M E 522 – Foundations of Fluid Mechanics II

Today, we will:

- Introduce the course and instructor: **John M. Cimbala, 863-2739, jmc6@psu.edu**
- Briefly go over the course website at **www.mne.psu.edu/me522**
- Begin where we left off in M E 521: **2-D laminar free shear layers**

... Continued from M E 521

VII Laminar Boundary Layers

A. Intro

B. Diff. eqs of motion

C. Blasius flat plate BL

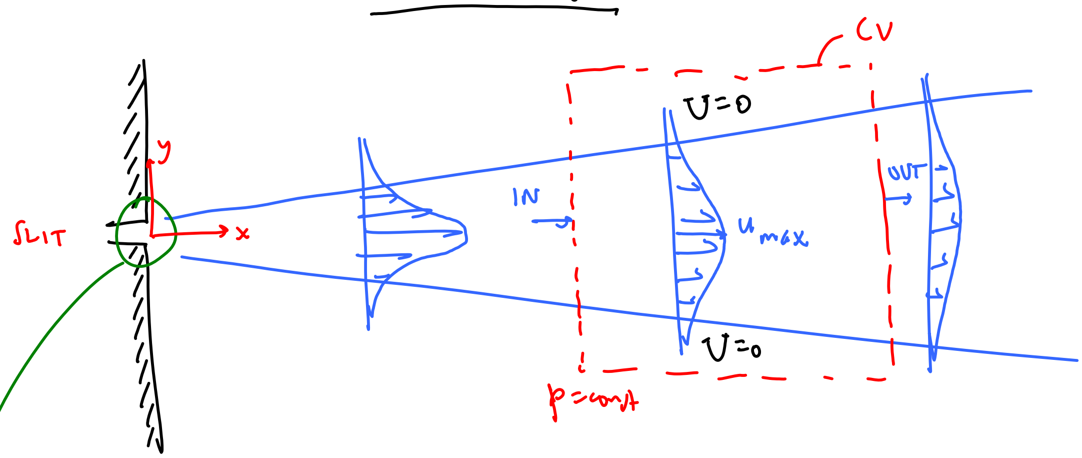
D. Pressure gradients in BLs

E. Falkner-Skan wedge flows

F. 2-D laminar free shear layers

1. Intro

2. 2-D Laminar jet (continued)



Assumptions / Approx.:
- steady
- incomp.
- 2-D
- large Re ⇒ jet is thin
- p ≈ const
- neglect gravity

We are not concerned with details @ the origin → We are interested in the far jet → We expect a similarity solution

a. C.V. (Integral analysis) →
$$\boxed{M = \text{momentum flux per unit span (into the page)} = \rho \int_{-\infty}^{\infty} u^2\,dy = \text{constant (independent of } x \text{ location)}}$$

b. Differential Analysis $\rightarrow$ We use the BL eqs

Here, $U\frac{dU}{dx} = 0$ since $U = 0$ & $\frac{dU}{dx} = 0$ (no outer flow)

Cont: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

x-mom: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$

Identical to Blasius eq's

Here $\rightarrow$ BC's will differ.

y-mom $\rightarrow$ $\frac{\partial p}{\partial y} \simeq 0 \rightarrow p = \text{constant everywhere}$

BCs: Need 4

v @ some y $\rightarrow$ $\boxed{@\ y=0,\ v=0}$ at all x (symmetry along x-axis)

u @ some y $\rightarrow$ $\boxed{@\ y=0,\ \frac{\partial u}{\partial y}=0}$ at all x (")

u is a local maximum

$\rightarrow$ @ $y=\infty$, $u = U(x) = 0$ $\boxed{@\ y\rightarrow\infty,\ u\rightarrow 0}$ at all x

u @ some x :

Could use the initial profile

OR since this is a similarity soln,

we use instead

$$\boxed{M = \rho\int_{-\infty}^{\infty} u^2\,dy = \text{constant @ any } x} \quad (1)$$

c. Similarity Solution

Schlichting (1933) $\rightarrow$ $\boxed{\psi = \text{stream fnc.} = a x^m f(\eta)}$

where $\boxed{\eta = \frac{y}{b x^n}}$ can call this δ_c characteristic BL thickness

· Continuity $\rightarrow$ identically satisfied by defn of ψ

<u>x-mom:</u> $u = \frac{\partial\psi}{\partial y} = a x^m f'(\eta) \frac{\partial \eta}{\partial y}$

$u = a x^m f' \frac{1}{b} x^{-n}$

$\frac{\partial \eta}{\partial y} = \frac{1}{b x^n} = \frac{x^{-n}}{b}$

$\frac{\partial \eta}{\partial x} = \frac{y}{b}(-n) x^{-n-1}$

$v = -\frac{\partial\psi}{\partial x} = -a m x^{m-1} f - a x^m f'(\eta) \frac{\partial \eta}{\partial x}$

$\boxed{u = \frac{a}{b} x^{m-n} f'}$ $\boxed{v = \frac{a n y}{b} x^{m-n-1} f' - a m x^{m-1} f}$

Plug these into x-mom eq. } algebra.

$$\boxed{\frac{ab}{\nu} x^{m+n-1} \left[(m-n)(f')^2 - m f f'' \right] = f'''} \qquad (2)$$

Have we achieved similarity?

We force x to drop out of the eq. $\therefore$ $\boxed{m+n-1=0}$ (3)

Also, $M = \rho \int_{-\infty}^{\infty} u^2 \, dy = \text{const}$ (indep. of x)

$dy = \underbrace{b x^n \, d\eta}$

use $u = \frac{a}{b} x^{m-n} f'$

$\therefore$ $M = \rho \int_{-\infty}^{\infty} \frac{a^2}{b^2} x^{2m-2n} (f')^2 \, b x^n d\eta$

or $M = \rho \frac{a^2}{b} x^{2m-n} \underbrace{\int_{-\infty}^{\infty} (f')^2 \, d\eta}_{\text{not a fnc. of } x} = \text{const.}$

$\boxed{\text{Must set } 2m-n=0}$

(4)

Solve (3) ; (4) simultaneously $\rightarrow$ $\boxed{m = \frac{1}{3} \quad n = \frac{2}{3}}$

★ These are the only values of m & n that lead to a similarity soln.

Result: $\eta = \frac{y}{\delta_c} = \frac{y}{bx^n} \rightarrow \delta_c \sim x^n$ or $\boxed{\delta_c \sim x^{2/3}}$

$u = \frac{a}{b} x^{m-n} f' \rightarrow u_{max}$ decays like $x^{m-n} \sim x^{-1/3}$

$\boxed{\text{jet velocity decays like } x^{-1/3}}$

Choose constants a & b $\rightarrow$ they are dimensional $\rightarrow$ also, $\frac{ab}{\nu}$ is nondimensional

We "pick" $\boxed{a = \left(\frac{9\nu M}{2\rho}\right)^{1/3}}$ $\boxed{b = \left(\frac{48\nu^2\rho}{M}\right)^{1/3}}$

plug into Eq. (2)

$$\boxed{f''' + 2f'^2 + 2ff'' = 0} \qquad (5)$$

Final similarity eq for $f(\eta)$

With BC's:

@ $y=0$, $\frac{du}{dy}=0 \rightarrow f''(0)=0$

@ $y=\infty$ $u=0 \rightarrow f'(\infty)=0$

@ $y=0$ $v=0 \rightarrow f(0)=0$

We used the 4th BC already

Similarity has been achieved $\rightarrow$ SOLN $\rightarrow$ $\boxed{f(\eta) = \tanh(\eta)}$

Finally,

$$\boxed{u = \underbrace{\left[\frac{3}{32}\frac{M^2}{\rho^2\nu}\right]^{1/3} x^{-1/3}}_{= u_{max} \text{ @ CL}} \operatorname{sech}^2\eta}$$ ★