Today, we will:

- Continue the example problem – Blasius flat plate laminar boundary layer

  1. **Step 1** - Solve for U(x)
  2. **Step 2** - thin BL
  3. **Step 3** - Solve BL off

3. Results

  1. **Step 1** - Calc. other properties of the BL

    A. B.L. thickness → Define \( f_\infty = f \) → where \( u = 0.99 U \)

    See Matlab soln: \( \frac{U}{U} = f' = 0.99 \) @ \( \eta = 4.91 \)

    \( \delta = \infty \) since \( \eta = y \sqrt{\frac{U}{2x}} \) → @ \( y = f_\infty \), \( \eta = 4.91 \)

    \[ f_\infty = 4.91 \sqrt{\frac{U}{U}} \]

    \[ \Rightarrow \delta \text{ grows like } \sqrt{x} \]

    Some authors round \( \delta_0 \) to 5.0

    \[ \delta_0 \approx \frac{5.0}{\sqrt{Re_x}} \]

**Step 5** - check if BL is thin

E.g., air \( \text{Re}_x \approx 10^5 \)

\[ \nu = 1.5 \times 10^{-6} \text{ m}^2/\text{s} \]

\[ \text{Here, } f = 4.91(x) \quad \Rightarrow \quad \frac{Re_x}{\sqrt{Re_x}} = 6.7 \times 10^5 \quad \Rightarrow \quad \delta = 6.0 \times 10^{-2} \text{ m} \]

\( \frac{\delta}{L} \ll 1 \)

At \( x = L = 1 \) m, \( \delta = 6 \text{ mm} \) → yes, \( \delta \) is small.
b. Displacement Thickness $\delta^*$

Define $\delta^*$: $\delta^*$ = distance that streamlines outside the BL are displaced away from the wall

OR

$\delta^*$ = the amount (thickness) you would have to displace the wall to get an equivalent new flow rate in a hypothetical frictionless flow above the wall

Application:
1) Improve the solution
2) Wind tunnel walls
It turns out that
\[ \int_{y=0}^{y=\eta} \left( 1 - \frac{y}{U} \right) \, dy \] at any \( x \)-location.

For a moving flat plate \( BL \)
\[ \int_{y=0}^{y=\eta} (1 - f') \, dy = \int_{y=0}^{\eta} \frac{\nu x}{U} \, dy \]
\[ \int_{y=0}^{\eta} \frac{\nu x}{U} \, dy = \left[ \frac{\nu x}{U} \right]_{0}^{\eta} \]
\[ \eta = \infty \quad n = 0 \quad f(\infty) = 0 \]

See Matlab code at "\( \infty \)" (large \( \eta \))
\[ (n-f)_{\infty} = 10 - 8.27521 = 1.7208 \]

Thus,
\[ \frac{\delta^y}{x} = \frac{1.7208}{\sqrt{Re_x}} \]
\[ \left[ \begin{array}{c} \text{Compare to } \delta : \\ \frac{\delta}{x} = \frac{4.91}{\sqrt{Re_x}} \end{array} \right] \]

C. Momentum Thickness \( \Theta \)

Refer to example problem of Integral (CV) flat plate (Week 4 notes)

Recall
\[ \frac{D}{b} = \text{drag on one side of plate} = \rho U^2 \int_{0}^{\infty} \frac{u}{U} \left( 1 - \frac{u}{U} \right) \, dy \]

\[ \rho U^2 \Theta \equiv \text{loss of momentum flux per unit depth due to the presence of the \( BL \).} \]

\[ \Theta = \text{momentum thickness} \]

\[ \text{D} \& \text{C} \text{U value for any } BL \]
For Blasius flat plate BL

\[ \theta = \int_0^\infty \frac{u}{V} (1 - \frac{u}{V}) \, dy = \frac{\nu x}{V} \int_0^\infty f'(1-f') \, dy \]

\[ \theta = \frac{\nu x}{V} \left\{ \left[ f(1-f') \right]_0^\infty + \int_0^\infty f'' \, dy \right\} \]

Apply limit:

\[ f'(\infty) = 1 \quad \text{as } \eta \to \infty \]

\[ f(0) = 0 \quad \text{as } \eta \to 0 \]

Blow up \( \eta \) to

\[ f'' + \frac{1}{2} f f'' = 0 \quad \Rightarrow \quad ff'' = -2f'' \]

\[ -2 \int_0^\infty f'' \, dy \]

\[ \theta = -2 \int \frac{\nu x}{V} f'' \, dy = -2 \frac{\nu x}{V} \left\{ f''(\infty) - f''(0) \right\} \]

\[ f''(\infty) = 0 \quad \text{see R-K calculation} \]

\[ f''(0) = 0.322 \]

\[ \frac{\theta}{x} = \frac{0.664}{\sqrt{Re_x}} \]

\[ \delta > \delta_x > \theta \]

\[ \frac{\delta}{x} \]

\[ \frac{f(x)}{V} \]

\[ \frac{\theta}{x} \]

\[ \frac{\delta_x}{x} \]

\[ \frac{\delta}{x} \]

\[ \frac{\theta}{x} \]

\[ x \]

\[ U \]
1. Shape Factor, $H$

\[ H = \frac{f_w}{\theta} \]

$[H \text{ must be } > 1 \text{ since } f' \text{ always } > \theta]$

Advice: $H$ is useful for predicting BL separation

- If $H$ is "small" [close to 1]

  \[ \mathcal{C}_w \text{ is big} \]

  Ready to separate

- If $H$ is "big" [$\approx 2 \text{ or } 4$]

  \[ \mathcal{C}_w \text{ is small} \]

  Ready to separate

\[ \boldsymbol{e. \text{ Wall shear stress}} \]

\[ \mathcal{C}_w = \mu \left( \frac{du}{dy} \right)_{\text{wall}} \]

\[ \downarrow \]

\[ \mathcal{C}_w = \mu U \sqrt{\frac{U}{u_x}} f''(0) \]

\[ \therefore \mathcal{C}_w = \frac{0.332 \rho U^2}{\sqrt{Re_x}} \]

$\left[ \mathcal{C}_w \downarrow \mu \times \uparrow \right]

\[ \mathcal{C}_w \text{ from our RK min } = 0.332 \]
Define \( C_f = \text{local skin friction coefficient} \)
\[
C_f = \frac{2\rho U^2}{\frac{1}{2} \rho U^2} \quad \rightarrow \quad C_f = \frac{0.664}{\sqrt{Re_x}}
\]

Notice: \( C_f = \frac{\Theta}{x} \) for a Blasius BL

[This is true only for a Blasius flat plate BL]

f. Drag coefficient
\[
C_D = \frac{D}{\frac{1}{2} \rho U^2 A}
\]
\( A = \) planform area = \( X \cdot b \)
\( b = \) width of plate

To calculate the drag coeff:
1. Integrate \( C_w \)
\[
D = \int \frac{C_w}{2} (dA) \quad \text{d}A = b \text{d}x
\]
\[
C_D = \frac{1.33}{\sqrt{Re_x}}
\]

or 2. From def of \( \Theta \)
\[
\frac{D}{b} = \rho U^2 \Theta
\]
Plug in \( \Theta \)
\[
C_D = \frac{1.33}{\sqrt{Re_x}}
\]
[on one side only]

Notice \( C_D = 2C_f \) for a Blasius flat plate BL

[not true in general for all BL's, only for Blasius]