Today, we will: • Go over Exam 1

- Continue to discuss ensemble average, notation, etc., and some rules about ensemble averaging
- Discuss statistical definitions used in turbulence

3. Other Definitions & Rules

a. The ensemble average of any fluctuating component is zero

$$\tilde{q} = Q + q$$

fluctuating (q); ensemble avg'd (Q)

[overbar = ensemble avg.]

take ensemble avg.

$$\overline{\tilde{q}} = \overline{Q} + \overline{q}$$

$$Q = Q + \overline{q} \rightarrow \therefore \boxed{\overline{q} = 0}$$

$$\tilde{u}_i = U_i + u_i \rightarrow \boxed{\overline{u_i} = 0}$$

$$\tilde{T} = \overline{T} + T' \rightarrow \boxed{\overline{T'} = 0}$$

b. "Commutative" rules

• Time derivative

$$\boxed{\overline{\frac{\partial \tilde{q}}{\partial t}} = \frac{\partial \overline{\tilde{q}}}{\partial t} = \frac{\partial Q}{\partial t}}$$

$\overline{\tilde{q}} = Q$

order does not matter

Spatial derivative – similarly,

$$\boxed{\overline{\frac{\partial \tilde{q}}{\partial x_i}} = \frac{\partial \overline{\tilde{q}}}{\partial x_i} = \frac{\partial Q}{\partial x_i}}$$

Similarly for integration

$$\boxed{\overline{\int \tilde{q}\, dx} = \int \overline{\tilde{q}}\, dx = \int Q\, dx}$$

- Addition: $\boxed{\overline{\tilde{q}+\tilde{p}} = \overline{\tilde{q}} + \overline{\tilde{p}} = Q+P}$

- Multiplication with a constant: $\boxed{\overline{c_1 \tilde{q}} = c_1 \overline{\tilde{q}} = c_1 Q}$

$$\boxed{\overline{Q\tilde{p}} = Q\overline{\tilde{p}} = QP}$$

But $\overline{\tilde{q}\tilde{p}} \neq \overline{\tilde{q}}\,\overline{\tilde{p}}$ can prove $\boxed{\overline{\tilde{q}\tilde{p}} = QP + \overline{qp}}$

STATISTICS Sec's 13.3 & 13.4 of Kundu

replace all u with $\tilde{u}$

" " v " $\tilde{v}$

[Notation → $\tilde{u} = \overline{\tilde{u}} + u = U + u$]

$\boxed{\text{Mean square value} \equiv \overline{\tilde{u}^2}}$

$\boxed{\text{Variance} \equiv \overline{u^2} = \overline{(\tilde{u}-U)^2}}$ standard definition (in T&L, other turbulence books)

Kundu's variance is defined as $\overline{\tilde{u}^2}$ (same as mean square value)

not standard

Root mean square (RMS) = $\sqrt{\text{mean square value}}$ $\boxed{\tilde{u}_{RMS} \equiv \sqrt{\overline{\tilde{u}^2}}}$

[Kundu calls it $\sqrt{\text{variance}}$]

Standard deviation (SD) = $\sqrt{\text{variance}}$ $\boxed{\tilde{u}_{SD} \equiv \sqrt{\overline{u^2}} = \sqrt{\overline{(\tilde{u}-U)^2}}}$

"Rms of the fluctuations" $\equiv$ SD

$$\boxed{u_{rms} \equiv \sqrt{\overline{u^2}}}$$

In papers many authors confuse "rms" & "standard deviation"

e.g. "The velocity was $\underbrace{10.3\ m/s}_{\tilde{\bar{u}} = U}$ with a $\underbrace{0.6\%\ rms}_{\text{standard deviation}}$"

Rms of the fluctuations only

Correlation of $\tilde{u}$ & $\tilde{v}$ $\equiv$ $\boxed{\overline{\tilde{u}\tilde{v}} = UV + \overline{uv}}$

Turbulence intensity I = overall turbulence intensity $\equiv$ square root of the ensemble average of the average squared velocity fluctuation divided by the magnitude of the mean velocity vector ($|\vec{U}|$)

(like a 3-D standard deviation – working with vectors rather than scalars)

for $\vec{u} = (u, v, w)$ = fluctuating velocity

"average squared velocity fluctuation" $= \frac{1}{3}(u^2 + v^2 + w^2)$ $\left[= \frac{|\vec{u}|^2}{3}\right]$

$$\star\ \boxed{I = \text{overall turbulence intensity} \equiv \frac{\sqrt{\frac{1}{3}\left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right)}}{|\vec{U}|}}$$