Today, we will:

- Continue our discussion of three-dimensional boundary layer coordinate systems
- Discuss secondary flow in three-D boundary layers
- Do an example problem – secondary flow on a flat plate

e.g.: a cone → can solve as a developable surface

b. Intrinsic coordinates – a "trick"

- Solve for $\vec{U}$ in any coord. system
- Pick the B.L. coordinate system such that
  - x is parallel to outer streamlines
  - z is normal " " "
- transpose $\vec{U}$ into x & z BL coordinates

Advantage → BCs are easy $U = |\vec{U}|$, $W = 0$

Disadvantage → h_x, h_z will not necessarily $= 1$

BL eqs more difficult to solve

4. SUMMARY – Why 3-D BLs are harder than 2-D BLs

a. Outer flow (harder to calculate)

→ two outer flow components $U(x,z)$, $W(x,z)$ instead of $U(x)$

b. Coordinate system

- not readily apparent – there are options

c. BL eqs. themselves are more complicated

- 2 mom. eqs. (x & z) instead of one (x)
- more terms & more complicated terms if $h_x, h_z \neq 1$
- h_x & h_z are embedded in the eqs.
  & they <u>change</u> with location on the body!

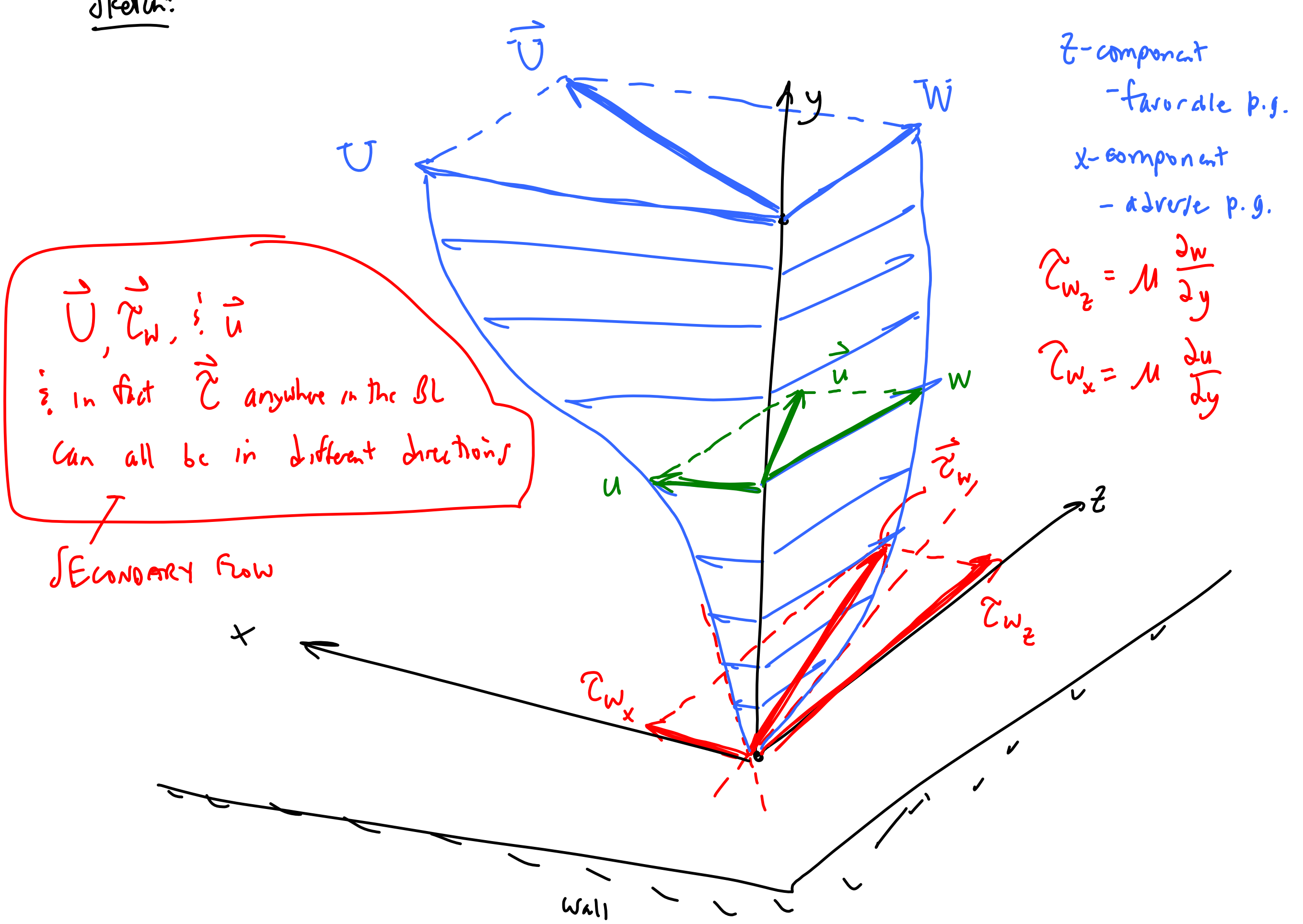
d. <u>Secondary flow</u> can occur

☆ The velocity within the BL does not have to be in the <u>same</u> <u>direction</u> as the outer flow

5. <u>Secondary Flow</u> →

a. <u>Defn</u>: → $\boxed{\text{When the flow inside the BL is in a different direction than the outer flow}}$

<u>Sketch</u>:

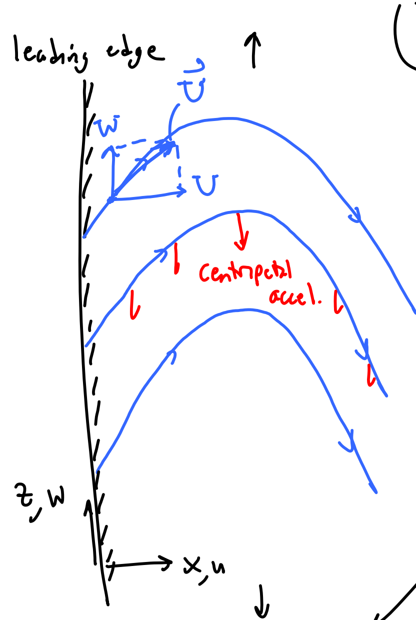


b. EXAMPLE PROBLEM

Given: · Semi-infinite flat plate with a constant transverse pressure gradient (in z-direction) $\left(\frac{\partial p}{\partial z} = \text{constant}\right)$

· zero pressure gradient in x-direction $\left(\frac{\partial p}{\partial x} = 0\right)$

Top view:



· Nothing special about any z-location

Outer Flow Velocity:

$$\boxed{\begin{aligned} U &= \text{constant} \\ W &= U(a-bx) \end{aligned}}$$

(a & b are constants)

generates parabolic streamlines as shown

This is a 2-D geometry, but it is a 3-D problem; will have secondary flow.

· Pick a BL coordinate system → choose x & z on the plate

$\therefore$ $\boxed{h_x = h_y = h_z = 1}$ → BL eqs are easy

· BL eqs:

We expect that $\frac{\partial}{\partial z}(\text{any velocity}) = 0$

But $\frac{\partial p}{\partial z} \neq 0$

Cont: $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \cancelto{0}{\frac{\partial w}{\partial z}} = 0$

x-mom: $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\cancelto{0}{\frac{\partial u}{\partial z}} = -\frac{1}{\rho}\cancelto{0}{\frac{\partial p}{\partial x}} + \nu\frac{\partial^2 u}{\partial y^2}$

z-mom: $u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\cancelto{0}{\frac{\partial w}{\partial z}} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\frac{\partial^2 w}{\partial y^2}$

y-mom: $\frac{\partial p}{\partial y} \simeq 0$ through the BL

Euler's Eq. for the outer flow:

x-mom: $U\cancel{\dfrac{\partial U}{\partial x}} + W\cancel{\dfrac{\partial U}{\partial z}} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$

$\underbrace{\qquad}_{U \approx \text{const}}$ $\qquad \dfrac{\partial p}{\partial x} = 0$

z-mom: $U\dfrac{\partial W}{\partial x} + W\cancel{\dfrac{\partial W}{\partial z}} = -\dfrac{1}{\rho}\dfrac{\partial p}{\partial z}$

Plug in $W = U(a - bx)$

$$\frac{\partial W}{\partial x} = -Ub$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = -U^2 b = \text{const}$$

BL eqs & BCs:

$$\boxed{\begin{array}{lll} \text{cont} & \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 & (1) \\ \text{x-mom:} & u\dfrac{\partial u}{\partial x} + v\dfrac{\partial u}{\partial y} = \nu\dfrac{\partial^2 u}{\partial y^2} & (2) \\ \text{z-mom:} & u\dfrac{\partial w}{\partial x} + v\dfrac{\partial w}{\partial y} = -U^2 b + \nu\dfrac{\partial^2 w}{\partial y^2} & (3) \end{array}}$$

BCs

$$\boxed{\begin{array}{ll} \text{no slip} \rightarrow & u = v = w = 0 \quad @\ y = 0 \\ \text{edge} \rightarrow & u \rightarrow U \quad @\ y \rightarrow \infty \\ & w \rightarrow W = U(a - bx) \quad @\ y \rightarrow \infty \end{array}}$$

W is uncoupled from Eqs (1) & (2) → We can solve (1) & (2) first & then solve (3) later

get u & v, then solve for w later

(1) & (2) + the BCs for u, v are identical to Blasius BL problem!

u & v components are already known!

(similarity soln)