Today, we will:

- Continue our discussion about the turbulent kinetic energy (tke) equation, and the "tke budget"
- Do a qualitative example problem – the turbulent jet
- Start discussing **order-of-magnitude analysis of turbulence** (fun!)

Comments about the tke eq. (continued)

(5) Terms II, III, & IV in Eq (3)

are usually lumped together & called "transport term" or "diffusion term"

↙

These terms redistribute tke spatially (i.e. diffuse it)

But they don't change the overall net amount of tke in the flow

(6) The tke eq. is often called the "tke budget"

Good analogy between a financial budget & the tke budget

money	⟺	tke
• Total rate of change of the amt of money that you have		• LHS of the eq = total rate of change of tke
• Production of $ (a check from a job or a stipend)		• Production of tke (from the mean shear → produces tke)
• Transport of $ (transfer from savings to checking or checking acct to cash, etc.) no net gain or loss		• Transport terms (II, III, IV) (no net production or destruction of tke – just move it around)
• Dissipation of $ (spend it)	⟹	• Dissipation of tke (term VII) (destroy tke due to viscosity)

Rewrite the eq. in word form for a stationary flow (steady in the mean)

$$\frac{D(q^2)}{Dt} = \frac{\partial q^2}{\partial t} + U_j \frac{\partial q^2}{\partial x_j} = \text{RHS terms}$$

$\frac{\partial q^2}{\partial t}$: 0 (stationary)

$U_j \frac{\partial q^2}{\partial x_j}$: ADVECTION

Let's move all terms to the RHS of the eq.:

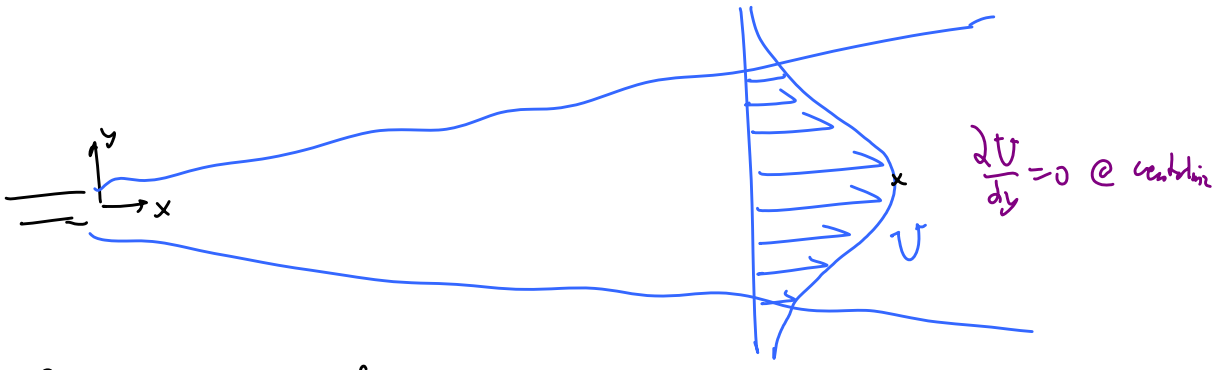
$$0 = -U_j \frac{\partial q^2}{\partial x_j} + \frac{\partial}{\partial x_j}\left[\sim\right] - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \varepsilon$$

as a word eq:

$$0 = -\text{Advection} + \text{Transport} + \text{Production} - \text{Dissipation}$$

☆ the budget

Qualitative e.g. — 2-D stationary turbulent jet



Do a BL-type of order-of-mag analysis (assume a thin jet) on the eq.

Dominant terms:

★ $$0 = -U\frac{\partial q^2}{\partial x} - V\frac{\partial q^2}{\partial y} + \frac{\partial}{\partial y}\left[-\frac{1}{2}\overline{u_i u_i v} - \frac{\overline{pv}}{\rho}\right] - \overline{uv}\frac{\partial U}{\partial y} - \varepsilon$$

the eq with BL approx for a thin BL-type problem (e.g. B.L. or jet)

- Advection term

+ TRANSPORT TERM

+ PRODUCTION

- DISSIPATION

Near centerline $U>0$, $\frac{\partial q^2}{\partial x}<0$ $\therefore -U\frac{\partial q^2}{\partial x} = (+ve)$

near edge of jet $U \to 0$ " $\to (0)$

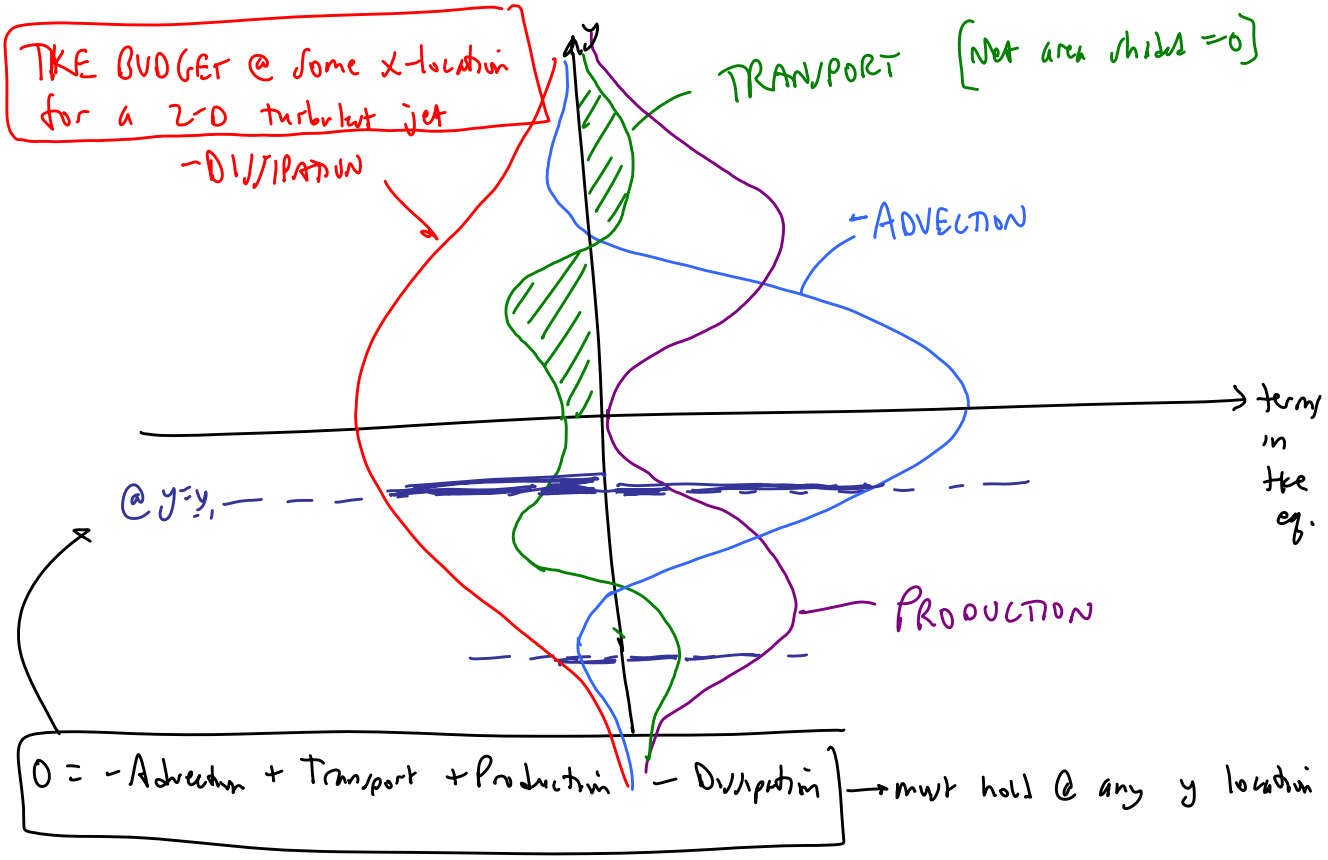
near centerline $V \to 0$ $\bar{V}=0$ on centerline (symmetry) $-V\frac{\partial q^2}{\partial y} \to (0)$

near edge $V<0$ (entrainment)

$\frac{\partial q^2}{\partial y}<0$ - decaying towards the edge of jet

$-V\frac{\partial q^2}{\partial y}$ w. (-ve)

TKE BUDGET @ some x-location for a 2-D turbulent jet



$0 = -\text{Advection} + \text{Transport} + \text{Production} - \text{Dissipation}$ → must hold @ any y location

D. Order-of-magnitude Analysis of Turbulence [some "fun"]

1. Intro

- physical & qualitative rather than mathematical
- provides a lot of understanding about turbulence

a. Notation

$u \sim U$ → means u is of order of magnitude U

T&L say if $0.2\,U \lesssim u \lesssim 5\,U$, then $u \sim U$

Also if $-5\,U \lesssim u \lesssim 0.2\,U$, then $u \sim U$

Sign does not matter!

Let's analyze turbulence this way (O.O.M. anal.)

b. Example — from §21 we did O.O.M. anal. for laminar BLs

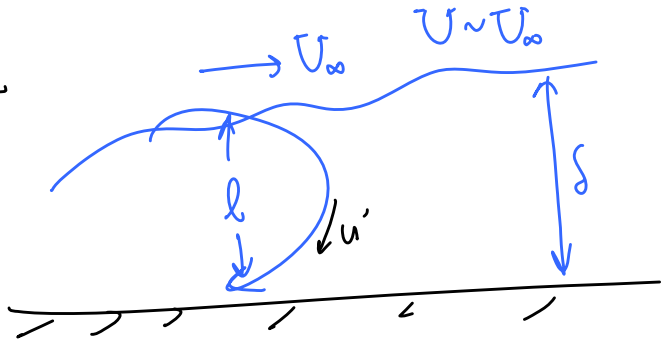
2. Eddy scales & the Energy Cascade

- In laminar flow BL anal., we had 2 length scales x, δ
- In turbulence, there are several length scales involved:

a. Largest eddies

- Let ℓ denote the characteristic length scale of the large-scale eddies
- Let u' denote the characteristic velocity scale of the large-scale eddies [T&L use u]
- U = char. mean shear velocity scale, & δ = char. mean length scale

e.g., Turbulent BL



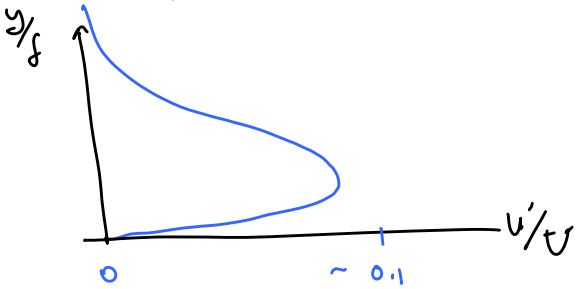
set $\boxed{\ell \sim \delta}$

What about u'? Typically set $u' \sim \sqrt{\overline{u^2}}$ or $\boxed{u' \sim \sqrt{q^2}}$

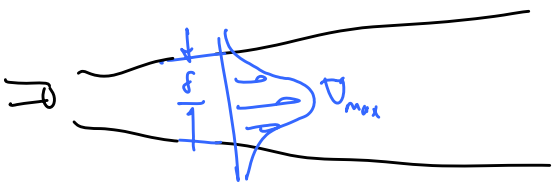
$q^2 = tke = \frac{1}{2}\overline{u_i u_i}$

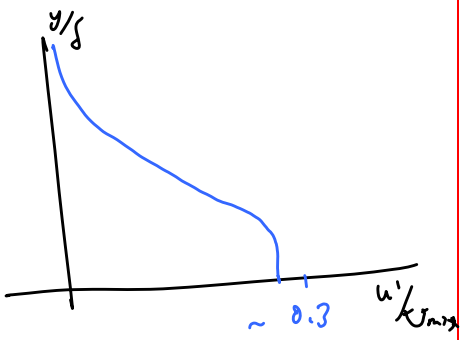
From experiments, we can get u'

Turb BL

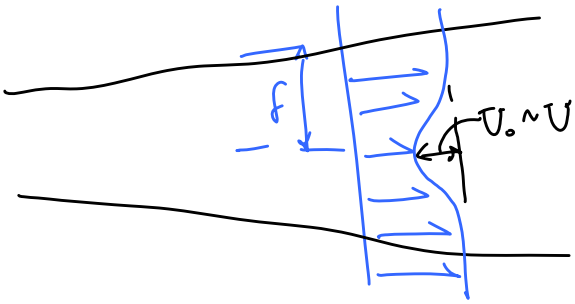


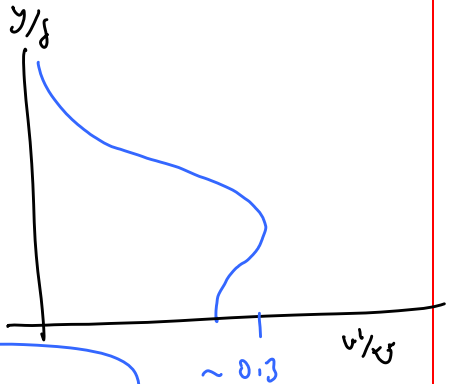
Jet





Wakes:





We will set $\frac{u'}{U} \sim 0.1$ for most turbulent flows ☆