

Instability of a Far Wake

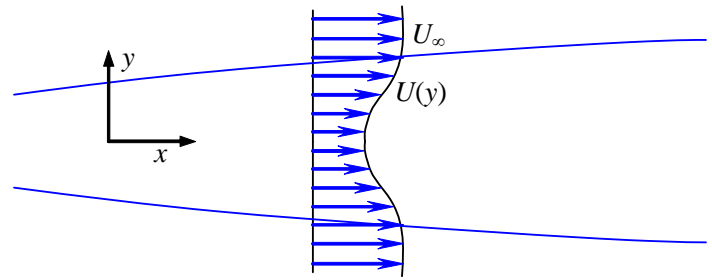
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1. Introduction

- We examine the stability of the two-dimensional, incompressible, nearly parallel, laminar far wake of a circular cylinder, as sketched.
- Since there is an inflection point (two of them actually), we expect that the basic state may be temporally unstable, even in the inviscid limit.
- For simplicity, we use the Rayleigh equation instead of the full Orr-Sommerfeld equation.

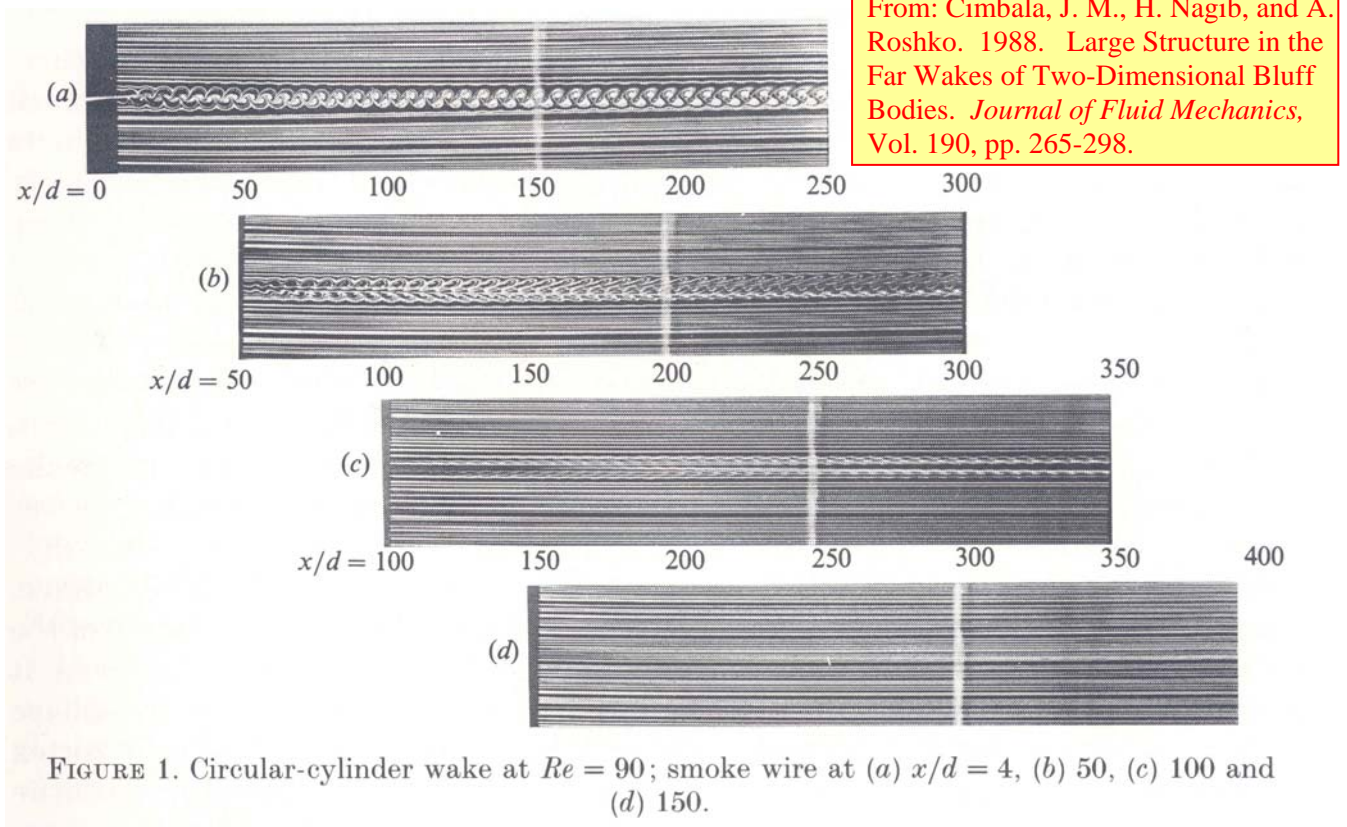
- The Reynolds number is defined as $Re = \frac{U_\infty d}{\nu}$,

where U_∞ is the free stream velocity and d is the cylinder diameter.



2. Experiments and Stability Analysis

- John M. Cimbala studied the stability of far wakes as his Ph.D. project at Caltech in the early 1980s.
- At Reynolds numbers between about 80 and 160, the wake is laminar and a Kármán vortex street forms immediately downstream of the cylinder.
- Inviscid stability analysis of the mean velocity profile downstream of the wake reveals that disturbances at the frequency of the Kármán vortex street are *stable*, and the Kármán vortex street is thus predicted to *decay* downstream.
- This is verified experimentally (Fig. 1).
- At Reynolds numbers lower than around 100, after the Kármán vortex street has decayed, no other large-scale vertical structures appear in the far wake (Fig. 1). Beyond about 150 diameters, the wake is steady and laminar.



- Only by introducing the streaklines far downstream is the decay of the Kármán vortex street fully appreciated. A smoke-wire was used to generate smoke streaklines in the experiments in a wind tunnel (Fig. 1).

- At Reynolds numbers between about 100 and 160, however, a secondary Kármán vortex street appears far downstream at a lower frequency (larger wavelength), as in Figures 2 and 3, after the original Kármán vortex street has disappeared.

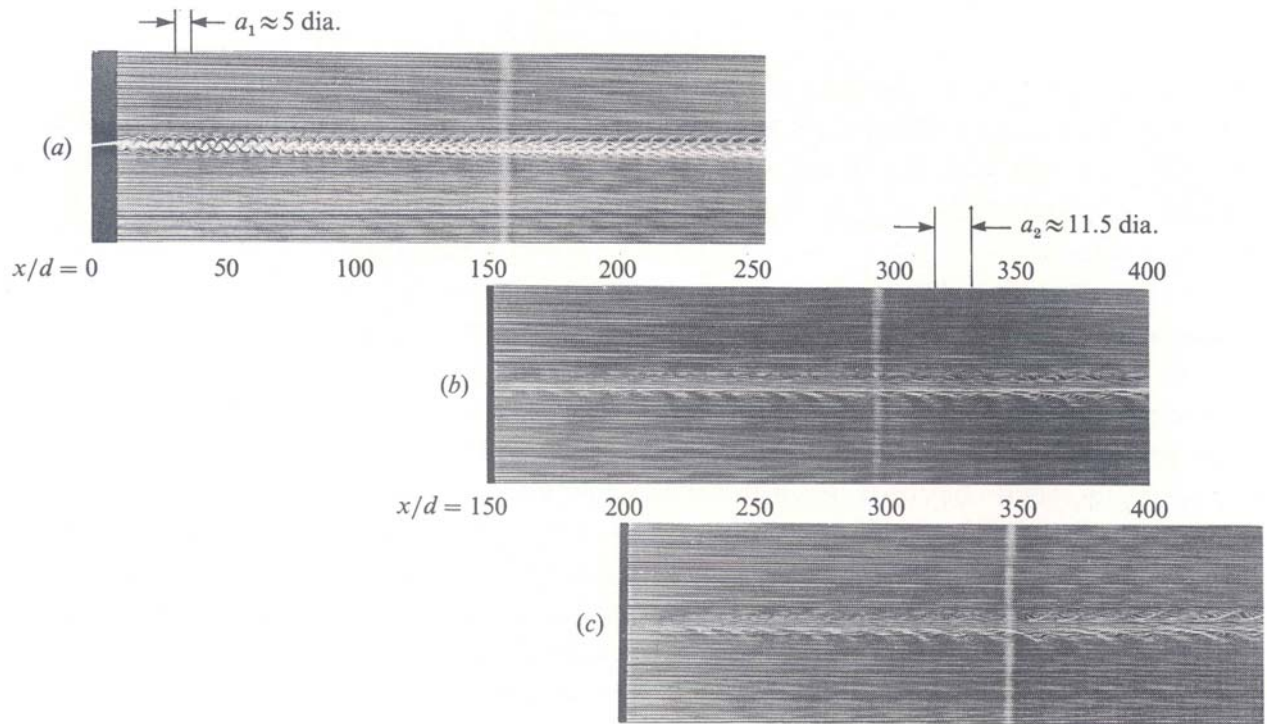


FIGURE 2. Circular-cylinder wake at $Re = 130$; smoke wire at (a) $x/d = 8$, (b) 150 and (c) 200.

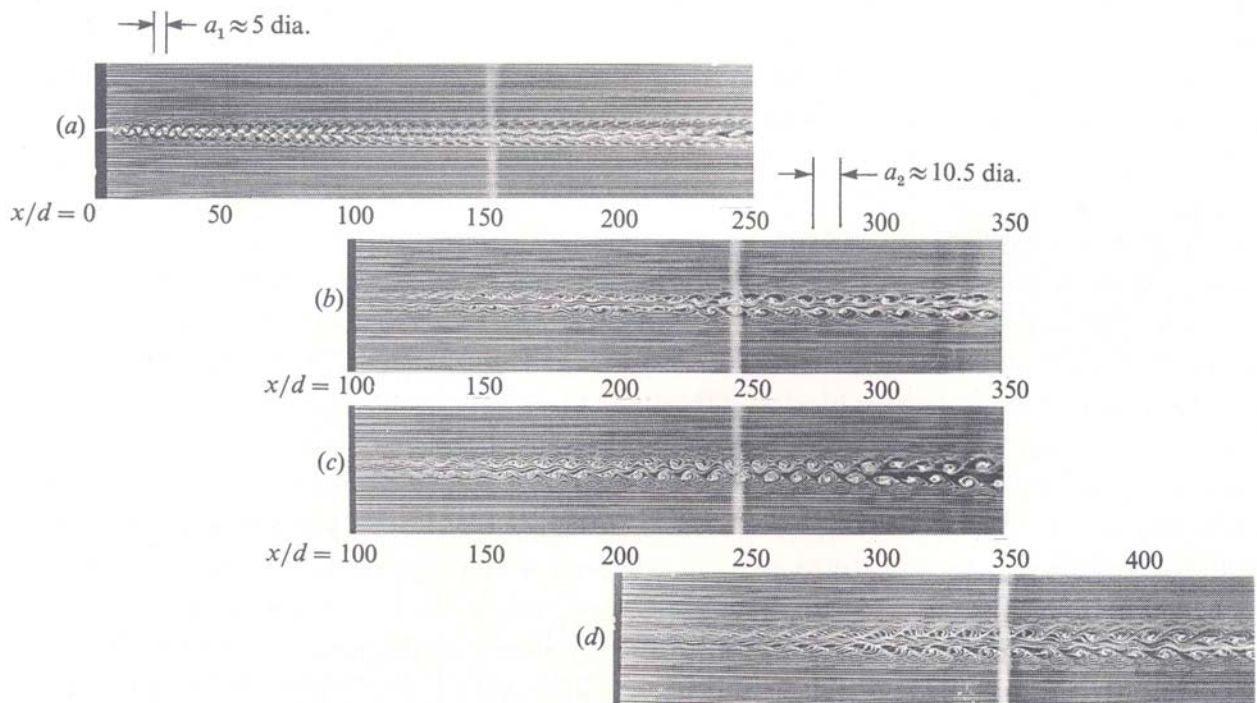


FIGURE 3. Circular-cylinder wake at $Re = 155$; smoke wire at (a) $x/d = 8$, (b) 100, (c) 100 and (d) 200.

- Can we predict the rise of this secondary vortex street? Yes, because inviscid stability analysis reveals that in the far wake at around $x/d = 100$, disturbances at lower frequency (larger wavelength) are unstable.
- Because of history effects and the fact that the actual wake is not parallel but grows downstream, the frequency of the Kármán vortex street (and subsequent Kármán vortex streets further downstream) tends to change in a step-like fashion (Fig 13 of the JFM article).

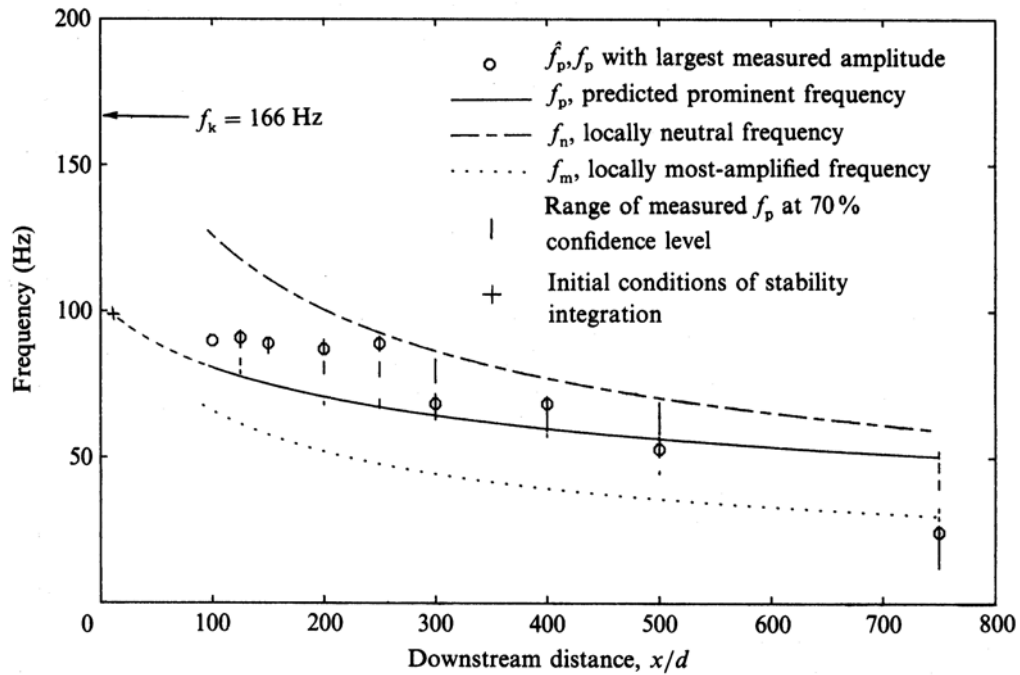


FIGURE 13. Prominent frequency versus downstream distance; circular-cylinder wake at $Re = 150$.

- At Reynolds numbers higher than about 160, the wake becomes turbulent. A Kármán vortex street is still formed downstream of the cylinder, but it quickly disappears. However, a new Kármán vortex street at lower frequency (larger wavelength) appears far downstream (Fig. 4), much as in the laminar wake case.
- The wake instability leading to a secondary Kármán vortex street exists even for turbulent flow. Stability analysis of the mean wake profile predicts the frequencies that are unstable, as also observed experimentally.
- In the wake of a porous flat plate with low solidity, there is no Kármán vortex street immediately behind the plate. However, farther downstream, a Kármán vortex street develops due to the instability of the wake, and this is also predicted by stability analysis (Fig. 5).

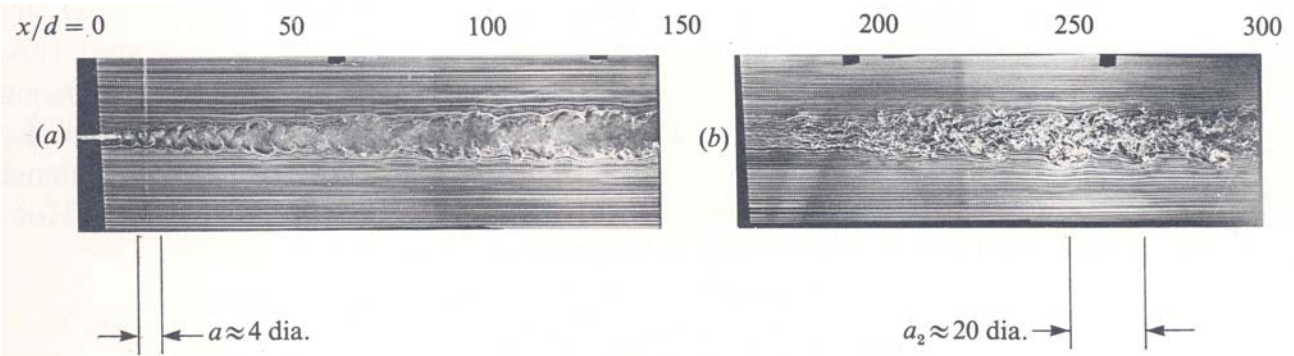


FIGURE 4. Circular-cylinder wake at $Re = 2200$; smoke wire at (a) $x/d = 1$ and (b) 160.

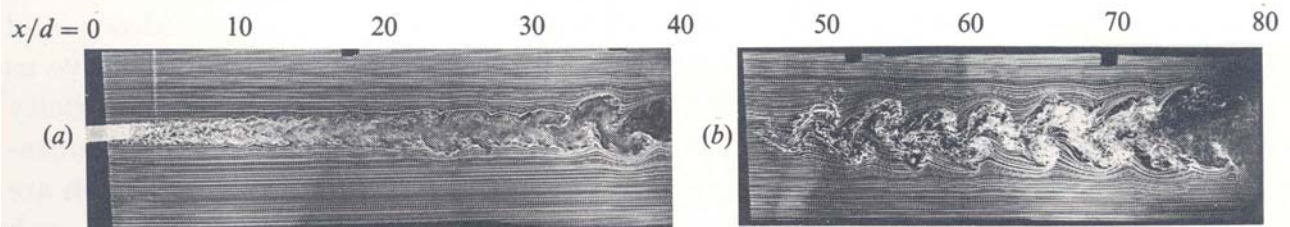


FIGURE 5. Porous-flat-plate wake at $Re = 6000$, $\sigma = 47\%$ smoke wire at (a) $x/d = 0$ and (b) 44.