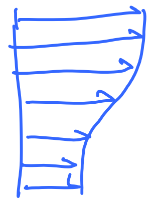
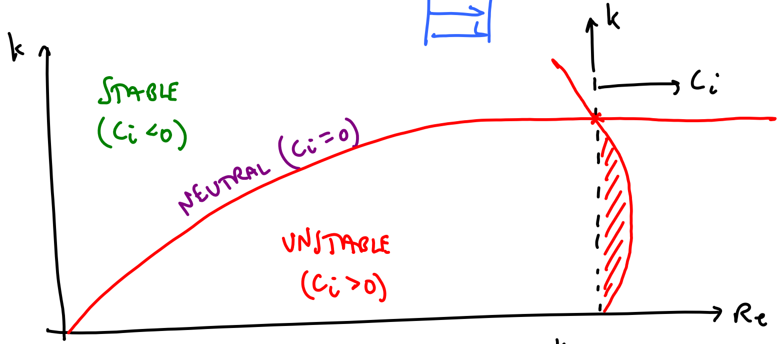
Today, we will:

- Discuss more examples of solutions to the O-S equation
- Discuss the Rayleigh eq. (inviscid O-S equation) in more detail, and look at some solutions

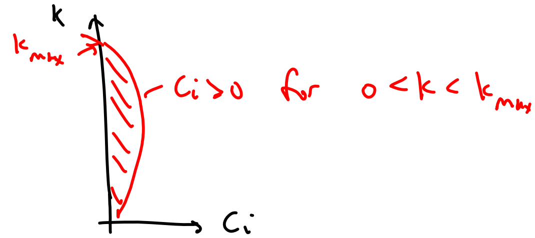
Last time: Examples of "thumb curves" (O-S solutions) (TEMPORAL ANALYSIS)

a) Smooth mixing layer





Rayleigh eq → get



(good agreement)

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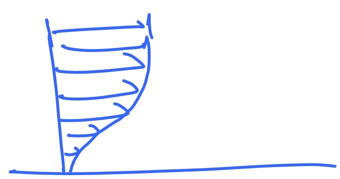
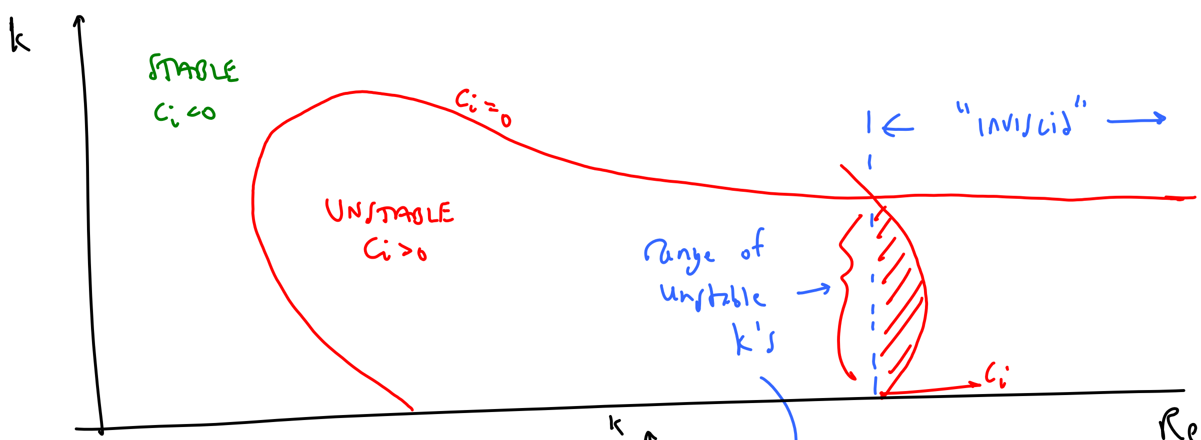
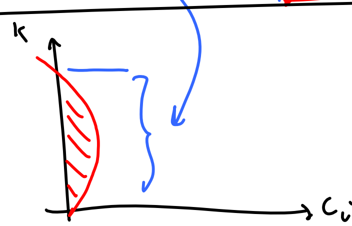
b) BL with an adverse pressure gradient
[or, similar behavior for wakes & jets]

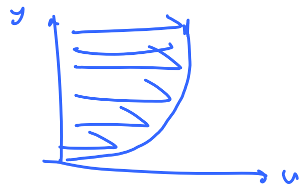


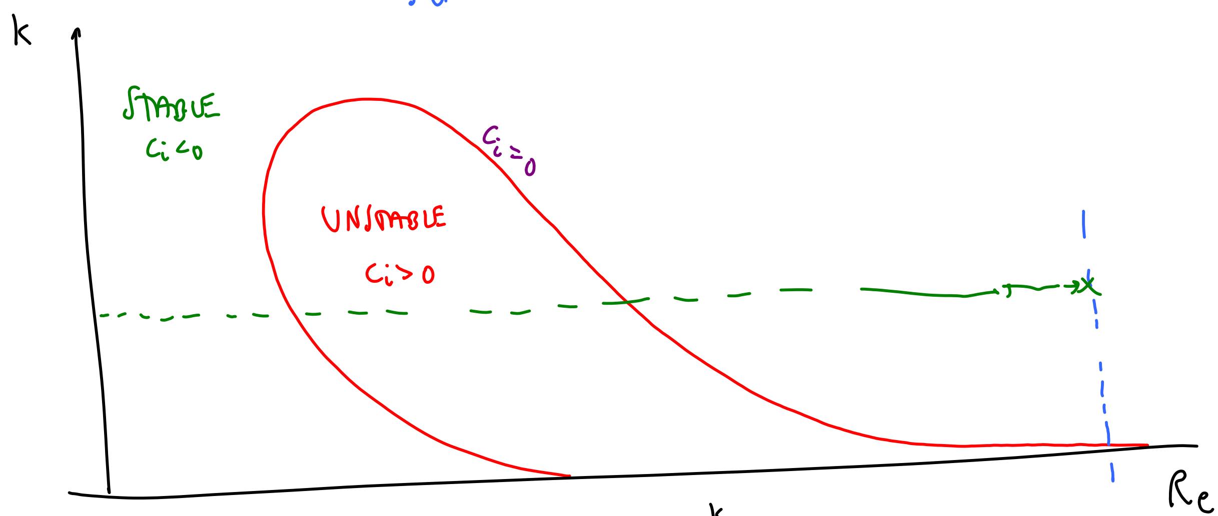


Rayleigh eq. solution (inviscid)

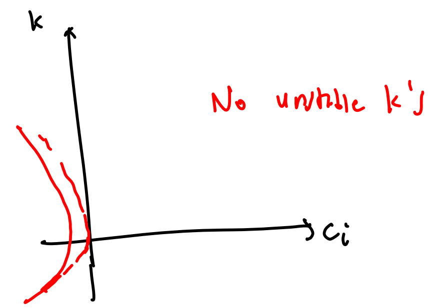


c) BL with favorable pressure gradient (e.g. front part of a body)





Rayleigh eq. (inviscid O-S eq) →



As $Re \to \infty$ we predict that the basic state is stable for any disturbance!

WHY DOES O-S SOLUTION PREDICT THAT HIGHER VISCOSITY FLUID (LOWER RE CASE) IS MORE UNSTABLE? [See mathematical argument in Kundu's book]

- We approximated flow as parallel — it is not in reality
- " " — temporal instability only — there may be spatial instabilities
- " " — linear stability → nonlinear effects may be important
- " " — 2-D → there may be 3-D effects
- To get to high Re, typically must pass through lower Re (where it is unstable) ☆

5. The Rayleigh Eq. (inviscid form of O-S eq.)

a. Derivation

- Consider 2-D parallel flow $U = U(y)$
- O-S eq. $(U-c)(\phi_{yy} - k^2\phi) - U_{yy}\phi = \frac{1}{ik\,Re}\left[\phi_{yyyy} - 2k^2\phi_{yy} + k^4\phi\right]$
- As $Re \to \infty$, RHS ≪ LHS → let's ignore the RHS

rewrite as

$$\boxed{\phi_{yy} - \left(\frac{U_{yy}}{U-c} + k^2\right)\phi = 0} \qquad (1) \quad \text{Rayleigh Eq.}$$

Note: we are assuming that the disturbances behave inviscidly, but the basic state itself can be a viscous flow

e.g.

Need 2 BCs since 2^{nd} order

We can no longer enforce the no-slip condition on the disturbances

BC's → $\hat{v}(y_1) = \hat{v}(y_2) = 0$ for flow between channels

$$\downarrow \quad \boxed{\phi(y_1) = \phi(y_2) = 0}$$

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b. Conditions for Instability (of the Rayleigh Eq.)

(1) Rayleigh's inflection pt. criterion

"manipulate" on Eq. (1) →

$$\boxed{c_i \int_{y_1}^{y_2} \frac{U_{yy}|\phi|^2}{|U-c|^2}\,dy = 0} \qquad (2)$$

(2) can be satisfied two ways:

(a) if $c_i = 0$ → neutrally stable, thus instability is not possible

(b) if $\boxed{\int_{y_1}^{y_2} \frac{U_{yy}|\phi|^2}{|U-c|^2}\,dy = 0}$; $c_i \neq 0$

☆ Necessary condition for instability

[The only way that c_i can be >0 (unstable) is if this integral $=0$]

$|\phi|^2$ is positive definite (always positive)

$|U-c|^2$ " "

U_{yy} can be ⊕ or ⊖

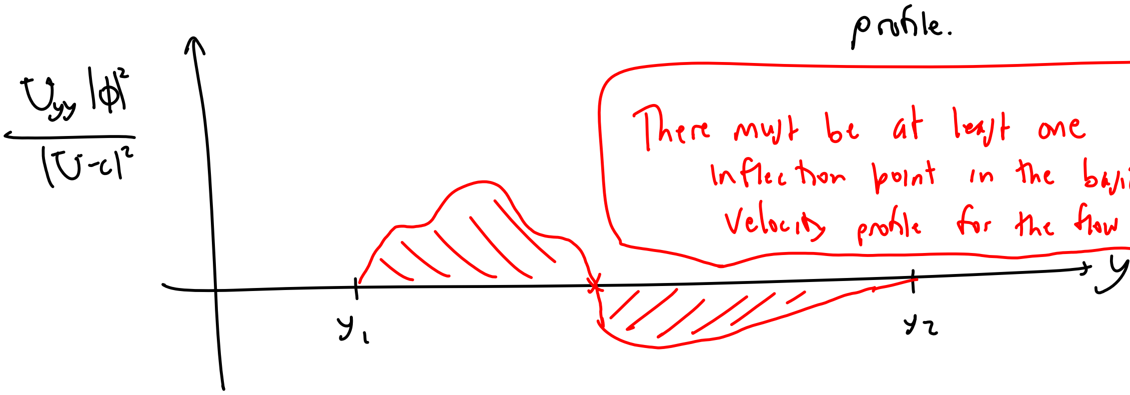
$\int$ can be zero only if :

- $U_{yy} = 0$ everywhere (trivial case – shear layer with constant slope $U_y = \text{const}$)

or

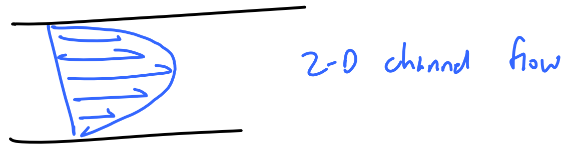
- U_{yy} must change signs somewhere within $y_1 < y < y_2$

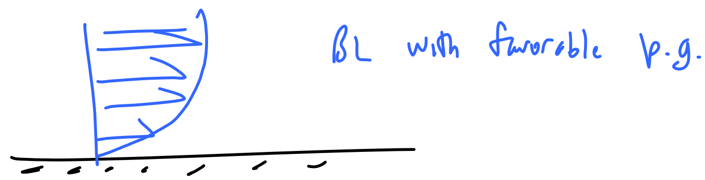
$U_{yy} = \frac{d^2U}{dy^2}$ → when $\frac{d^2U}{dy^2} = 0$, this is an inflection pt in the basic state velocity profile.



☆ There must be at least one inflection point in the basic state velocity profile for the flow to be unstable e.g.s

STABLE BASIC STATES (no inflection pt.)





★ Inviscid stability analysis (Rayleigh eq.) predicts that these basic states are stable for <u>all</u> wavenumbers k

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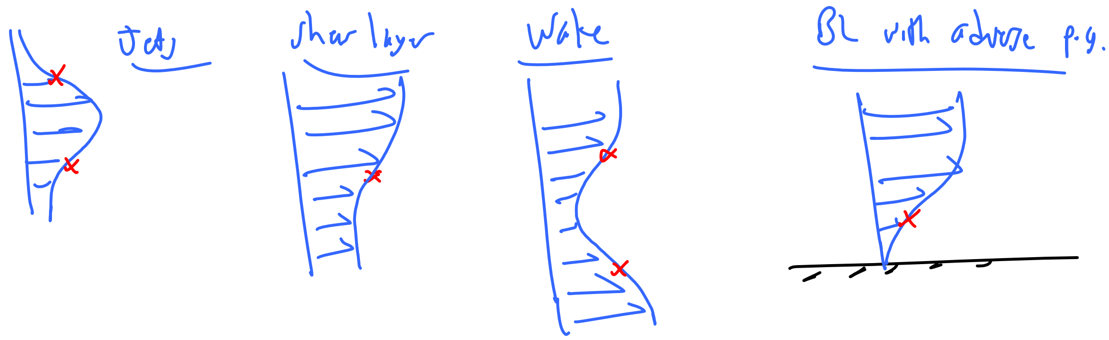
POTENTIALLY UNSTABLE BASIC STATES



★ Note: Rayleigh's inflection pt. criterion is <u>necessary</u> but not sufficient for instability