Today, we will:

- Discuss propulsion of fish, birds, and sailboats
- Do Candy Questions for Candy Friday
- Begin a new major topic: Laminar Flow Solutions

4. Propulsion of fish & birds (see Sec. 15.13 in book)
   a. Fish - neutrally buoyant - need a thrust, but do not need lift

   Frame of ref: relative to the fish's body
   • \( U \)  
   • \( V \)

   Net hydrodynamic force
   • Net force: thrust
   • Net force: lift
   • Net force: drag

   "Upstroke" - reverse the fin orientation

   Frame of ref: relative to the fish's tail fin
   • \( U_r \) - relative velocity that fin "sees"
b. Birds: Must generate **thrust & lift**

**Downstroke:** Pretty much same as the fish

**Upl stroke:** - basically do nothing (thrust or lift)

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5. Force on a sailboat → How to sail “into the wind”

Look at frame of ref. of the sail, consider aerodynamic force
Frame of ref. of the keel (in the video)

\[ \mathbf{\tilde{F}}_{\text{aero}} = -\mathbf{\tilde{F}}_{\text{hydro}} \]
Laminar Flow Solutions (Ch. 9)

A. Intro i. review

"Nearly incompressible" flow $\rightarrow$ Liquids $\rightarrow$ $\rho = \text{const. everywhere}$

$\nu$ $\rightarrow$ Very small Mach #

$\therefore$ Density changes are negligible

ii. Buzgany $\rightarrow$ Due to temperature effects

(hot fluid wants to rise)

iii. Boussinesq approx $\rightarrow$ Let $\rho = \text{constant} = \rho_0$ everywhere except in the gravity term

where $\rho = \text{func. of } T$

Temperature $T$ then gets coupled into the mom. eq.

SEE HANDBOUT FOR THE EQUATIONS

(also copied on next page)
Assumptions and Approximations
- The fluid is **Newtonian** with constant properties ($\mu, \nu, k, \alpha, \kappa, C_p$).
- The flow is **laminar** rather than transitional or turbulent.
- The fluid is **nearly incompressible** – either an incompressible liquid or an ideal gas at very low Mach numbers.

Differential Equations of Motion for Nearly Incompressible Flow (general review)
- **Conservation of mass:**
  \[
  \frac{\partial u_i}{\partial x_i} = 0.
  \]
- **Momentum equation:**
  \[
  \rho \frac{Du_i}{Dt} = \rho \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \rho g_i + \mu \frac{\partial^2 u_i}{\partial x_i \partial x_j}.
  \]
- **Conservation of energy (first law): For incompressible liquid:**
  \[
  \rho C_p \frac{DT}{Dt} = \kappa \frac{\partial^2 T}{\partial x_i \partial x_i} + \phi, \quad \text{where} \quad \phi = 2\mu \varepsilon_0.
  \]
  For ideal gas at very low Ma:
  \[
  \rho C_p \frac{DT}{Dt} = \frac{\kappa}{\rho_c} \frac{\partial^2 T}{\partial x_i \partial x_i} \quad \text{or} \quad \frac{DT}{Dt} = \frac{k}{\rho_c} \frac{\partial^2 T}{\partial x_i \partial x_i} \quad \text{(where} \quad \kappa = \frac{k}{\rho_c} = \text{thermal diffusivity}).
  \]
- **Vorticity equation:**
  \[
  \frac{D\omega_k}{Dt} = \omega_j \frac{\partial u_k}{\partial x_j} + v \frac{\partial^2 \omega_k}{\partial x_j \partial x_j}.
  \]

Differential Equations of Motion for Nearly Incompressible Flow with Buoyancy
- **Boussinesq approximation** (See Kundu, Section 4.18): Assume that changes in density $\rho$ are negligible everywhere except in the gravity term (buoyancy), where we let $\rho = \rho_0 [1 - \alpha (T - T_0)]$ where $\alpha$ is the thermal expansion coefficient, $\alpha = \frac{-1}{\rho} \left( \frac{\partial \rho}{\partial T} \right)$ (for an ideal gas, $\alpha = \frac{1}{T}$), and $\rho_0$ is a reference density corresponding to reference temperature $T_0$. ($T$ is assumed to vary only slightly from $T_0$, so that density is nearly constant, but does lead to buoyancy in the flow.) The density is assumed to equal $\rho_0$ in all other terms in the equation.
- The continuity and vorticity equations are the same as above, since density does not appear in these equations.
- The momentum equation becomes
  \[
  \frac{\rho Du_i}{Dt} = \frac{\rho_0 Du_i}{Dt} \left[ \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right] = -\frac{\partial p}{\partial x_i} + \rho_0 [1 - \alpha (T - T_0)] g_i + \mu \frac{\partial^2 u_i}{\partial x_i \partial x_j},
  \]
  or
  \[
  \frac{Du_i}{Dt} = \left( \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \right) = -\frac{1}{\rho_0} \frac{\partial p}{\partial x_i} + \left[ 1 - \alpha (T - T_0) \right] g_i + \mu v \frac{\partial^2 u_i}{\partial x_i \partial x_j},
  \]
  where $v = \frac{\mu}{\rho_0}$.
- The energy equation then becomes: For incompressible liquid:
  \[
  \rho_0 C_p \frac{DT}{Dt} = \kappa \frac{\partial^2 T}{\partial x_i \partial x_i} + \phi.
  \]
  For ideal gas at very low Mach number:
  \[
  \frac{DT}{Dt} = \frac{k}{\rho_0 C_p} \frac{\partial^2 T}{\partial x_i \partial x_i}, \quad \text{where} \quad \kappa = \frac{k}{\rho_0 C_p} \quad \text{& viscous dissipation is negligible.}
  \]

Solution Technique for Nearly Incompressible Laminar Flow without Buoyancy
1. Write the continuity and momentum equations. (*Note: The energy equation is uncoupled if there is no buoyancy. If buoyancy is important, the energy equation must be solved simultaneously with mass and momentum.*)
2. Simplify the equations as much as possible (cross off zero terms, etc. – always justify your simplifications).
3. Solve for $u_i$ and $p$. (This step will generate some constants from the integration).
4. Apply BCs (on $u_i$ and $p$) to solve for the unknown constants. (Now $u_i$ and $p$ are known everywhere.)
5. Write the energy equation. (uncoupled)
6. Simplify as much as possible.
7. Solve for $T$. (This step will generate some constants from the integration).
8. Apply BCs (on $T$) to solve for the unknown constants. (Now $T$ is known everywhere, and we are finished.)
Mathematically, for buoyant, nearly incomp. flow,

\[ \begin{align*}
5 \text{ unknowns: } & U_i, p, T \\
\rho &= \text{fre. of } T \text{ is not an "unknown"} \\
5 \text{ eqs: } & \begin{align*}
\text{mass} (1) \\
\text{mom.} (2) \\
\text{en.} (3)
\end{align*}
\]

For flow w/o buoyancy,

\[ \begin{align*}
4 \text{ eqs: } 4 \text{ unknowns: } & U_i, p, \text{ (remove } T \text{)} \\
\text{ (remove energy)}
\end{align*} \]

Then, we can solve energy separately. \( \Rightarrow \) solve for \( T \) once we know \( U_i \) and \( p \)

**How to solve these diff. eq's?**

a) **Analytic solution** — possible only for very simple geometries.

b) **Similarity solutions** — but more complex than a)

c) **Computational solution** — solve PDEs directly

Also often involve a computer, but solve ODE's not PDE's