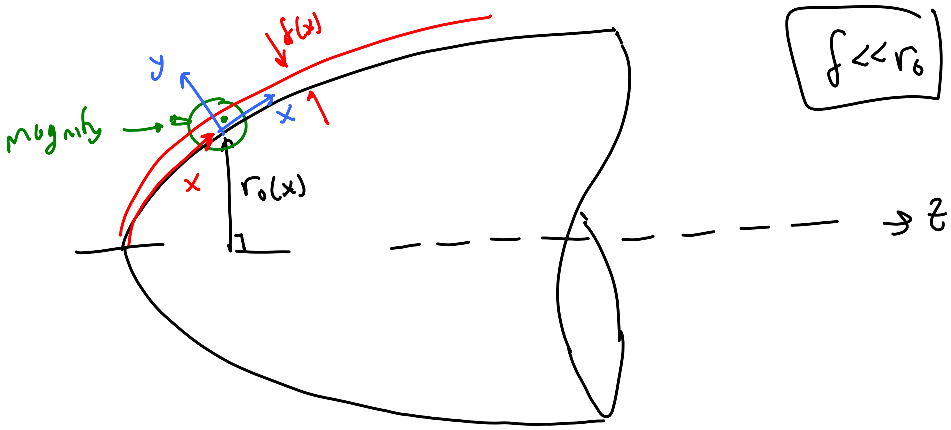
Today, we will:

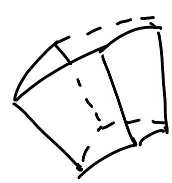
- Discuss the axisymmetric boundary layer equations (Mangler)
- Discuss the Mangler transformation
- Do an example problem – axisymmetric stagnation point flow
- Do Candy Questions for Candy Friday

2. Axisymmetric BL equations (over a body)



$\delta \ll r_0$

Fluid element in the BL



Get continuity eq:

$$\boxed{\frac{\partial}{\partial x}(r_0 u) + r_0 \frac{\partial v}{\partial y} = 0} \quad (1)$$

Axisymm. BL. conti. eq.

Mangler (1945)

Axisymm. B.L. eqs of Mangler: [steady, incompressible, thin BL over a solid body]

y-mom: $\boxed{\frac{\partial p}{\partial y} \approx 0}$ [same as 2-D]

OR, this term = $U \frac{dU}{dx}$ (just as in 2-D)

x-mom: $\boxed{u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2}}$ [same as 2-D]

- BCs → same as 2-D case @ $y=0$, $u=v=0$ (no slip)
  @ $y \to$ "∞" $u \to U(x)$

  also need a starting profile
  @ some x — typically @ stag. pt. ($x=0$)

  ∴ march in x

---

3. The Mangler Transformation

a. Intro → Mangler (1945) → You can transform any steady, incomp, axisymm. BL problem (over a body) into an equivalent 2-D problem

b. Derivation

Let $\boxed{x' = \frac{1}{L^2}\int_0^x r_0^2\, dx}$ where L = some characteristic length scale in the problem.

Let $\boxed{y' = \frac{r_0 y}{L}}$

It turns out that $\boxed{u' = u}$ ∴ $\boxed{U'(x') = U(x)}$

Plug these into the BL eqs for axisymm. BL.

$\big\}$ algebra

Get $\boxed{\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0}$ cont.

& $\boxed{u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = U'\frac{dU'}{dx'} + \nu\frac{\partial^2 u'}{\partial y'^2}}$ x-mom

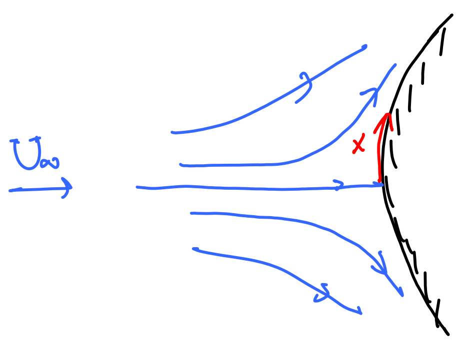
Summary – transform from axisymm. B.L. problem

(x, y, U, r_0, u, v)

Into an equivalent 2-D problem

(x', y', U', u', v')

C. Example: Given: Axisymm. stagnation pt. flow



To do: Generate the nose shape of the equivalent 2-D body using Mangler transf.

Sol'n: Need $U(x)$ for the axisymm. case

It turns out that $\boxed{U(x) = Cx}$

C depends on the body shape & the freestream velocity U_∞

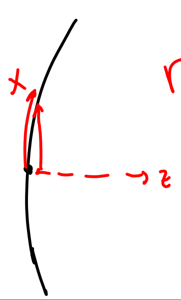
$$x' = \frac{1}{L^2}\int_0^x r_0^2\,dx \qquad y' = \frac{r_0 y}{L}$$

L is some characteristic length of the body

e.g. – sphere, perhaps R or D

Let's just call it L

Calc. x' & y' in terms of $x, y,$ & L



$r_0 = x$ close to the stag. pt.

$$x' = \frac{1}{L^2}\int_0^x x^2\,dx \rightarrow \boxed{x' = \frac{x^3}{3L^2} \quad y' = xy/L} \rightarrow x = (3L^2 x')^{1/3}$$

$$y' = r_0 y/L = xy/L$$

We have $U'(x') = U(x) = Cx = \underbrace{C(3L^2}_{\text{constants}} x')^{1/3}$

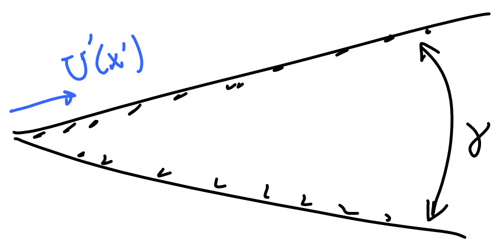
$$\therefore \boxed{U'(x') = K(x')^{1/3}}$$

☆ Irrotational outer flow of the equivalent 2-D nose shape

Recall (521) → power law function & conformal mapping

recall wedge flow

2-D

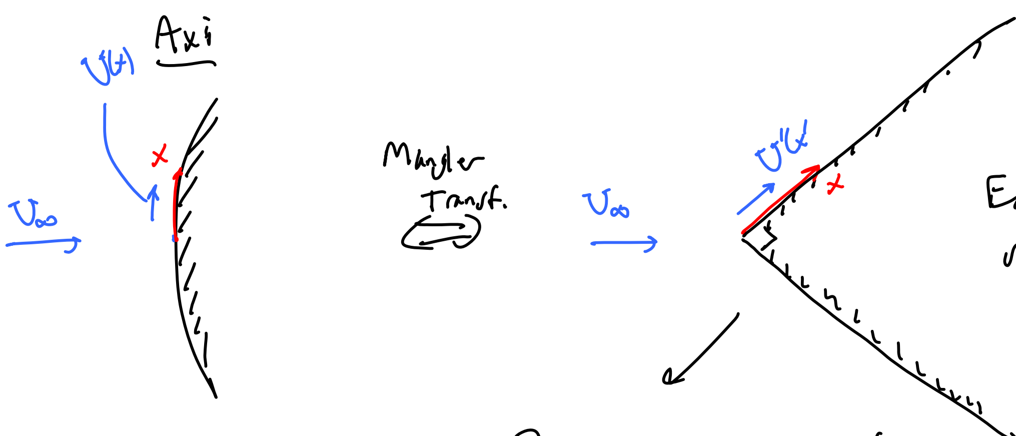


recall, for 2-D wedge flow $U'(x') = \text{const}\,(x')^{\frac{\gamma}{2\pi-\gamma}}$

Here we know that $U'(x') = \text{const}\,(x')^{1/3}$

Equate exponents:

& solve for γ → $\boxed{\gamma = \frac{\pi}{2}}$ $(90°)$



Equiv. 2-D nose shape!

This is a special case of Falkner-Skan wedge flow

Recall, we had $\boxed{U'(x') = \text{const}\,(x')^m}$

Here, $\boxed{\begin{array}{l} m = 1/3 \\ \beta = \dfrac{2m}{1+m} = \dfrac{1}{2} \end{array}}$

So.. add to your ME 521 notes another special case

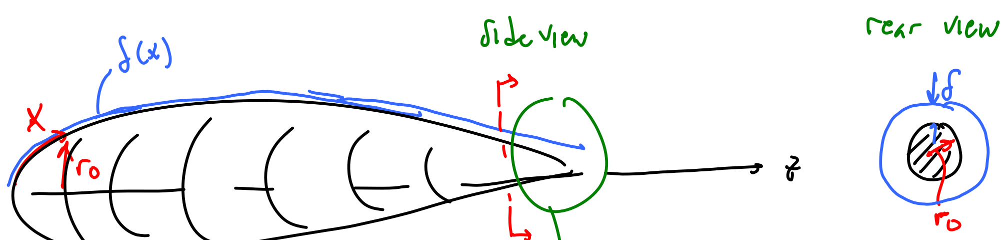
$\beta = \frac{1}{2}$ → corresponds to axisymm. stag. pt. flow !

4. General Axisymmetric BL's

a. Intro → Mangler transf. is useful only for solid bodies

→ What about free shear layers, jets, wakes

→ Also, Mangler transf. breaks down when δ gets big.



Mangler transf. assumes that $\delta \ll x$ and $\delta \ll r_0$

transverse curvature

near the tail of the body, $\delta \not\ll r_0$

$r_0 \to 0$ ∴ δ is "big"

Mangler transf. is not valid near the tail of the body