Today, we will:

- Discuss the small-scale eddies in turbulence; Kolmogorov's contributions
- Discuss how to estimate ε, ℓ, η, etc. in turbulent flows using order-of-magnitudes
- Do some example problems – order-of-magnitude problems

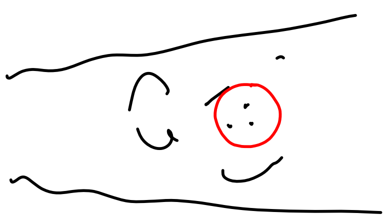
C. Smallest eddies (continued) Comments:

1) the smallest turbulent eddies → molecular diffusion smears out velocity fluctuations

2) " → size determined by viscosity

3) → statistically independent of large-scale turbulent eddies or mean flow

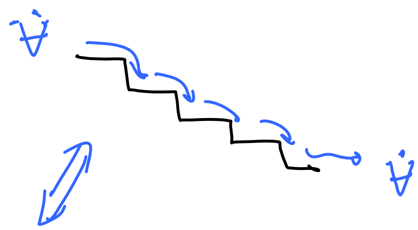
4) " → become nearly isotropic



How then are large scale eddies & small scale eddies related?

Ans. → through the kinetic energy associated w/ their fluctuations

i.e., the energy cascade – Analogy to a cascading waterfall



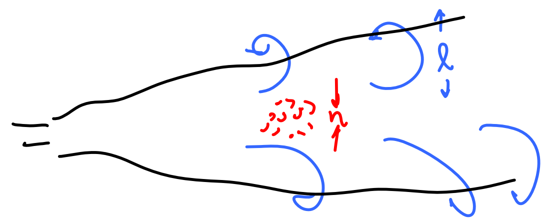
turbulence → the only parameter "shared" by the largest & smallest eddies is the kinetic energy of their fluctuations

3. Kolmogorov's contribution (1941, Russian)

a. **Kolmogorov Universal Equilibrium Hypothesis** – The turbulent kinetic energy supplied by the mean flow (through interaction with the large-scale eddies) equals the $\underbrace{\text{dissipation of t.k.e.}}_{\varepsilon}$ due to viscosity (by the smallest-scale eddies)

[Production of tke = Dissipation of tke in a stationary flow]

e.g. turbulent jet:

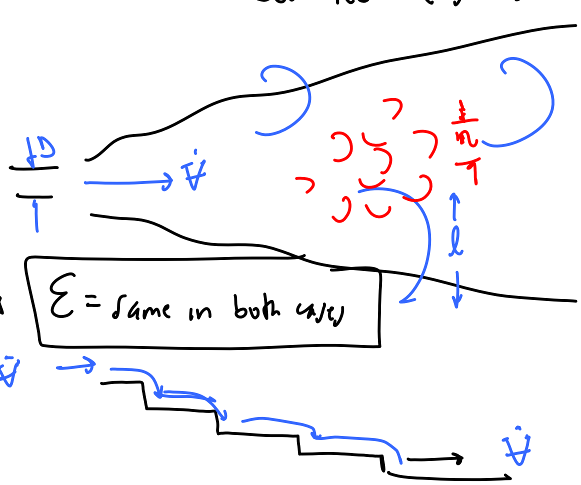


- The size of the large scale eddies is determined by the mean flow
- The size of the smallest eddies adjusts itself automatically due to the viscosity & the kinetic energy that needs to be dissipated.

In movie, two identical jets, one low viscosity fluid
one high " "

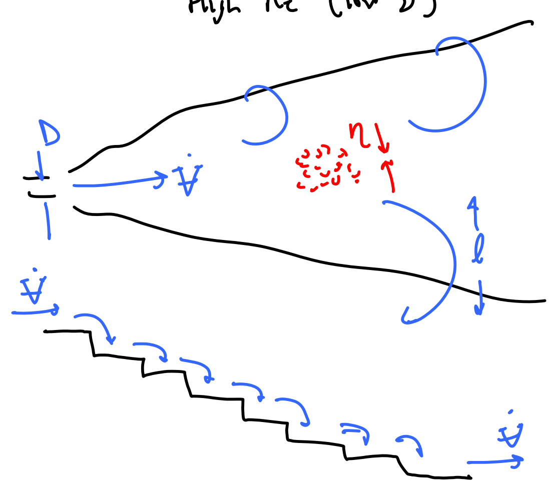
Velocity & size of the jets are identical – mean flow is identical

Low Re (high ν)



$\boxed{\varepsilon = \text{same in both cases}}$

High Re (low ν)



b. Kolmogorov Microscales → Kolmogorov predicts the size of the smallest eddies

Assume $\boxed{\eta = \text{fnc. of } (\varepsilon, \nu) \text{ only}}$

$\eta \neq$ fnc. of ℓ, υ ...

	Dimensions
η = length scale of smallest (energy dissipating) eddies	$\{L\}$
ε = kinetic energy dissipation rate per unit mass [recall, the eq. $\varepsilon = 2\nu \overline{e_{ij} e_{ij}}$]	$\{L^2/t^3\}$
ν = kinematic viscosity	$\{L^2/t\}$

Kolmogorov used dimensional analysis & O-O-M analysis

$$\{\varepsilon\} = \frac{\text{energy}}{\text{time} \cdot \text{mass}} = \frac{\frac{mL}{t^2} \cdot L}{t \cdot m} = \frac{L^2}{t^3}$$

We get:

$$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} = \text{Kolmogorov length scale (of smallest eddies)}$$

$$\tau \sim \left(\frac{\nu}{\varepsilon}\right)^{1/2} = \text{" time " "}$$

$$\upsilon \sim (\nu \varepsilon)^{1/4} = \text{" velocity " "}$$

Kolmogorov microscales of turbulence

Comments: 1) Reynolds # of smallest eddies

$$\boxed{Re_{\text{Kolmogorov}} = \frac{\eta \upsilon}{\nu} \sim 1}$$

inertial effects ~ viscous effects

2) η is the smallest possible o.o.m. length scale in the flow

3) The 3 scales are related to each other

e.g. 1 turnover time for a Kolmogorov eddy

$$t \sim \frac{\text{length}}{\text{Velocity}} \sim \frac{\eta}{v} \sim \frac{(\nu^3/\varepsilon)^{1/4}}{(\nu\varepsilon)^{1/4}} \sim \underline{\underline{\left(\frac{\nu}{\varepsilon}\right)^{1/2} = \tau}}$$

☆ 4) To estimate η, we <u>must</u> estimate ε

How?

4. How to Estimate ε?

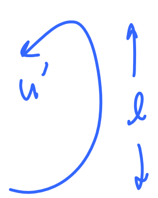
a. O.O.M. analysis → Use <u>Kolmogorov's Universal Equilibrium Hypothesis</u>

ε is the <u>link</u> between the largest scale eddies & the smallest eddies

ε = two things :
- rate of kinetic energy per unit mass dissipated by the smallest eddies
- rate of kinetic energy per unit mass supplied by the mean flow via the largest eddies

lets we <u>this</u> one to estimate ε

O.O.M. analysis of ε using the large eddies



u' = velocity scale of the large eddies

l = length " " "

$\frac{l}{u'}$ = time " " " (turn over time)

$\frac{1}{2}u'^2$ = kinetic energy per unit mass of the large eddies

Let's set k.e. per unit mass of the large eddies $\sim u'^2$ (don't care about the ½)

Let's do some "shoveling" or hand waiving here.

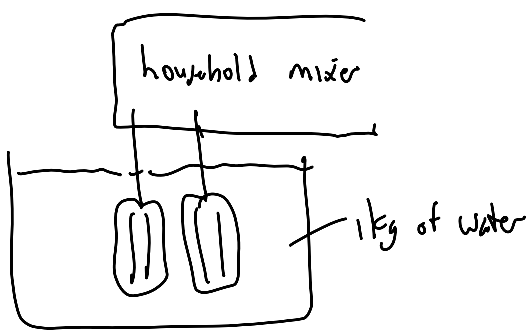
$$\mathcal{E} \sim \frac{\text{k.e. per unit mass of the large scale eddies}}{\text{time scale of the " " "}}$$

$$\mathcal{E} \sim \frac{u'^2}{\ell/u'} \sim \frac{u'^3}{\ell} \qquad \boxed{\mathcal{E} \sim \frac{u'^3}{\ell}}$$

The <u>viscous</u> dissipation rate is determined by the <u>inviscid</u> dynamics of the large scale turbulence. (no ν here)

This allows us to do a lot of approximate o.o.m. analyses of turbulent flows — "shovel problems"!

b. <u>Examples</u>



<u>Given</u>: Supply 100 W of power to the mixer blades

<u>To do</u>: Calculate (estimate) $\mathcal{E}, \eta$

<u>Soln</u>:
- All of supplied energy is turned into turbulence
- All of the turbulence is dissipated as heat $\rightarrow \boxed{\mathcal{E} \approx 100 \frac{W}{kg}}$ *
- $\nu = 1\times10^{-6}\ m^2/s$

$$\eta \sim \left(\frac{\nu^3}{\mathcal{E}}\right)^{1/4} \sim \left[\frac{(1\times10^{-6}\ m^2/s)^3}{(100\ W/kg)\left(\frac{N\cdot m}{s\cdot W}\right)\left(\frac{kg\cdot m}{s^2\cdot N}\right)}\right]^{1/4} \sim 1\times10^{-5}\ m \qquad \boxed{\eta \approx 0.01\ mm}$$ *

NOTE: • η is <u>very small</u>! • REPORT ANSWERS TO ONLY 1 SIGNIFICANT DIGIT!