M E 522 Spring 2008 Professor John M. Cimbala Lecture 25 03/19/2008

Today, we will:

- Discuss more definitions and notation in turbulence (turbulent kinetic energy, intermittency)
- Discuss the difference between homogeneous turbulence and isotropic turbulence
- Start talking about the equations of motion for turbulent flow; Reynolds decomposition

f. Turbulent Kinetic Energy ("tke")

- Recall k.e. per unit mass of a fluid particle $= \frac{1}{2}\left|\vec{\tilde{u}}\right|^2$
- For turbulence, let's look at the k.e. associated with the turbulent fluctuations only
- Define

$$q^2 = \overline{K'} = k = tke \equiv \text{turbulent kinetic energy per unit mass}$$

(Kundu)

$$\equiv \frac{1}{2}\overline{u_i u_i}$$

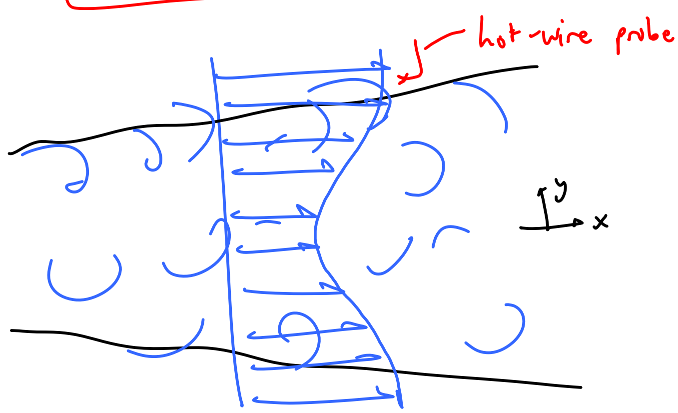
$$q^2 = \frac{1}{2}\overline{\left(u^2 + v^2 + w^2\right)} = \frac{1}{2}\left(\overline{u^2} + \overline{v^2} + \overline{w^2}\right)$$

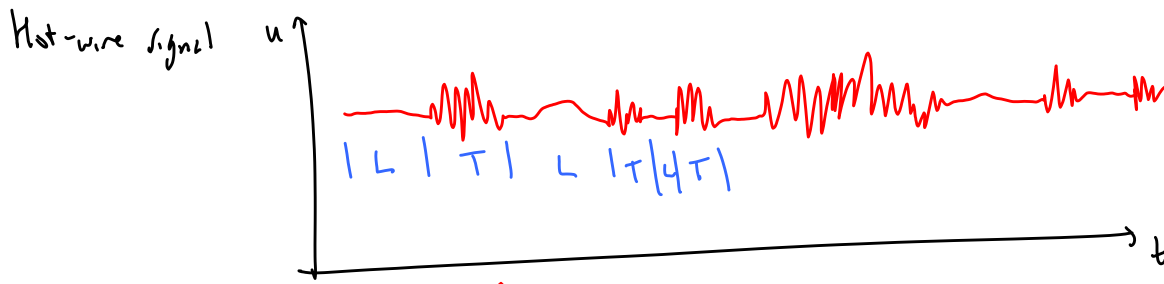
This is the k of the famous k-ε turbulence model

g. Intermittency, γ

$\gamma \equiv$ The fraction of time that the flow at a point is turbulent

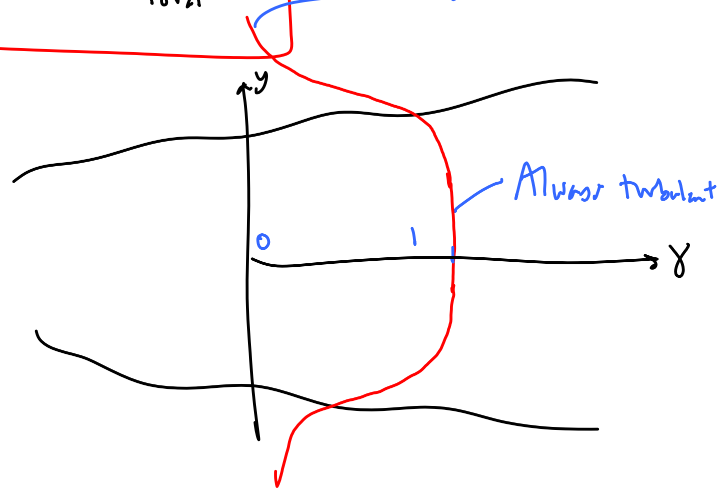
e.g. a turbulent wake





$$\boxed{\gamma = \frac{\sum t_{turbulent}}{t_{total}}}$$

$$\boxed{0 \le \gamma \le 1}$$



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h. <u>Homogeneous Turbulence</u> – [very useful for <u>calibrating</u> turbulence models]

↓

An <u>approximation</u> → $\boxed{\text{Assume no spatial dependence of ensemble-averaged turbulent quantities}}$ ✱

i.e., $\boxed{\frac{\partial}{\partial x_j}\overline{(\ )} = 0}$

any turbulence quantity

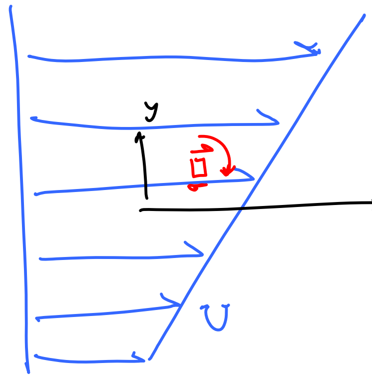
e.g. Velocity → · U (mean velocity) <u>can</u> vary with space

But · ensemble avg's turbulent quantities like $\overline{u^2}$ cannot.

· However, $\frac{\partial}{\partial t}\overline{(\ )}$ is not necessarily zero

Classic Example

- Homogeneous Turbulent Shear Layer $\rightarrow$ $U = \dfrac{dU}{dy}\, y$, $\dfrac{dU}{dy}$ = constant



The mean flow has non-zero vorticity

- Locally, $\overline{u^2}$, $\overline{uv}$, etc. are constant in any direction — no spatial dependence

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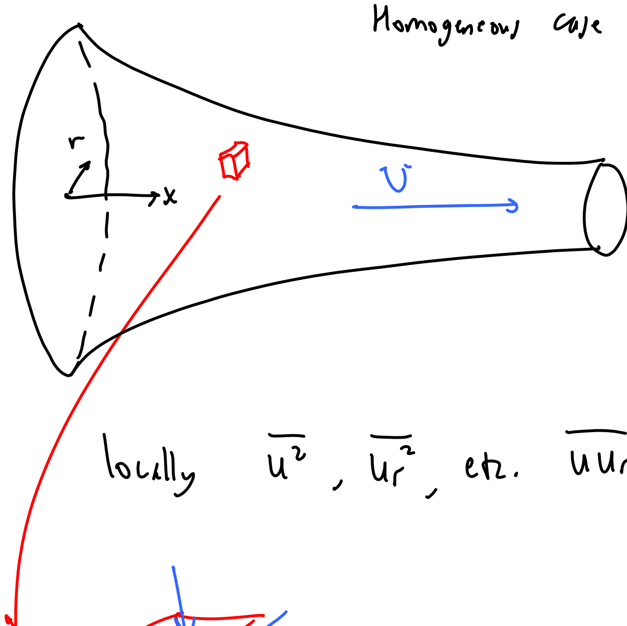
- Axisymmetric Expansion or Contraction

Homogeneous case — Ignore wall effects (BL's) $\rightarrow$ assume inviscid walls

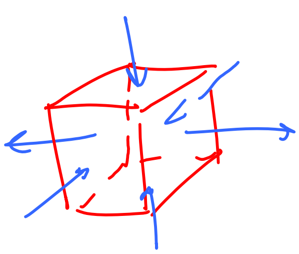


$U \uparrow$ (accelerates) with $x \uparrow$

U_r is negative everywhere

locally $\overline{u^2}$, $\overline{u_r^2}$, etc. $\overline{uu_r}$, etc. are constant in space

an approximation



There are 3 strains in all 3 directions

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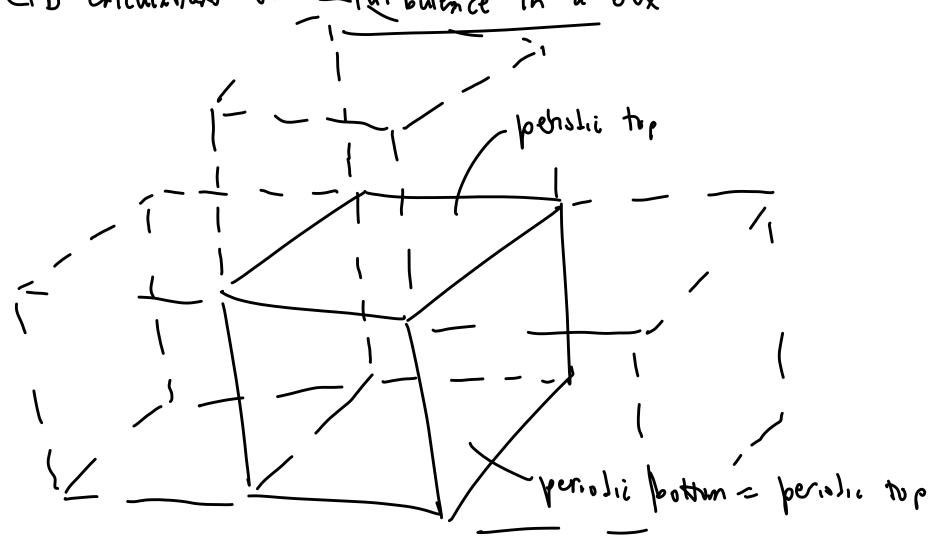
i. ISOTROPIC TURBULENCE — A special case of homogeneous turbulence

— Ensemble-avg'd turbulent quantities are independent of space (homogeneous)

And — " " " " " " " orientation of the axes

For isotropic turbulence $\quad \sqrt{\overline{u^2}} = \sqrt{\overline{v^2}} = \sqrt{\overline{w^2}}$ [no preferred direction]

e.g. CFD calculation of "turbulence in a box"



e.g. physical experiment – turbulence decaying behind a screen

Note: All isotropic flows are homogeneous
But not all homogeneous flows are isotropic

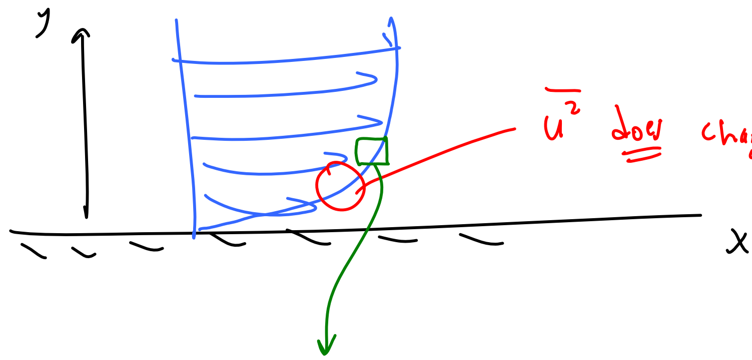
[Isotropic is a special case of homogeneous]

j. Inhomogeneous Turbulence: $\frac{\partial}{\partial x_j}\overline{(\ \ )}$ not necessarily 0

any turbulence quantity

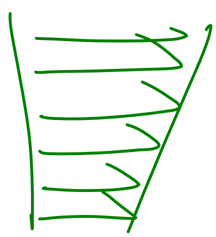
e.g. — all real engineering flows

e.g. BL's, jets, wakes, etc.



$\overline{u^2}$ does change with spatial location

May assume locally homogeneous turbulence in a small region under study



An approximation — but useful for analysis

C. Equations of Motion for Turbulent Flow (see handout)

1. Navier-Stokes eq's in Boussinesq. approx. → See Eqs (1)–(3) on handout

Very hard to solve these eq's "as is"

- p.d.e's → functions of (x, y, z, t)
- unsteady
- 3-D → u, v, & w components — can't ignore any of them
- nonlinear
- elliptic in space — can't "march" in one direction like in a BL

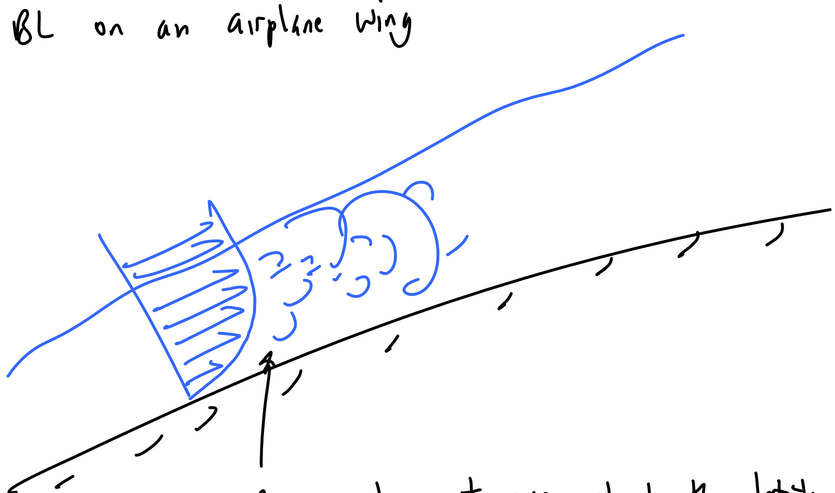
real killer → ✱ → • smallest eddies are orders of mag. smaller than the largest eddies

↳ ∴ would need extremely fine grids in a CFD calculation

We cannot solve the exact eqns for total flow variables for most real-life engineering problems.

So → instead we want to solve for the mean flow [ensemble avg'd]

e.g. Turbulent BL on an airplane wing



engineer does not care about the details of all the turbulent eddies

We want to solve for the mean quantities like U_i

like τ_w — shear stress at wall (mean)

2. Reynolds Decomposition → a technique to obtain Eqns for the mean flow quantities

SEE HANDOUT ☆

We same notation as previously

$$\boxed{\begin{aligned} \tilde{u}_i &= U_i + u_i \\ \tilde{p} &= P + p \\ \tilde{T} &= \overline{T} + T' \end{aligned}}$$

Plan of attack:

Split the N-S eq's into
- mean eqs
- eqs for the fluctuations