Blasius-secondary flow flat plate boundary layer similarity solution

J. M. Cimbala

The equations to solve are f''' + cff'' = 0, where prime denotes d/dη, and h'' + 0.5fh' - f'h + 1 - (f')^2 = 0.

Here, let c = 1/2, following Kundu's book. $c := 0.5$

The boundary conditions are f'(0)=1, f(0)=1, f'(∞)=1, h(0)=0, and h(∞)=0.

Since two of these are at infinity, f''(0) and h'(0) need to be guessed until the boundary conditions at infinity are satisfied.

First define a vector Y which contains five unknowns, Y_1 = f'', Y_2 = f', Y_3 = f, Y_4 = h', and Y_5 = h.

$\text{ORIGIN} := 1$

Known boundary conditions:

$Y_2 := 0 \quad Y_3 := 0 \quad Y_5 := 0$

Guessed boundary conditions:

$Y_1 := 0.332057 \quad Y_4 := 1.085973166$

Verify the vector:

$$Y = \begin{pmatrix} 0.33206 \\ 0 \\ 0 \\ 1.08597 \\ 0 \end{pmatrix}$$

Now define the derivative vector D which contains the first derivative with respect to η of each variable in the Y vector. This derivative vector D is needed for the Runge-Kutta solution.

Now calculate the solution as η marches from ηstart to ηend. Here Z is the solution matrix, where the first column is η, the second column is Y_1, the third column is Y_2, the fourth column is Y_3, the fifth column is Y_4, and the last column is Y_5.

$$D(\eta, Y) := \begin{bmatrix} -c \cdot Y_3 \cdot Y_1 \\ Y_1 \\ Y_2 \\ -1 + (Y_2)^2 + Y_2 \cdot Y_5 - \frac{1}{2} \cdot Y_3 \cdot Y_4 \\ Y_4 \end{bmatrix}$$

Here the function Rkadapt is used, which is similar to rkfixed except it internally uses adaptable spacing instead of fixed spacing (more accuracy where needed). It reports at fixed spacing however.

$\eta start := 0 \quad \eta end := 10 \quad num\_steps := 2000 \qquad Z := \text{Rkadapt}(Y, \eta start, \eta end, num\_steps, D)$

Top portion of Z:

Z =

	1	2	3	4	5	6
1	0	0.33206	0	0	1.08597	0
2	$5 \cdot 10^{-3}$	0.33206	$1.66028 \cdot 10^{-3}$	$4.15071 \cdot 10^{-6}$	1.08097	$5.41737 \cdot 10^{-3}$
3	0.01	0.33206	$3.32057 \cdot 10^{-3}$	$1.66028 \cdot 10^{-5}$	1.07597	0.01081
4	0.015	0.33206	$4.98085 \cdot 10^{-3}$	$3.73564 \cdot 10^{-5}$	1.07097	0.01618
5	0.02	0.33206	$6.64114 \cdot 10^{-3}$	$6.64114 \cdot 10^{-5}$	1.06597	0.02152
6	0.025	0.33206	$8.30142 \cdot 10^{-3}$	$1.03768 \cdot 10^{-4}$	1.06098	0.02684
7	0.03	0.33206	$9.96171 \cdot 10^{-3}$	$1.49426 \cdot 10^{-4}$	1.05598	0.03213
8	0.035	0.33206	0.01162	$2.03385 \cdot 10^{-4}$	1.05098	0.0374
9	0.04	0.33206	0.01328	$2.65646 \cdot 10^{-4}$	1.04598	0.04264
10	0.045	0.33206	0.01494	$3.36208 \cdot 10^{-4}$	1.04098	0.04786

Bottom portion of Z (to verify BCs):

Z =

	1	2	3	4	5	6
1991 [clipped]	9.95	$1.03781 \cdot 10^{-8}$	1	8.22921	$-2.12027 \cdot 10^{-9}$	$1.43271 \cdot 10^{-6}$
1992 [clipped]	9.955	$1.01667 \cdot 10^{-8}$	1	8.23421	$-1.69195 \cdot 10^{-9}$	$1.4327 \cdot 10^{-6}$
1993 [clipped]	9.96	$9.95948 \cdot 10^{-9}$	1	8.23921	$-1.27186 \cdot 10^{-9}$	$1.4327 \cdot 10^{-6}$
1994 [clipped]	9.965	$9.75637 \cdot 10^{-9}$	1	8.24421	[clipped] $8.59864 \cdot 10^{-10}$	$1.43269 \cdot 10^{-6}$
1995 [clipped]	9.97	$9.55729 \cdot 10^{-9}$	1	8.24921	$-4.558 \cdot 10^{-10}$	$1.43269 \cdot 10^{-6}$
1996 [clipped]	9.975	$9.36215 \cdot 10^{-9}$	1	8.25421	[clipped] $5.95158 \cdot 10^{-11}$	$1.43269 \cdot 10^{-6}$
1997 [clipped]	9.98	$9.17088 \cdot 10^{-9}$	1	8.25921	[clipped] $3.29137 \cdot 10^{-10}$	$1.43269 \cdot 10^{-6}$
1998 [clipped]	9.985	$8.9834 \cdot 10^{-9}$	1	8.26421	[clipped] $7.10305 \cdot 10^{-10}$	$1.43269 \cdot 10^{-6}$
1999 [clipped]	9.99	$8.79965 \cdot 10^{-9}$	1	8.26921	$1.08413 \cdot 10^{-9}$	$1.43269 \cdot 10^{-6}$
2000 [clipped]	9.995	$8.61955 \cdot 10^{-9}$	1	8.27421	$1.45076 \cdot 10^{-9}$	$1.4327 \cdot 10^{-6}$
2001 [clipped]	10	$8.44303 \cdot 10^{-9}$	1	8.27921	$1.81033 \cdot 10^{-9}$	$1.43271 \cdot 10^{-6}$

Now generate a plot of the similarity variables: $n := 1 .. num\_steps$