Today, we will: • Do some more "shovel" examples.

- Discuss the question: Can turbulent flow be solved *exactly* on a computer? [direct numerical simulation]
- Start to discuss turbulence modeling
- Do Candy Questions for Candy Friday

Example

Given: A cloud

$\ell \sim 1$ km

$u' \sim 2$ m/s

@ high elevation $\nu = 8\times10^{-5}$ m²/s

To do: Estimate η (in mm)

$$Re_\ell \sim \frac{u'\ell}{\nu} \sim 2.5\times10^{7}$$

$$\frac{\eta}{\ell} \sim (Re_\ell)^{-3/4} \sim \underline{2.83\times10^{-6}} \rightarrow \eta = 2.83\times10^{-3}\ m$$

$$\boxed{\eta \sim 3\ mm}$$

OR

$$\varepsilon \sim \frac{u'^3}{\ell} \sim 8\times10^{-3}\ \frac{m^2}{s^3}$$

$$\eta \sim \left(\frac{\nu^3}{\varepsilon}\right)^{\frac{1}{4}} \sim \underline{2.83\times10^{-3}\ m}$$

Given: A volcano

Approx. steam jet

$V \sim 50$ m/s

$D \sim 300$ m

HDTV camera full 1080 resolution → 1920 × 1080 pixels

(a) Estimate the size of the smallest eddies you can resolve with this camera

Smallest resolvable eddy $\sim \left(\frac{300\ m}{480\ pixels}\right) \times \left(\frac{2\ pixels}{eddy}\right) = 1.25\ \frac{m}{eddy} \sim \boxed{1\ m}$

Nyquist criterion

(b) • Build a $\frac{1}{7500}$ scale model → $D = 4.0$ cm (0.04 m)

• Take video in a water tank, dye

$\nu = 1 \times 10^{-6}\ m^2/s$

• Use same camera & same "framing"

Calculate the minimum required V of the jet to properly "fool" the audience

Hint: use $u' \sim \frac{V}{10}$

Soln: $\ell \sim \frac{D}{2} \sim 0.02\,m$

$u' \sim \frac{V}{10} \rightarrow V = 10u'$

set η to $\left(\frac{0.04}{480\ pix}\right)\left(\frac{2\ pixel}{eddy}\right) = 0.000167\ m$

$$R_\ell \sim \frac{u'\ell}{\nu} \sim \left(\frac{\eta}{\ell}\right)^{-4/3} \qquad \left[\frac{\eta}{\ell} \sim (R_\ell)^{-3/4}\right]$$

$$u' \sim \left(\frac{\eta}{\ell}\right)^{-4/3} \frac{\nu}{\ell} \qquad \boxed{V \sim 10u' \sim 10\left(\frac{\eta}{\ell}\right)^{-4/3} \frac{\nu}{\ell}}$$

$V \sim 0.296\ m/s$

$\boxed{V \sim 0.3\ m/s}$

In practice, I would run V higher than this to be sure

6. Can turbulence be solved exactly on a computer?

DNS Direct numerical simulation

Consider incompressible, Newtonian fluid

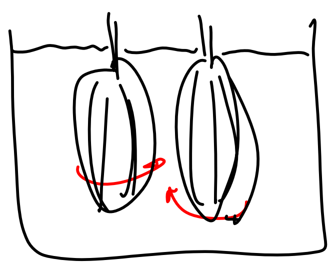
4 unknowns $\tilde{u}_i$, $\tilde{p}$

4 eqs cont + 3 mom

BC's are known

N-S computer code

Our mixer example from previously:



we had $\varepsilon = 100 \frac{W}{kg}$

$\forall = (0.1\ m)^3$ [1 Liter]

$\ell \sim 0.05\ m$

$u' \sim 2\ m/s$

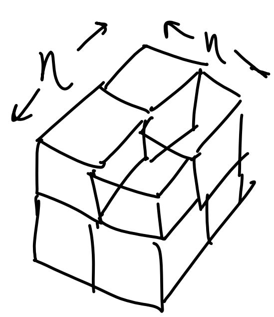
$\nu \sim 1\times10^{-6}\ m^2/s$ (water)

$\eta \sim 0.01\ mm = 1\times10^{-5}\ m$

$$R_\ell = \frac{\ell u'}{\nu} \sim \frac{(0.05\,m)(2\ m/s)}{1\times10^{-6}\ m^2/s} \sim 100{,}000$$

$$\text{So,}\quad \frac{\eta}{\ell} \sim R_\ell^{-3/4} \sim (100{,}000)^{-3/4} \sim 0.0002$$

- To resolve the smallest eddies, Nyquist criterion says we need cells smaller than $\eta/2$



need 8 cells in η^3 volume

Volume of one computational cell $\sim \left(\frac{\eta}{2}\right)^3 \sim 1 \times 10^{-16}\ m^3$

Total volume = 1 liter $\sim 0.001\ m^3$

$\therefore$ need $N = \#\text{ cells} \sim \frac{V_{total}}{V_{cell}} \sim \frac{0.001\ m^3}{1 \times 10^{-16}\ m^3} \sim \boxed{1 \times 10^{13} \text{ cells required}}$

Furthermore, @ each cell, we need to store in RAM

four variables: $\tilde{u}, \tilde{v}, \tilde{w}, \tilde{p}$

If we Real*8 arithmetic, each variable requires 8 bytes of RAM

Memory requirement $\sim \left(1 \times 10^{13}\ \cancel{\text{cells}}\right)\left(4\ \frac{\cancel{\text{variables}}}{\cancel{\text{cell}}}\right)\left(\frac{8 \text{ bytes}}{\cancel{\text{variable}}}\right) \sim \underline{\underline{3 \times 10^{14} \text{ bytes}}}$

1 Gbyte $= 10^9$ bytes $\rightarrow$ $\boxed{\text{Need } 300{,}000 \text{ Gbytes of RAM}}$!

Fastest : Biggest computers today have ~ 100,000 Gbytes → we are close

Suppose we have enough memory.

Time required

$$\text{Need} \sim \underset{\substack{\| \\ \#\text{ cells}}}{N^2}\ \frac{\text{floating point operations}}{\text{time step} \cdot \text{variable}}$$

FLOPS

Here, we need $\sim 4N^2$ FLOPS per time step

$$\sim 4\left(1 \times 10^{13}\right)^2 \sim \frac{4 \times 10^{26} \text{ FLOPS}}{\text{time step}}$$

Today's fastest computers $\sim 500 \times 10^{12}$ Flops = 500 teraflops

or 0.5 petaflops

One time step would take $\dfrac{4 \times 10^{26} \ \frac{\text{Flops}}{\text{time step}}}{500 \times 10^{12} \ \text{Flops}} \sim \dfrac{8 \times 10^{11} \ \text{s CPU time}}{\text{time step}}$

$\sim$ 20,000 yrs per time step !

How many time steps are required?

Bare minimum → for some statistics, need ~10 large eddy turnover times

$$t_{total} \sim 10 \frac{\ell}{u'} \sim 10 \frac{0.05 \text{ m}}{2 \text{ m/s}} \sim \boxed{0.2 \text{ s}} \quad \text{(physical time)}$$

How much is one time step? → what is required Δt ?

$$\Delta t \sim \underset{\substack{\uparrow \\ \text{Nyquist}}}{\tfrac{1}{2}} \tau$$

turnover time for Kolmogorov eddy

$$\tau \sim \left(\frac{\nu}{\varepsilon}\right)^{1/2} \sim 0.0001 \text{ s}$$

so $\boxed{\Delta t \sim 5 \times 10^{-5} \text{ s}}$

$$\text{Total \# time steps} = \frac{t_{total}}{\Delta t} \sim \frac{0.2 \text{ s}}{5 \times 10^{-5} \text{ s}} \sim 4000 \text{ time steps}$$

TOTAL CPU TIME ON THE WORLD'S BIGGEST & FASTEST COMPUTER

$$\sim (4000 \text{ time steps})\left(\frac{20000 \text{ yr}}{\text{time step}}\right) = 80 \text{ million yrs !}$$

Conclude: DNS will "never" be a practical Engineering tool

→ We must come up with some models for turbulence

Next week → we will start to discuss

TURBULENCE MODELS