vorticity is not important, just its slope.
- There are three cases of interest:
  1. \( \frac{\partial \omega_z}{\partial y} \) wall < 0 Here, the vorticity flux at the wall is positive, and there is a flux of positive vorticity from the wall to the fluid. Since vorticity is being added to the fluid, we think of this as production of positive vorticity at the wall.
  2. \( \frac{\partial \omega_z}{\partial y} \) wall > 0 Here, the vorticity flux at the wall is negative, and there is a flux of negative vorticity from the wall to the fluid. Since vorticity is being removed from the fluid, we think of this as production of negative vorticity at the wall.
  3. \( \frac{\partial \omega_z}{\partial y} \) wall = 0 Here, the vorticity flux at the wall is zero, and there is no net flux of vorticity from the wall to the fluid or from the fluid to the wall. In this case, no new vorticity is being added or removed from the fluid. There is no production of vorticity at the wall, but simply a redistribution of vorticity that already exists in the fluid.