Today, we will:

- Discuss turbulence models – why we need them, desired attributes, classification of turbulence models
- Discuss the eddy viscosity and Prandtl's mixing length theory

E. TURBULENCE MODELS

1. Intro

a. Why do we need a turbulence model?

- $\eta << \ell$ ⇒ too many grid pts. ∴ too much computer time to solve DNS (exactly) for engr. problems
- closure problem ⇒ can't even solve the mean flow because of extra Reynolds stress terms

Reynolds stress tensor $-\rho_0 \overline{u_i u_j}$ = 6 extra independent unknowns

⇓

Goal of a turbulence model is to model the Reynolds stress

b. Eqs of motion:

Simplest case – incompressible flow $\rho = \rho_0 = \text{const}$

No buoyancy

No energy eq. to worry about

Use Ensemble-Averaged Reynolds Eqs

- Mean cont. eq. $$\boxed{\frac{\partial U_i}{\partial x_i} = 0} \qquad (1\text{ m})$$

- Mean mom. eq $$\boxed{\frac{DU_i}{Dt} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{1}{\rho}\frac{\partial}{\partial x_j}\left[\mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j}\right]} \qquad (2\text{ m})$$

☆

· Also have Eqs for turbulent fluctuations — see previous handouts & notes

- · Cont eq. for the turbulent fluctuations
- · Mom " "

can manipulate to get · tke eq.

· Reynolds stress transport eq. — but, contains even more new unknowns!

MUST GENERATE SOME KIND OF MODEL FOR THESE TERMS & Reynolds stress terms, etc.

C. Desired Attributes of a Turbulence Model

(1) Physically based — should justify the model by physical arguments [although the shovel comes in handy]

(2) Simple as possible — "economy of computer time"
- eqs can get "stiff"

(3) Robust → should not cause computer to "bomb" or "crash"
e.g. → avoid infinite loops, $\sqrt{-}$, etc.

(4) "Realizable" → model should not be capable of generating impossible (non-realizable) values.

e.g., $\overline{u^2} \geq 0$ → model should not allow $\overline{u^2} < 0$

$\overline{u^2}\,\overline{v^2} \geq (\overline{uv})^2$ → " " " ... $(\overline{uv})^2 < \overline{u^2}\,\overline{v^2}$

(5) Universal → should work for all kinds of flows!

This is the killer

- · There are constants in any turb. model
- · constants are fitted to experiments
- · But - same constants may not work in other flows

d. Classification of Turbulence Models

- We have 4 Eqs & 4 primary unknowns (U_i, P)

- $\boxed{\text{Turbulence models are classified according to the number of additional transport eqs. that we use to "close" the problem mathematically}}$

transport eq. → $\boxed{\dfrac{D}{Dt}(\sim) = \sim\sim\sim}$

(1) Zero-equation turbulence models – no new transport eqs

- Model Reynolds stress terms as algebraic functions of the mean flow (very simple closure)

but – limited applications – usually tied into certain types of flows

– Solve for U, V, W, P (4 unknowns)

(2) One-equation turbulence models – add one new transport eq.

typically for q^2 or K (i.e., tke)

↓

solve one additional transport eq. for K

$$\frac{D}{Dt}(K) = \sim\sim\sim$$

Reynolds stress terms = fncs (U, V, W, P, K)

– Solve for U, V, W, P, K 5 eqs – 5 unknowns

(3) Two-eq. turb. models – Add 2 additional transport eqs.

5 common ones:

(a) K (or q^2) (tke) & ℓ length scale of the turb. eddies

transport eq for ℓ

(b) K & $\overline{\omega_i \omega_i}$ = enstrophy ω_i = fluctuating vorticity

(c) K & ε – most popular

☆

2 transp. eqs · for K $\frac{DK}{Dt} = \qquad - \varepsilon$

· for ε $\frac{D\varepsilon}{Dt} =$

"K-ε model"

There are constants in these modeled eq's – <u>not universal</u>

(d) K-ω model → define ω = specific dissipation rate

$$\boxed{\omega = \frac{\varepsilon}{\beta^* K}}$$

β^* = const = 0.09

$\frac{D\omega}{Dt} =$

(e) q-ω model use $\boxed{q = \sqrt{K} = \sqrt{q^2}}$

& use $\boxed{\omega = \frac{\varepsilon}{K}}$ (no constant)

$\frac{Dq}{Dt} = \qquad + \qquad \frac{D\omega}{Dt} =$

(3.5) <u>ASM</u> → <u>Algebraic stress models</u>

2 transport eqs – typically K & ε

<u>Plus</u> – <u>Algebraic</u> eqs for the components of Reynolds stress – allows for anisotropy

(4) Reynolds Stress Models (RSM)

- Write 6 additional transport eqs for the 6 indep. R.S. components

$$\frac{D}{Dt}(\overline{u^2}) = \sim\sim\sim$$

$$\frac{D}{Dt}(\overline{uv}) = \sim\sim\sim$$

$$\frac{D}{Dt}(\overline{vw}) = \sim\sim\sim$$

etc.

- The RHS's also contain terms with ε

So, add $\frac{D\varepsilon}{Dt} = \sim\sim\sim$

$\boxed{\text{7-eq. turbulence model}}$

compared to laminar flow 4 eq, 4 unkn.

here 4+7= 11 eq, 11 unkn.

"Second-order closure"

or "Second-moment closure"

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(5) Large Eddy Simulation (LES)

Physical argument: – large-scale eddies are problem-dependent & non-isotropic

– small-scale eddies are problem-independent & nearly isotropic

So, make grid fine enough to resolve the large & intermediate scale eddies

but models the small-scale eddies → $\boxed{\text{"sub-grid scale modeling"}}$ ☆

Example Application – Atmospheric Turbulence Research — LES is used a lot