Today, we will:

- Go over Exam 1
- Continue to discuss ensemble average, notation, etc., and some rules about ensemble averaging
- Discuss statistical definitions used in turbulence

3. Other Derivation Rules

a. The ensemble average of any fluctuating component is zero

\[ \overline{q} = \overline{Q + q} \]

\[ \overline{Q} = \overline{Q} + \overline{q} \]

\[ \overline{q} = 0 \]

\[ \overline{u_i} = \overline{u_i} + \overline{u_i} \rightarrow \overline{u_i} = 0 \]

\[ \overline{T} = \overline{T} + \overline{T'} \rightarrow \overline{T'} = 0 \]

b. "Commutative" rules

- Time derivative

\[ \frac{d \overline{q}}{dt} = \frac{d \overline{q}}{dt} = \frac{d \overline{Q}}{dt} \]

- Spatial derivative

\[ \frac{d \overline{q}}{dx_i} = \frac{d \overline{q}}{dx_i} = \frac{d \overline{Q}}{dx_i} \]

- Similarity for integration

\[ \int \overline{q} \, dx = \int \overline{q} \, dx = \int Q \, dx \]
- Addition: 
\[ \vec{q} + \vec{p} = \overline{\vec{q}} + \overline{\vec{p}} = \overline{\vec{q} + \vec{p}} = \overline{\vec{Q} + \vec{P}} \]

- Multiply with a constant: 
\[ c \overline{\vec{q}} = c \overline{\vec{q}} = c \overline{\vec{Q}} \]

\[ \overline{Q \vec{p}} = \overline{Q \vec{p}} = \overline{Q \vec{P}} \]

But \( \overline{\vec{q} \vec{p}} \neq \overline{\vec{q}} \overline{\vec{p}} \)

Can prove: \( \overline{\vec{q} \vec{p}} = \overline{Q \vec{P} + \vec{q} \vec{p}} \)

\[ \text{STATISTICS} \quad \text{Sec' 13.3 & 13.4 of Kundu} \]

Replace all \( u \) with \( \tilde{u} \)

\[ \tilde{u} = \tilde{u} + u = \tilde{u} + u \]

\[ \text{Notation} \rightarrow \tilde{u} = \tilde{u} + u = \tilde{u} + u \]

Mean square value: \( \overline{\tilde{u}^2} \)

Variance: \( \overline{u^2} = (\tilde{u} - u)^2 \)

Standard deviation (in T.I.L. other turbulence books)

Kundu’s variance is defined as \( \tilde{u}^2 \) (same as mean square value)

\[ \text{Root mean square} = \overline{\text{mean square value}} \] (RMS)

\[ U_{\text{RMS}} = \overline{\tilde{u}^2} \]

Kundu calls it \( \overline{\text{variance}} \)

\[ \text{Standard deviation (SD)} = \overline{\text{variance}} \]

\[ \tilde{u}_{SD} = \overline{\tilde{u}^2} = \overline{(\tilde{u} - u)^2} \]
In many papers, the authors confuse "rms" with "standard deviation".

Example: "The velocity was 10.3 m/s with a 0.6% rms." 

\[ \bar{U} = U \quad \text{(Standard Deviation)} \]

\[ \text{RMS of the fluctuation only} \]

\[ \text{Correlation of } \bar{u} \text{ and } \bar{v} = \bar{u} \bar{v} = UV + uv \]

Turbulence intensity: \[ I = \text{overall turbulence intensity} \equiv \text{square root of the ensemble average of the average squared velocity fluctuation divided by the magnitude of the mean velocity vector} \]

\[ \frac{1}{|\vec{U}|} \]

For \( \vec{U} = (u, v, w) = \text{fluctuating velocity} \)

"Average squared velocity fluctuation" = \[ \frac{1}{3} (u^2 + v^2 + w^2) \left( = \frac{|\vec{u}|^2}{3} \right) \]

\[ I = \text{overall turbulence intensity} \equiv \sqrt{\frac{\frac{1}{3} (u^2 + v^2 + w^2)}{|\vec{U}|}} \]