Today, we will:

- Discuss the Baldwin-Lomax turbulence model (a popular zero-eq. turbulence model)
- Discuss "half-eq." turbulence models
- Discuss one-eq. turbulence models, and how to model the tke equation
- Do Candy Questions for Candy Friday

Zero-eq. models (cont'd)

(4) The Baldwin-Lomax model

Valid for BL (wall-bounded) flows only

If we vortex to get the eddy viscosity, $v_{eddy}$, near

Two-layer model

Inners: $M_e = \rho \frac{u^2}{\nu} \left| \bar{\omega} \right|

Outer: $M_e \sim \text{complex form}

C. Summary of two-eq. eddy-viscosity - mixing length turbulence models

Advantages

1) simple - no "extra" transport eqs

2) we can calculate $\nu_m$ easily from experiments:

3) works quite well for turbulent BLs

but not with strong adverse pressure gradients
Quasi-one 1) Valid only for BL's i.e. shearing [not for 3D flow]

2) \(- \overline{uv} = 0 \) when \( \frac{\partial U}{\partial y} = 0 \) Not physical!

\[ \text{e.g. wall jet} \]

\[ \text{But, } \overline{uv} \neq 0 \text{ from experiment} \]

3) It is only a local model - \( \overline{uv} \) determined by

\[ \text{local velocity flow only.} \]

Not physical since turbulence has a “memory” of

upstream effects

4) Not universal!

How do we get around the “memory” problem?

\[ \text{Add } \text{we another transport eq. for a turbulence quantity} \]

\[ \text{\downarrow} \]

\[ \text{one-eg. turbulence models} \]

3a. Closure for “half-eq. models”

\[ \text{e.g. Johnson & King} \rightarrow 2-layer model for BL flow} \]

\[ \text{Me = fre. of } \eta, y, Z_{\text{max}} \text{ + some control} \]

\[ Z_{\text{max}} \text{ = maximum Reynolds}\text{ or } Z_{\text{max}} \text{ in the BL} \]

\[ \text{Solve an ODE for } Z_{\text{max}} \text{ - not a transport eq.} \]
4. Closure for one-\(e\) turbulence models

a. Intro

- add one additional transport eq. to our primary set of eqs

- It is still an eddy viscosity model

\[
\overline{-\rho \nu_{ij}'} = -\frac{2}{3} K \delta_{ij} + 2 \nu_e E_{ij} \quad \text{[Bowin\text{\textregistered}]
}

- Still we \( \nu_e = \rho \cdot \text{const.}\ \nu_m' \)

\[
\nu_e = C_p \nu_m' \quad \text{[production constant]}
\]

kneanie eddy viscosity, \( \nu_e = C_p \nu_m' \)

We still need to specify \( \nu_m \) (algebraically)

- But, for \( u' \) we use the \( \text{the eq.} \) \[
\nu_e = C_p \delta K \nu_m
\]

Recall \( \text{the} = K = \nu^2 = \frac{1}{2} \overline{u_i u_i} \sim (u')^2 \)

So let \( \overline{u'} = \delta K \Rightarrow \quad \text{\( \nu_e = C_p \delta K \nu_m \) }

One-\(e\) models are often called "mixing length - the model"

[or \( \nu_m - q \) or \( \nu_m - K \) models]

\( \text{Don't confuse with 2-\(e\) models like K-E, K-W} \)
"One -q" = the eq. , but it must be modeled

Exact the eq. (w/o buoyancy)

\[ \frac{dK}{dt} + U_j \frac{dK}{dx_j} = \frac{2}{dx_j} \left( \frac{1}{\rho} \frac{\partial p}{\partial y_j} - \frac{1}{2} \frac{\partial u_i^2 y_j}{\partial y_j} + 2 \nu \frac{\partial u_i}{\partial y_j} \right) - \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} - E \]

Total rate of change of \( K \)
Transport or diffusion of \( K \)

But to mathematically close the problem, we need to deal with these extra terms, \( u_i u_j, u_i u_j, u_i u_j, E \)

We must model these!

First, compute o.m. of term \( \text{IV} \), \( \text{VII} \):

\[ \frac{2}{dx_j} \left( \nu \frac{\partial u_i}{\partial y_j} \right) \]

\[ \frac{1}{\nu} \frac{\partial u_i}{\partial x_j} \left( \frac{u_i^3}{\nu} \right) \]

\[ \frac{1}{\nu} \frac{\partial u_i}{\partial x_j} \left( \frac{u_i^2}{\nu} \right) \]

\[ \frac{1}{\nu} \frac{\partial u_i}{\partial x_j} \left( \frac{u_i^2}{\nu} \right) \]

(Re for large turb. shear & u_i big, Re >> 1)

\( \therefore \) Term \( \text{IV} \) is negligible except very close to the wall

The:

\[ \frac{dK}{dt} + U_j \frac{dK}{dx_j} = -\frac{2}{dx_j} \left( \frac{1}{\rho} \frac{\partial p}{\partial y_j} + \frac{1}{2} \frac{\partial u_i^2 y_j}{\partial y_j} \right) - \nu \frac{\partial u_i}{\partial x_j} \frac{\partial u_i}{\partial x_j} - E \]

LHS does not need modeled
Distribution or transport
Production term
Away from wall.
\[ \text{Product term: } -u_i v_i \frac{du_i}{dx_j} \]

\[ \text{we bound } u_i v_i = \nu_e \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \frac{2}{3} \nu K f_{ij} \]

\[ \text{Prod. term } = \nu_e \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{du_i}{dx_j} + \frac{2}{3} \nu K f_{ij} \frac{du_i}{dx_j} \]

\[ \frac{2 u_i}{dx_i} > O \text{ (cont)} \]

**Diffusion term**

\[ \text{We use Gradient Diffusion Model (a cornerstone of turbulence modeling)} \]

\[ \frac{2}{dx_j} \]

In 2D flow, recall x-v_m BL eq.

\[ U \frac{dU}{dx} + V \frac{dU}{dy} = \frac{1}{\rho} \frac{dP}{dx} + V \frac{d^2 U}{dy^2} + \frac{2}{\gamma} (-\overline{wv}) \]

Model:\n
\[ \frac{2}{dy} \left( \nu_e \frac{dU}{dy} \right) \]

\[ \text{We derive } U \text{ and } \frac{dU}{dy}, \text{ to show that this term is proportional to the gradient of } U, \frac{dU}{dy} \]
Extend this to any turbulence property $\Phi$. $\Phi$ can be $K$, $\varepsilon$, $w_j$, etc.

Transport of $\Phi$:

$$\frac{D \Phi}{Dt} = \nabla \cdot (\text{some ugly term}) + \ldots$$

Transport or diffusion

Gradient Diffusion Model

$$\frac{\partial}{\partial x_j} \left( \nu \frac{\partial \Phi}{\partial x_j} \right)$$

$\nu$ = turbulent transport coeff. for $\Phi$

Here, for the eq., let $\Phi = K$

$\implies$ we model

$$\frac{\partial}{\partial x_j} \left( \text{some ugly term} \right) = \frac{\partial}{\partial x_j} \left( \nu_k \frac{\partial K}{\partial x_j} \right)$$

Terms II+III

$\nu_k$ = turbulent transport coeff. for t.k.e. ($K$)

Let $\nu_k = \frac{\nu_e}{\nu_k}$ = ratio of eddy viscosity to the transport coeff.

$\nu_e$ = eddy viscosity

Physicist $\rightarrow$ $\chi$

Argue that the large eddies transport momentum in the same way as they transport the (or any other $\Phi$)

Expect: that $\nu_k = 1$