Today, we will:

- Begin a new topic - Instability
- Make the "ball on the floor analogy
- Discuss several examples of instability in fluid mechanics
- If time, begin to discuss linear stability theory

VIII INSTABILITY (Ch. 12 in Kundu's book)

A. Introduction

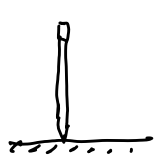
- So far -- laminar flow, mostly steady
- Under some circumstances, laminar, steady sol'ns are predicted, but such flows are not observed in real life
- Why not? — Mathematically, the sol'n satisfies the eq's & BCs but it may be unstable

For a flow to exist in real life, it must be stable

PROCEDURE:
- Start with a basic state (a predicted laminar flow)
- Add a small disturbance to the basic state
- Watch if the disturbance grows or decays
  - grows → basic state is unstable → Will not be observed in real life
  - decays → basic state is stable for this disturbance

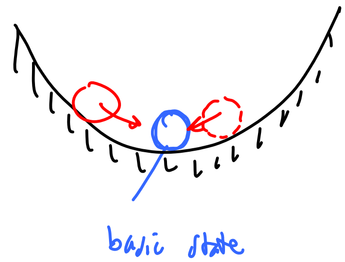
if it is unconditionally unstable → is unstable for any disturbance

e.g. pencil on its tip



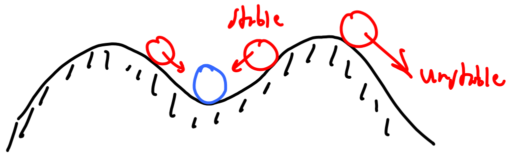
1. Examples of Instabilities

a. Ball on the floor

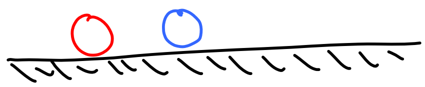


Unconditionally stable

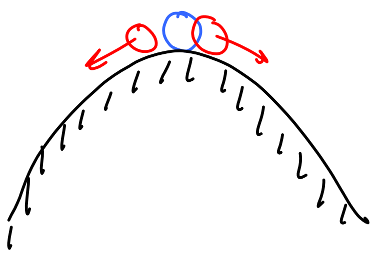
(ball always returns to its basic state)



Conditionally stable or **nonlinearly stable**



Neutrally stable



Unstable (unconditionally unstable)

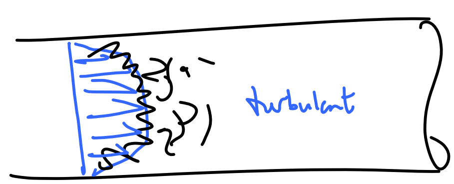
---

b. Unstable Flows

1) Pipe flow

$Re \lesssim 2300$

we observe laminar pipe flow

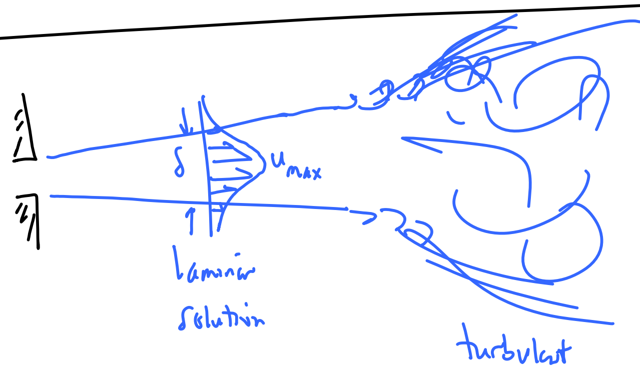


$Re \gtrsim 2300$

↑ Re_{crit}

---

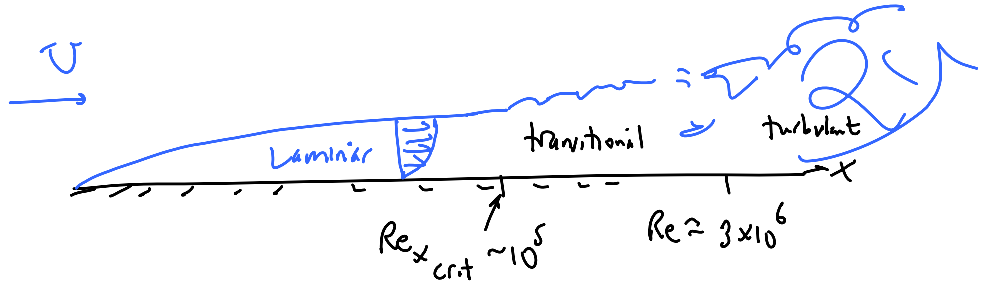
2) jets



$$Re = \frac{u_{max}\,\delta}{\nu}$$

$$Re_{crit} \sim 10$$

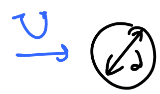
3) Blasius BL



Not all instabilities lead to turbulence → sometimes a different steady basic state (still laminar)

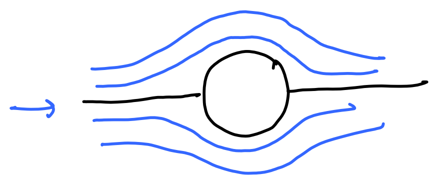
or a periodic state

4) circular cylinder $Re = \frac{Ud}{\nu}$



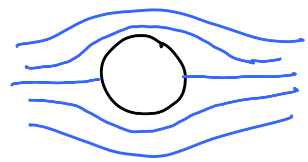
2-D cyl.

$Re \lesssim 1$ (creeping flow)

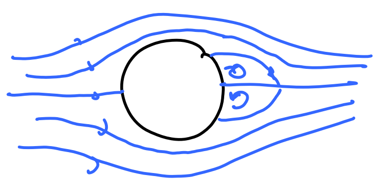


steady, laminar flow

$1 \lesssim Re \lesssim 3$ (unsymmetric flow)

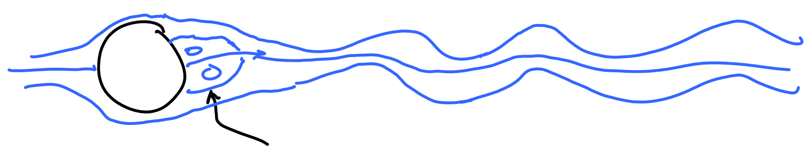


$3 \lesssim Re \lesssim 30\text{-}40$ (separation bubble)



$30\text{-}40 \lesssim Re \lesssim 80\text{-}90$ (laminar wake instability)

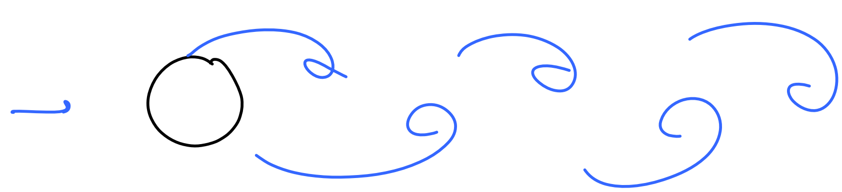
laminar but not steady (periodic)



$80\text{-}90 \lesssim Re \lesssim 160$ lose the closed separation bubble

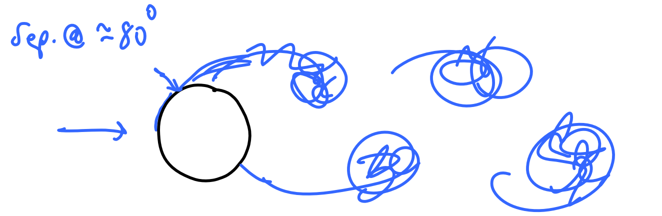
get Karman vortex shedding

vortex "street"



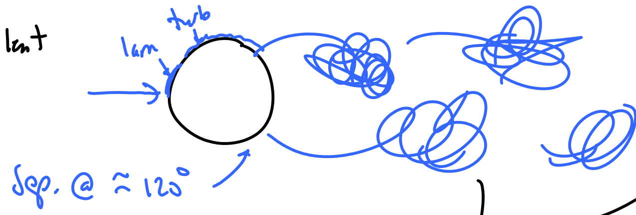
$160 \lesssim Re \lesssim 2\times10^5$

Laminar BL on the cylinder
Laminar BL separation
But turbulent wake



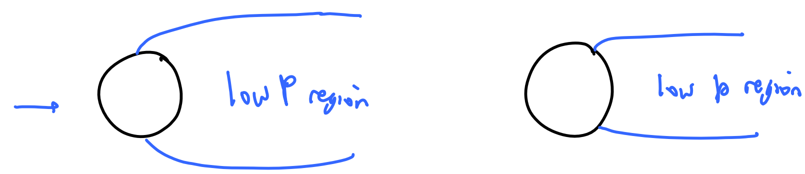
$Re \gtrsim 2.5\times10^5$

BL on cylinder becomes turbulent before it separates



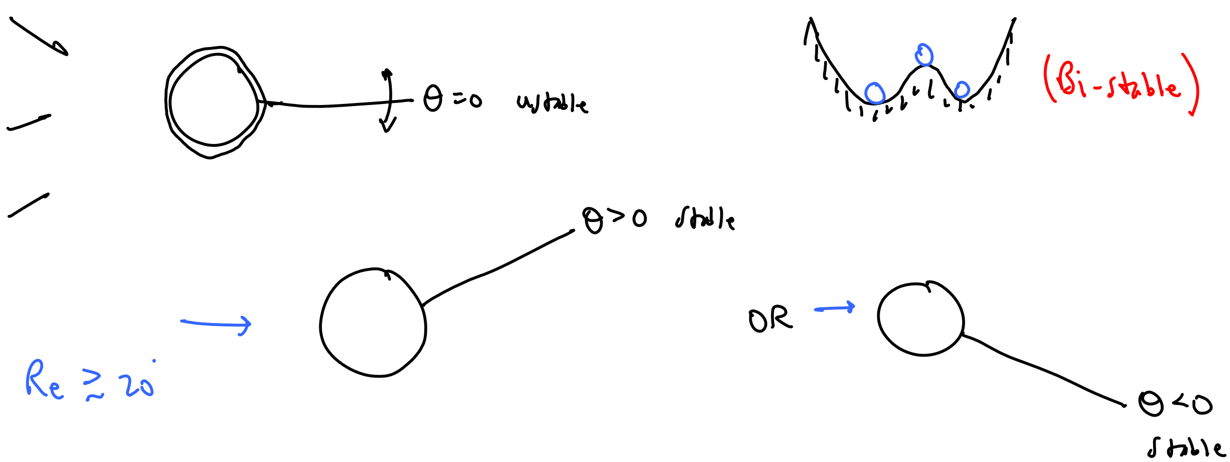
Drag drops suddenly
"Drag crisis"

time-averaged views:



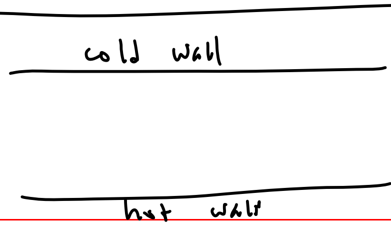
---

5) Freely rotatable cylinder with splitter plate



---

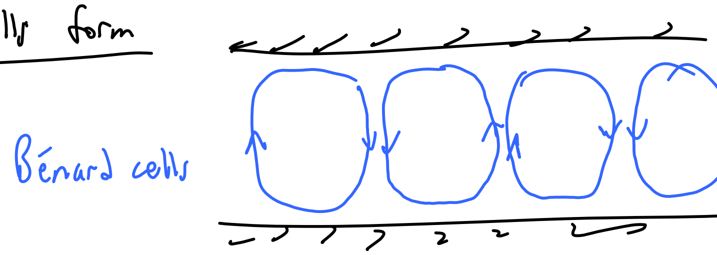
6) Buoyancy-driven thermal instability (Bénard convection)



$$\text{Rayleigh \#} = \frac{\text{Buoyancy force}}{\text{viscous force}} = Ra$$

$$Ra_{crit} \simeq 1700$$

For $R_a \gtrsim 1700$, convection cells form

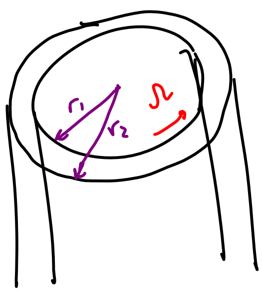


7) Also occurs with a free surface → Surface tension-driven Bénard convection

See pictures on website

---

8) Taylor-Couette Flow



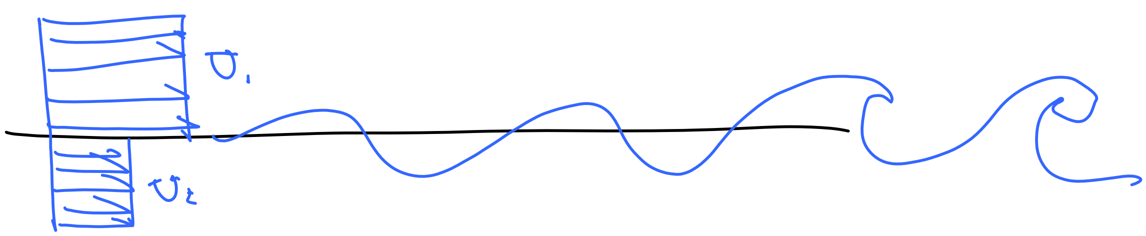
- Inner cylinder rotates
- Outer cylinder is stationary

} Goes unstable if Ω is large enough

See website for pictures

---

9) Kelvin-Helmholtz Instability (Free shear layer instability)



See website for pictures