Today, we will:

- Continue to discuss potential flow using complex variables: flow over a circular cylinder, lift, conformal mapping
- Do some more example problems using complex variables

Recall from last lecture: Potential flow over a circular cylinder

\[
w(z) = Uz + \frac{\mu}{z}, \quad \frac{dw}{dz} = U - \frac{\mu}{z^2}, \quad a = \text{cylinder radius} = \frac{\mu}{U},
\]

Along the surface, \( V(s) = 2U \sin \frac{s}{a} \)

Also — I will review the midterm exam

**Results:**

- Mean = 86.2
- Median = 88
- Low = 67
- High = 100

Velocity field is symmetric both top & bottom & left & right

Also, the pressure is symmetric...

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Bottom line -> **NO DRAG**!
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C. *O'Almert's Paradox:*

The drag on any closed 2-D body in steady potential (irrotational) flow is zero!

\[ \text{In real life, we know that } D_{\text{drag}} \neq 0 \]
So... For blunt bodies that have flow separation, potential flow is not very useful.

But for streamlined bodies (no flow separation), potential flow is very useful.

For instance:

Procedure:
1) Calculate potential flow, neglecting the boundary layer (BL)

2) Fit in a very thin BL. (so thin that it does not affect the outer potential flow)

3) Solve BL equation: from this, can predict skin friction drag on body

You can even predict lift on arbitrary other body — but you must add the appropriate circulation.

1. Flow past a circular cylinder with circulation

Recall, a line vortex has circular streaming.

Let's suppose a vortex on our 2-0 cylinder flow.

One of the streamlines of the line vortex will align with the (cos)

"surface" of the cylinder.

Is suppose uniform stream + doublet + line vortex at origin.
Notation: Kundu changes notation here — let $\Gamma$ be $-\Gamma$ instead

My notation: $\Gamma$ = positive clockwise $\Rightarrow$ (math. correct)

let $\Gamma_a$ = positive clockwise $= -\Gamma$ $\Rightarrow$ $[\Gamma_a = -\Gamma]$

From pg. 168 on, change $\Gamma$ to $\Gamma_a$ i.e., there will be no sign error

$\Gamma_a = 0$

$\rho U \Gamma_a = 0$

$\frac{d}{dt} L = 0$

$L = 0$

$0 < \Gamma_a < 4\pi a U$

$\rho U \Gamma_a > 0$

$L = \rho U \Gamma_a$

$\Gamma_a = 4\pi a U$

$\Gamma_a > 4\pi a U$

Streamline around cylinder

May fall down off the cylinder

Model a spinning cylinder in a freestream flow

day a fairly good job
(2) Lift force on 2-D bodies

**The Kutta–Zhukovskii Lift Theorem:**

For any 2-D closed body (whether it is symmetric or not) in potential flow, let:

- Drag \( D = 0 \)
- Lift \( L = \rho U \Gamma_a \)

Foundation of Aerodynamics

K-J Lift theorem applies to airfoils (wings)

**Airfoil:**

- \( \Gamma_a \) (no vortex)
- Add a line vortex (upstream) \( \Gamma_a \)

\( L = \rho U \Gamma_a \)

\( D = 0 \)
\( L \neq 0 \)

*Note: Must add just the right \( \Gamma_a \) to get the streamlines to flow smoothly over the sharp trailing edge*