Today, we will:

- Briefly discuss axisymmetric irrotational (potential) flow
- Briefly discuss three-dimensional irrotational (potential) flow
- Discuss lift and drag on 3-D bodies, e.g., finite-span wings, induced drag
- Do Candy Questions for Candy Friday

### E. Axisymmetric Irrotational (Potential) Flow

1. Intro.

\[ \vec{U} = \nabla \phi \]

- \( \phi \) can be defined (even in 3-D)
- \( \psi \) can be defined for 2-D flows (flow depends on only 2 independent variables)
- \( \psi \) can be defined for axisymmetric flow

In the x-y plane,

- e.g., bulbs, torpedoes, etc.
- Sphere
2. Equations (see App. B2)

\[ \frac{1}{r} \frac{d}{dr} \left( r^2 u_r \right) + \frac{1}{r \sin \theta} \frac{d}{d\theta} \left( u_\theta \sin \theta \right) = 0 \] (1)

Define \( \Psi (r, \theta) \) such that it exactly satisfies (1)

Stream function

\[ u_r = \frac{1}{r \sin \theta} \frac{d\Psi}{d\theta}, \quad u_\theta = \frac{1}{r \sin \theta} \frac{d\Psi}{dr} \]

Velocity Potential \( \Phi \rightarrow
\begin{align*}
U_r &= \frac{d\Phi}{dr} \\
U_\theta &= \frac{1}{r} \frac{d\Phi}{d\theta}
\end{align*} \]

3. Similarity in difference between 2-D and axisymmetric

[see HW problem]

4. Simple Axisymmetric. Irrotational Flow (building block flow)

See text for details

\[ \Psi = \frac{U r^2}{2} \sin^2 \theta \]

\[ \Psi = -\frac{Q}{4\pi} \cos \theta \] x

\[ \Psi = -\frac{m}{r} \sin^2 \theta \]

5. Superposition:

Qualitative — see text for more details

\[ \text{Uniform stream} + \text{source at origin} \]
b. Flow over a sphere — Uniform stream + doublet (at origin)

c. Torpedo-shaped body

Superpose uniform stream + distribution of sources along the x-axis

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E. 3-D IRROTATIONAL FLOW (CHAP. 15)

1. Intro:
   - For 3-D, cannot define \( \Psi \)
   - Can define \( \phi \)
     \[
     \mathbf{u} = \nabla \phi
     \]
     \( \text{since} \ \nabla \times \mathbf{u} = 0 \)
   - Continuity
     \[
     \nabla \cdot \mathbf{u} = 0
     \]
   - Can still use superposition for \( \phi \)

2. Lift & Drag in 3-D irrotational flow

For a closed 3-D non-lifting body \( (L=0) \), \( D=0 \) (no drag)

Generalize O'Malley's paradox

E.g., 3-D (finite spin) wing at zero angle of attack
For a closed 2-D lifting body \((L \neq 0, \alpha \neq 0)\)

This drag is called \textbf{Induced Drag}\* (induced by the fact that there is lift)

e.g. a finite span wing @ angle of attack

\[ L \approx \int_{y=-s/2}^{y=s/2} \rho U \Gamma_a \, dy \]

For \(2D\) wing, \(\Gamma_a = \text{constant along the span}\)

\[ L \approx \rho U \Gamma_a s \]

Approximate (not exact) because of behavior at the wing tips

\[ y = \pm s/2 \]
Recall Helmholtz Theorem #3 — A vortex tube cannot end within the fluid.

From the rear — looking at the T.E. of the airfoil.

Trailing vortex or tip vortex.