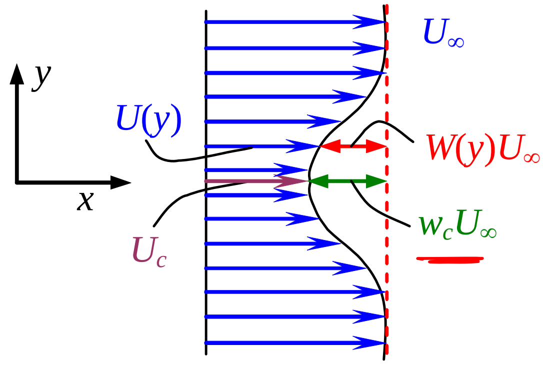
M E 522 Spring 2008 Professor John M. Cimbala Lecture 20 02/29/2008

Today, we will:

- Continue to discuss the inviscid stability of a 2-D wake
- Discuss transition to turbulence
- Do Candy Questions for Candy Friday

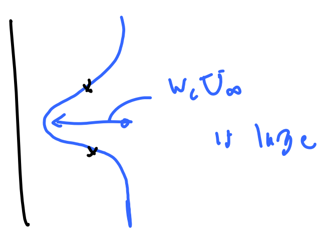


Last time …

We manipulated the Rayleigh equation into the ***universal wake Rayleigh equation for temporal instability***: $\boxed{(W-c_u)\left(\phi_{yy}-k^2\phi\right)-W_{yy}\phi=0}$ - EXACTLY SAME FORM AS ORIGINAL RAYLEIGH EQ. EXCEPT c_u & W

where $W(y)$ is the ***normalized wake defect*** $\boxed{W(y)=\dfrac{U-U_\infty}{U-U_c}}$, w_c is the ***nondimensional velocity defect at the centerline***, and c_u is the ***universal wave speed*** $\boxed{c_u=\dfrac{1}{w_c}(1-c)}$ (c_u may be complex). Experiments reveal that $W(y)$ is closely approximated by $\boxed{W(y)=\exp\left(-ay^2\right)}$, where a = ln2 and we note that y is nondimensional.

This eq. (one eq.) is valid for any x location in the wake



near wake

far wake

Need BCs: (need 2) $\boxed{@\ y\to\infty,\ \phi\to 0}$

& symmetry condition @ $y=0 \to \phi_y=0$

Temporal mode: k is real, c_u is complex $\Big\}$ Solve & get c_{u_i} as a fn. of k

This one soln c_u vs k. is valid for any x-location in the wake

Calc. c_i as a fn. of c_{u_i} & w_c where $c_u=\dfrac{1}{w_c}(1-c)$

$C = 1 - w_c C_u$

$$C = C_r + i C_i = \underbrace{1 - w_c C_{u_r}}_{C_r} + i \underbrace{(-w_c C_u)}_{C_i} \quad \therefore \boxed{C_i = -w_c C_{u_i}}$$

when C_{u_i} is negative, then $\underbrace{C_i \text{ is positive}}_{\downarrow}$

UNSTABLE

I used R-K: Split into real & imag. parts

let:

$F_1 = \phi_r \longrightarrow D_1 = \text{derivative of } \phi_1 = \phi_{r_y} = F_2$

$F_2 = \phi_{r_y} \longrightarrow D_2 = \phi_{r_{yy}} \longrightarrow$ get this from the modified Rayleigh eq.

$F_3 = \phi_i \longrightarrow D_3 = \phi_{i_y} = F_4$

$F_4 = \phi_{i_y} \longrightarrow D_4 = \phi_{i_{yy}} \longrightarrow$ - - - - - - -

Split Rayleigh eq. into real & imag. parts

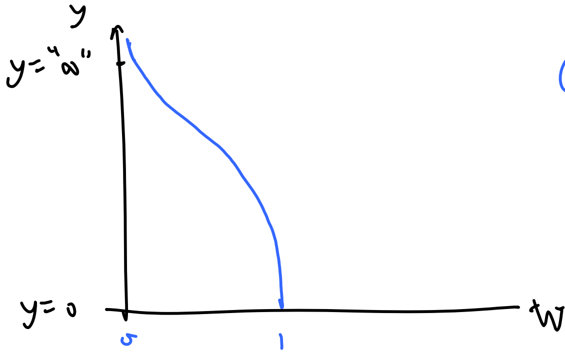
$\phi = \phi_r + i\phi_i$

$\phi_{yy} = \phi_{r_{yy}} + i\phi_{i_{yy}}$

algebra

$$\boxed{\begin{aligned} &\phi_{r_{yy}} - \left[\frac{W_{yy}(W - C_{u_r})}{\text{Denom}} + k^2\right]\phi_r + \left[\frac{W_{yy} C_{u_i}}{\text{Denom}}\right]\phi_i = 0 \\ &\phi_{i_{yy}} - \left[\qquad\qquad\right]\phi_i - \left[\qquad\right]\phi_r = 0 \\ &\text{Where Denom} = (W - C_{u_r})^2 + C_{u_i}^2 \end{aligned}}$$

Now we R-K to march from "∞" (far away - large y) to $y = 0$



@ $y=\infty$, $W=0$ & $W_{yy}=0$

recall $W = e^{-ay^2}$

$W_y = -2ay\,e^{-ay^2}$

$W_{yy} = -2ay(-2ay)e^{-ay^2} - e^{-ay^2}(2a)$

$$\boxed{W_{yy} = 2a\left(2ay^2-1\right)e^{-ay^2}}$$

@ $y\to\infty$ since $W_{yy}\to 0$, Rayleigh eq reduces to

$$\boxed{\phi_{r_{yy}} - k^2\phi_r = 0 \quad \text{as } y\to\infty}$$

2^{nd}-order ODE with const. coeff.

Soln :

$\phi_r = c_1 e^{-ky}$

$\phi_{r_y} = -k c_1 e^{-ky}$

as $y\to\infty$

Similarly

$\phi_i = c_2 e^{-ky}$

$\phi_{i_y} = -k c_2 e^{-ky}$

as $y\to\infty$

Use THESE AS THE BC's @ $y\to\infty$ (at the "edge" of the wake)

For a given k,

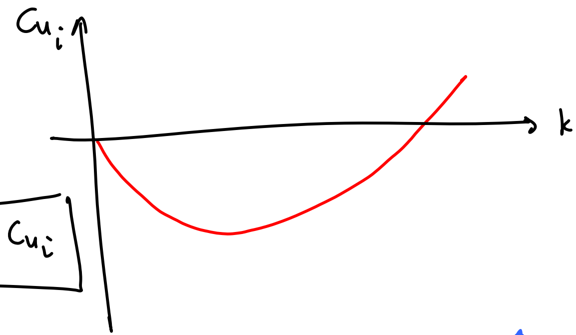
- Guess c_{u_r} & c_{u_i}
- March with R-K from edge to the middle of the wake
- check if F_2 & F_4 go to zero

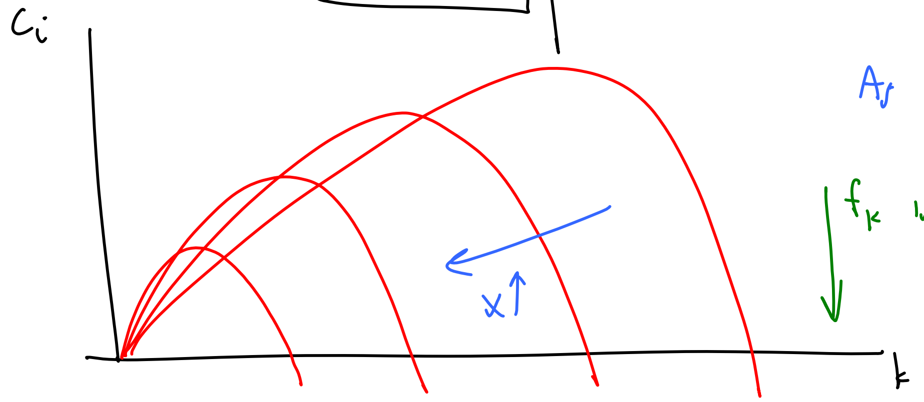
$F_2 \to \phi_{r_y}$, $F_4 \to \phi_{i_y}$

Results: Cu_i vs k

translate to c_i

$\boxed{c_i = -W_c Cu_i}$





As $x\uparrow$, $W_c\downarrow$ & $b\uparrow$

f_k is way up here

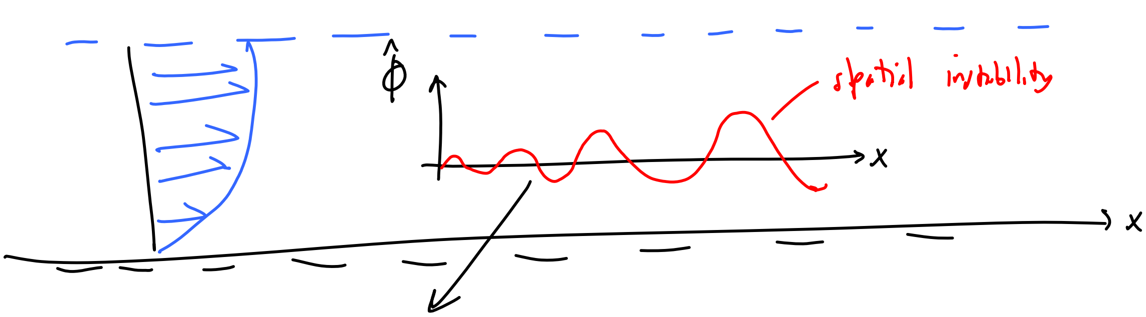
E. TRANSITION (Sec. 12.13 Kundu)

- Linear stability → onset of instability

  Can Predict
  - critical Re or Ra
  - wavenumber most likely to become unstable
  - initial growth rate of the instability

Transition in a flat plate BL

assume locally parallel flow & use O-S

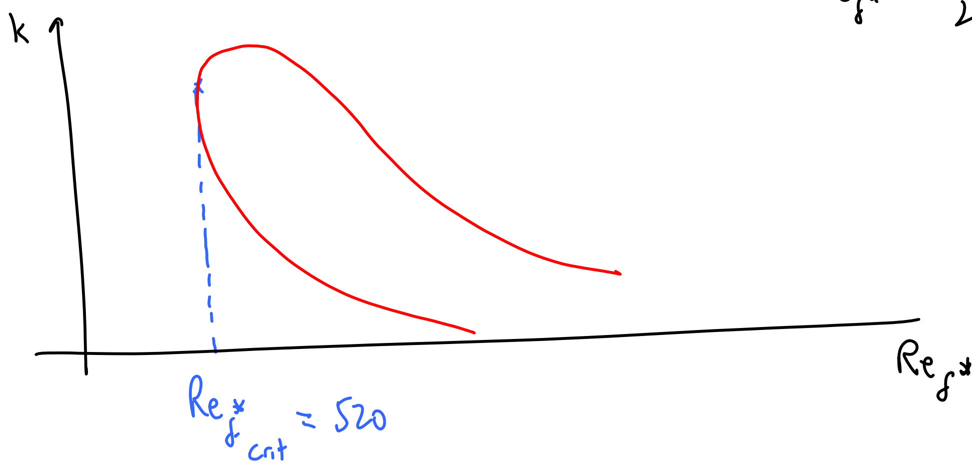


nonlinear terms become non-negligible as the disturbance grows — terms like $v\frac{du}{dy}$ become non-negligible

Also, 3-D disturbances start to appear & amplify

Blasius BL → Do an O-S analysis

$$Re_{\delta^*} = \frac{U\delta^*}{\nu}$$



Recall from Blasius soln → $\dfrac{\delta^*}{x} = \dfrac{1.72}{\sqrt{Re_x}}$

Calculate $Re_{x_{critical}} = \dfrac{U x_{crit}}{\nu} = \dfrac{U\delta^*}{\nu} \dfrac{\delta^*}{x}$

$$Re_{x_{crit}} = Re_{\delta^*_{crit}} \frac{\sqrt{Re_{x_{crit}}}}{1.72}$$

$$Re_{x_{crit}} = \left(\frac{Re_{\delta^*_{crit}}}{1.72}\right)^2 = 91,400$$

first inkling of transition to turbulence

At $Re_x \approx 10^5$ (100,000), transition starts on a smooth laminar flat plate

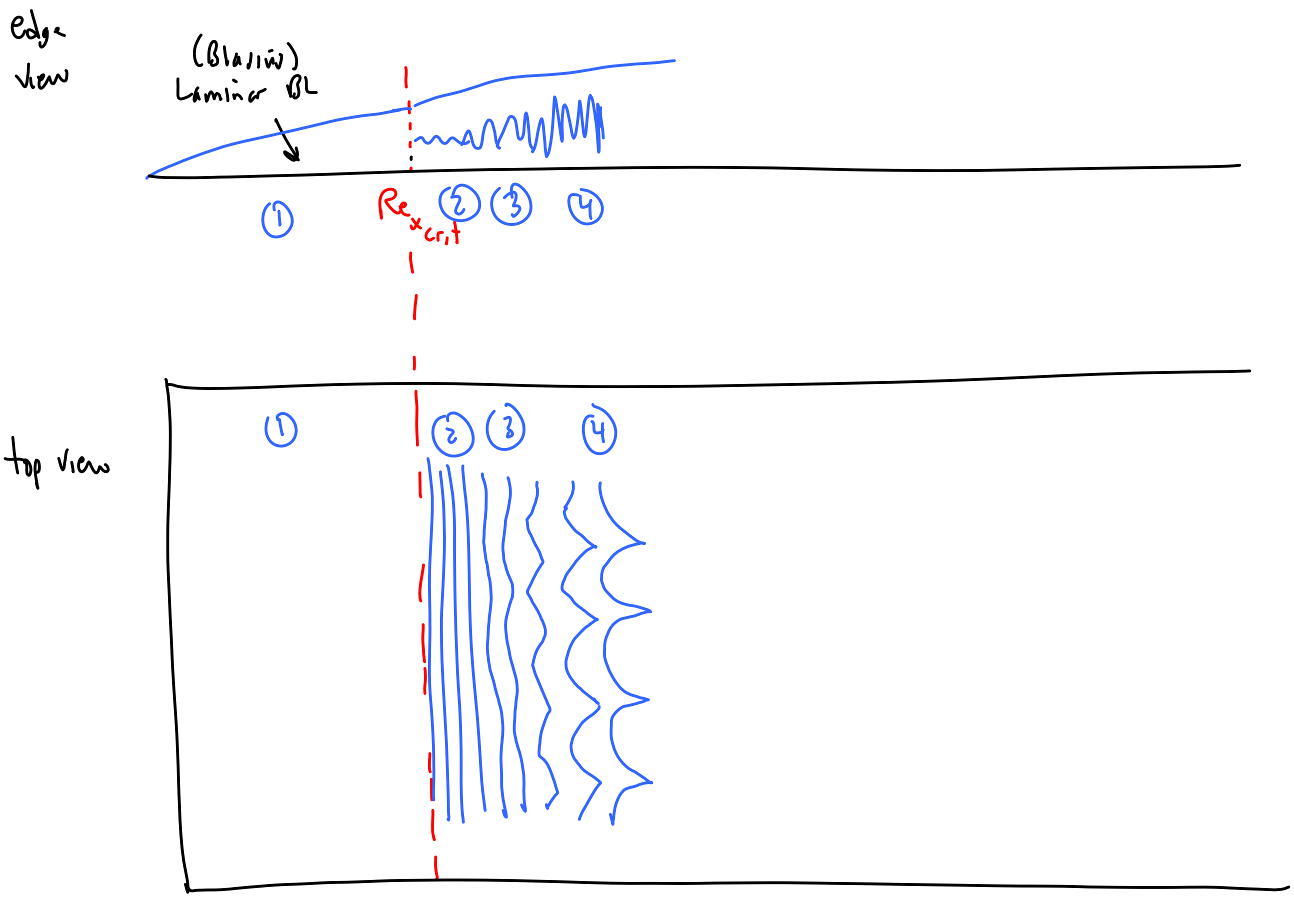
Squire's Thm → assures us that the first disturbances that are amplified are 2D

↓

we expect T-S (Tollmein-Schlichting) waves to appear

↳ predicted in 1929, verified experimentally in 1947

Qualitative Description of Transition on a Flat Plate BL



Region	Description
1	stable laminar flow $Re_x < Re_{x_{crit}}$ (Blasius)
2	First instability appears @ $Re_x = Re_{x_{crit}}$ 2-D T-S waves appear & grow [≈ 1% of U_∞]
3	3-D waves appear on these 2-D T-S waves (secondary instability)
4	3-D "hairpin eddies" form