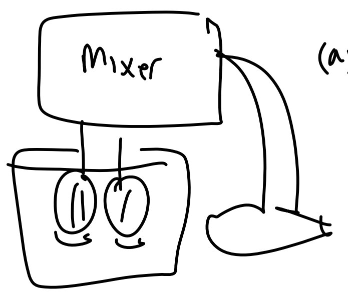
**Today, we will**:

- Continue the example problem from last time – household mixer
- Discuss the turbulent Reynolds number
- Do some example problems – o.o.m analysis & movie-making




(a) $\varepsilon = 100 \frac{W}{kg}$

$\nu = 1\times10^{-6}\ m^2/s$

$$\eta = \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \sim 1\times10^{-5}\ m \approx \underline{0.01\ mm}$$

(b) Estimate $\ell$ – char. length scale of the large-scale eddies

Soln. 1 Liter of water = $(0.1\,m)^3$

$\ell \sim$ half of the size of the container

$$\boxed{\ell \sim 0.05\ m}$$

(c) Estimate $u'$ – char. velocity scale of the large-scale eddies

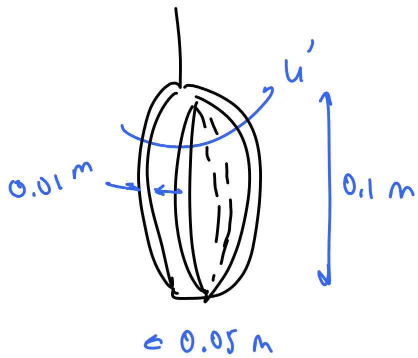
Soln: Two methods:

A. $\varepsilon \sim \frac{u'^3}{\ell} \rightarrow u' \sim (\varepsilon \ell)^{1/3}$

$$u' \sim \left[\left(\frac{100\ W}{kg}\right)(0.05\ m)\left(\frac{N\cdot m}{s\cdot W}\right)\left(\frac{kg\cdot m}{s^2\cdot N}\right)\right]^{1/3} = 1.71\ m/s$$

$$\boxed{u' \sim 2\ m/s}$$ [one sig. digit]

B. Let's look at the blades themselves




4 blades per beater

∴ 8 blades total

$$\text{Power} = 8 \cdot \text{Drag}_{\text{one blade}} \cdot u'$$

$$= 8\left(\tfrac{1}{2}\rho u'^2 C_D A\right) u'$$

$$\therefore \quad u' = \left(\frac{\text{Power}}{4\rho C_D A}\right)^{1/3}$$

Power: 100 W

$C_D$: ~1

$A = (0.1\text{ m})(0.01\text{ m})$ = frontal area

$$u' = 2.9 \text{ m/s} \quad \text{or} \quad \boxed{u' \sim 3 \text{ m/s}}$$

Agrees with method A

since $\boxed{2 \sim 3}$

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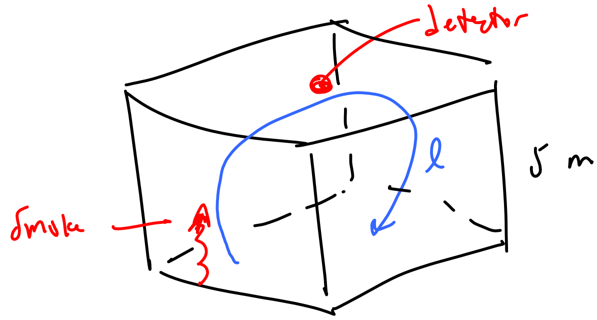
Example:
- room 5×5×5 m
- temperature-sensitive smoke detector on ceiling
- hot smoke @ one corner of the room



To do: Compare how long it takes for the detector to sense the smoke:

(a) laminar diffusion i.e. stagnant air in room

(b) turbulent " with $u' \sim 0.1$ m/s

Soln: (a) Laminar diffusion eq

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x_i \partial x_i}$$

$$\frac{\partial T}{\partial t} = K \frac{\partial^2 T}{\partial x_i \partial x_i} \qquad K = \frac{k}{\rho C_p}$$

o.o.m. anal.

$$\frac{\Delta T}{t_{lam}} \sim K \frac{\Delta T}{L^2}$$

$K_{air} \approx 2\times10^{-5}$ m²/s

$$t_{lam} \sim \frac{L^2}{K} = \frac{(5\text{ m})^2}{2\times10^{-5}\text{ m}^2/\text{s}} = 1.25\times10^{6}\text{ s}$$

$$\boxed{t_{lam} \sim 300 \text{ hours}}$$

(b) Turbulent $u' \sim 0.1$ m/s (given)

$l \sim 5$ m

One eddy turnover time $\sim t_{turb}$

$$t_{turb} \sim \frac{l}{u'} \sim \frac{5\text{ m}}{0.1\text{ m/s}} \approx 50\text{ s}$$

$$\boxed{t_{turb} \sim 1 \text{ minute}}$$

(c) Estimate Kolmogorov scales

Soln: $\ell \sim 5\ m$

$u' \sim 0.1\ m/s$

$$\varepsilon \sim \frac{u'^3}{\ell} \sim \frac{(0.1\ m/s)^3}{5\ m} \sim 0.0002\ m^2/s^3$$

$$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{1/4} \sim \left(\frac{(1.5\times10^{-5}\ m^2/s)^3}{0.0002\ m^2/s^3}\right)^{1/4} \sim \boxed{0.002\ m \sim \eta}$$

$$\tau \sim \left(\frac{\nu}{\varepsilon}\right)^{1/2} \sim \boxed{0.3\ s \sim \tau}$$

$$v \sim (\nu\varepsilon)^{1/4} \sim \boxed{0.007\ m/s \sim v}$$

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5. Turbulent Reynolds Number, $R_\ell$

Define $\boxed{R_\ell \equiv \frac{u'\ell}{\nu}}$

$\varepsilon \sim \frac{u'^3}{\ell}$

$$\frac{\eta}{\ell} \sim \frac{(\nu^3/\varepsilon)^{1/4}}{\ell} \sim \frac{\left(\frac{\nu^3\ell}{u'^3}\right)^{1/4}}{\ell} \sim \frac{\left(\frac{\nu}{u'\ell}\right)^{3/4}\cancel{\ell}}{\cancel{\ell}} \sim R_\ell^{-3/4}$$

$$\boxed{\frac{\eta}{\ell} \sim R_\ell^{-3/4}}$$ ☆

$R_\ell$ is typically large

$\therefore \frac{\eta}{\ell} \ll 1$

The bigger $R_\ell$ is, the bigger the gap between $\eta$ & $\ell$

b. Examples - movie making

real life

$\eta$ ~ very small compared to $\ell$

$\ell$

$\eta \ll \ell$

$R_\ell \gg 1$

scale model

$\eta$

$\ell$

$\eta < \ell$

$R_\ell$ not as large

[looks fake]

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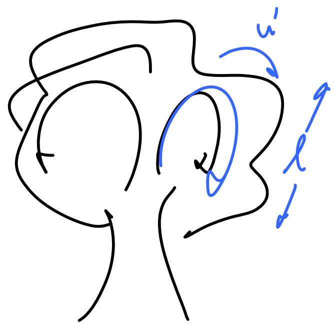
Eg. "The Day After" movie 1983

Dan Nosenchuck

real life



$u'$

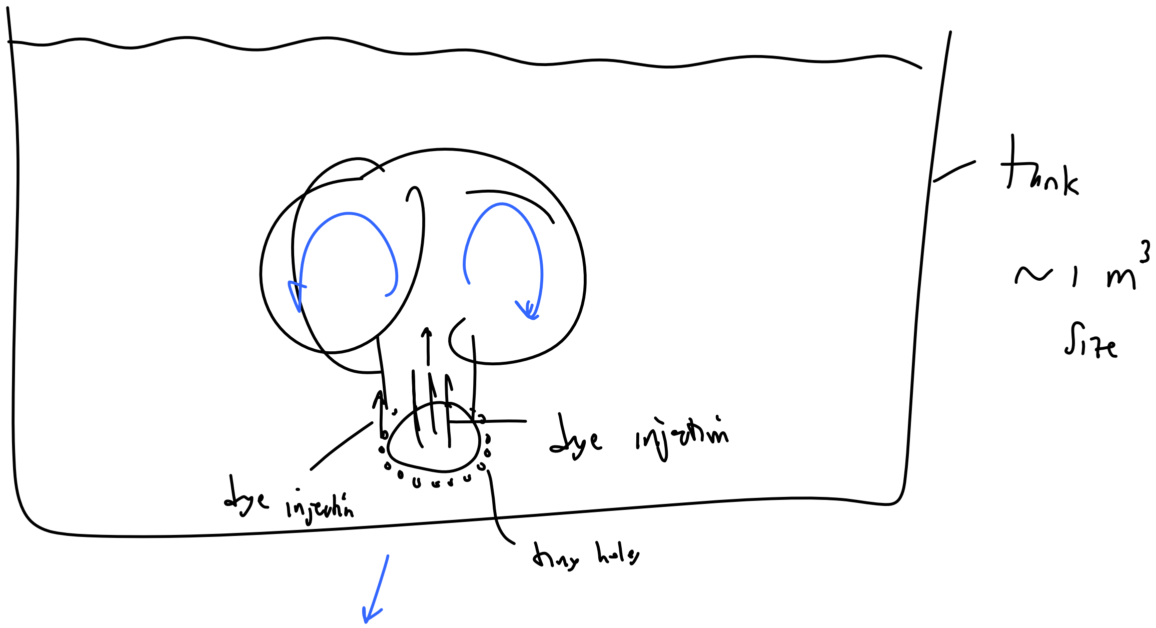
$\ell$

$$\left.\begin{aligned} u' &\sim 100 \text{ m/s} \\ \ell &\sim 1 \text{ km} \\ \nu &\sim 1.5\times10^{-5} \text{ m}^2/\text{s} \end{aligned}\right\} R_\ell \sim 7\times10^{10}$$

$$\eta/\ell \sim R_\ell^{-3/4} \sim 4\times10^{-8}$$

$\eta \sim 4\times10^{-5}$ m

0.04 mm




- Dan used tiny holes around the perimeter of the main jet to produce small-scale eddies
- The effect was fairly convincing, especially at TV resolution [it was a "made for TV" movie - filmed in video format, not movie film format]

[The resolution of standard TV is only ~500 horizontal lines]