Today, we will:

- Finish discussing the classic example problem – Blasius flat plate boundary layer
- Discuss boundary layers with pressure gradients
- If time, begin a discussion of Falkner-Skan wedge flows

**New Eq sheet for final exam is posted on website [on Exam fall](#)**

\[ \text{Blasius Solution (continued)} \]

9. Velocity normal to the plate \( (v) \)

Recall, \( v = U \frac{d \xi}{dx} (\eta f' - f) \)

\[ v \to \frac{1}{2} \sqrt{\frac{\nu U}{x}} (\eta f' - f) \]

Outside the BL \( (\eta \to \infty) \),

\[ v \to 0.860 \sqrt{\frac{\nu U}{x}} \quad \text{or} \quad \frac{v}{U} \to \frac{0.860}{\sqrt{Re_x}} \]

**Number:** \( U = 10 \text{ m/s}, \quad x = 1 \text{ m} \)

Outside the BL

Outside \( 0.860 \sqrt{\frac{1.5 \times 10^{-5} \text{ m}^2}{\text{m}}} (10 \%) \)

\[ = 1.22 \times 10^{-2} \% < U = 10 \% \]

\[ v \approx 0.12\% \text{ of } U \text{ outside the BL} \quad v \ll U \]
0. BL's with pressure gradient

1. Intro: Blowing flat plate, \( \rho = \text{constant} \)

\[ \omega_z = -\frac{2u}{dy} \] near the wall

\[ \text{slope = constant near wall} \]

Zero slope near wall

(\text{vorticity = zero near the wall})

No vorticity production at the wall

2. Eq. at the wall

x-momentum BL eq

\[ \rho \left( \frac{du}{dx} + v \frac{dv}{dy} \right) = -\frac{dp}{dx} + \mu \frac{d^2 u}{dy^2} \]

At the wall \((y=0)\): no slip \( U = V = 0 \)

\[ \frac{d^2 u}{dy^2} \text{ wall} = \frac{1}{\mu} \frac{dp}{dx} \]

At the wall \( (1) \)

Curvature of velocity profile

Pressure gradient

3. Examples

- Blowing \( \frac{dp}{dx} = 0 \) \( \Rightarrow \) \( \frac{d^2 u}{dy^2} \text{ wall} = 0 \)

\( H = 2.59 \) no vorticity production

- Accelerating flow

\( \frac{dp}{dx} < 0 \) \( \Rightarrow \) favorable pressure gradient

\[ \frac{d^2 u}{dy^2} \text{ wall} < 0 \]

\( \left[ \frac{d^2 u}{dy^2} < 0 \text{ everywhere in } BL \right] \)

\( H \) is small

\( H \approx 1.3 \)
Negative vorticity ($\omega_z$) is being produced at the wall

- Decelerating flow $\rightarrow \omega_y < 0$, $b^+$ 
  
  Advise or unfavorable pressure gradient

Eq (1) $- \left( \frac{\partial^2 u}{\partial y^2} \right)_{\text{wall}} > 0$

But $\frac{\partial^2 u}{\partial y^2}$ must be $< 0$ near the edge of the BL since $U \rightarrow U$

There must be an inflection pt. somewhere in the BL

\[ \frac{\partial^2 u}{\partial y^2} = 0 \]

If $H$ is "large" $\geq 3.5$

There is production of $\omega_z$ vorticity at the wall

If there is a very strong adverse pressure gradient

$\frac{\partial b}{\partial x} > 0$

\[ \left( \frac{\partial^2 u}{\partial y} \right)_{\text{wall}} > 0 \rightarrow \omega_z = \frac{\partial u}{\partial y} = 0, \quad \omega_z = \left( \frac{\partial^2 u}{\partial y^2} \right)_{\text{wall}} = 0 \]
As an even stronger adverse pressure gradient, BL has separated, we are in a separation bubble.

The BL. egs are not valid beyond the separation point.

Why not?
- $f$ gets by very rapidly — BL may no longer be thin.
- Reverse flow destroys the parabolic nature of the egs, (becomes elliptic)

\[
\left[ \frac{d^2u}{dx^2} \text{ term is no longer negligible} \right]
\]

- Irrotational outflow might change if there is flow separation.

BL egs are useful up to the separation point — can be used to predict the separation location, but are not valid beyond the separation point.