Today, we will:

- Discuss the Baldwin-Lomax turbulence model (a popular zero-eq. turbulence model)
- Discuss "half-eq." turbulence models
- Discuss one-eq. turbulence models, and how to model the tke equation
- Do Candy Questions for Candy Friday

Zero-eq. models (continued)

(4) The Baldwin-Lomax model

- Valid for BL (wall-bounded) flows only
- It uses vorticity to get the eddy viscosity

Two-layer model

Inner: $\mu_e = \rho l_m^2 |\vec{\omega}|$ mag. of vorticity vector

$\therefore l_m = K y \left[1 - e^{-y^+/A_o^+}\right]$ constants

$$\boxed{y^+ = \frac{y u^*}{\nu}}$$

[u^* = friction velocity]

Outer: $\mu_e = \sim$ complicated fn.

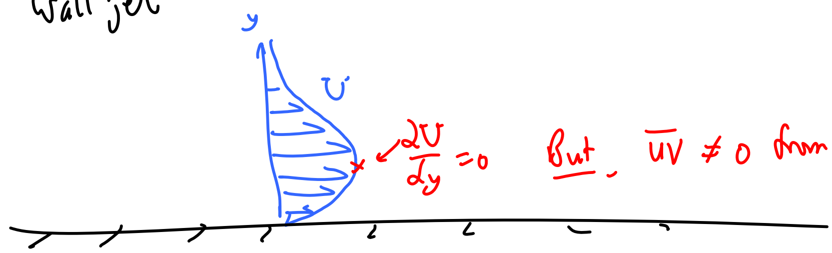
C. Summary of zero-eq. eddy-viscosity - mixing length turbulence models

- Advantages
  1) Simple – no "extra" transport eqs
     ↓ fast on a computer
  2) we can calculate l_m easily from experiments:
     use $\boxed{-\overline{uv} = l_m^2 \left|\frac{\partial U}{\partial y}\right| \frac{\partial U}{\partial y}}$
  3) Works quite well for turb. BL's
     but not with strong adverse pressure gradients

Disadvantages 1) Valid only for BL's & shear flows [not for 3-D flows]

2) $-\rho\overline{uv} = 0$ when $\frac{\partial U}{\partial y} = 0$ Not physical!

e.g. wall jet



But, $\overline{uv} \neq 0$ from experiments

3) It is only a local model → $\overline{uv}$ determined by local velocity values only.

↓

not physical since turbulence has a "memory" of upstream effects

☆ 4) Not universal!

How do we get around the "memory" problem?

Ans → Use another transport eq. for a turbulence quantity

⇓

one-eq. turbulence models

3a. Closure for "half-eq. models"

e.g. Johnson & King → 2-layer model for BL flows

$$\mu_t = \text{fnc. of } l_m,\ y,\ \tau_{max} + \text{some constants}$$

τ_{max} = maximum Reynolds stress τ_{xy} in the BL

↓

Solve an ODE for τ_{max} — not a transport eq.

4. Closure for one-eq. turbulence models

a. Intro

- add one additional transport eq. to our primary set of eqs
- It is still an eddy viscosity model

$$\boxed{-\rho\overline{u_i u_j} \approx -\frac{2}{3}K\delta_{ij} + 2\mu_e E_{ij}} \quad [\text{Boussinesq}]$$

i. still use $\mu_e = \rho \cdot \text{const} \cdot l_m u'$

$\text{const} = C_p \longrightarrow$ "production constant"

Kinematic eddy viscosity, $\boxed{\nu_e = C_p l_m u'}$

This is still a mixing-length model

We still need to specify l_m (algebraically)

- But, for u' we use the tke eq. [our additional transport eq.]

Recall $\text{tke} = K = q^2 = \frac{1}{2}\overline{u_i u_i} \sim (u')^2$

so Let $\boxed{u' = \sqrt{K}} \Rightarrow \therefore \boxed{\nu_e = C_p \sqrt{K}\, l_m}$

One-eq models are often called "mixing length - tke models"

[or l_m-q or l_m-K models]

don't confuse with 2-eq. models like K-ε, K-ω

"One-eq" = the eq., but it must be modded

Exact the eq. (w/o buoyancy)

$$\boxed{\underbrace{\frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j}}_{\text{I}} = \frac{\partial}{\partial x_j}\left(\underbrace{-\frac{1}{\rho}\overline{p u_j}}_{\text{II}} \underbrace{- \frac{1}{2}\overline{u_i^2 u_j}}_{\text{III}} \underbrace{\boxed{+ 2\nu \overline{u_i e_{ij}}}}_{\text{IV}}\right) \underbrace{- \overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{\text{V}} \underbrace{\boxed{- \varepsilon}}_{\text{VII}}}$$

I: total rate of change of K

II–IV: transport or diffusion of K

V: Production

VII: Dissipation

BUT to mathematically close the problem, we need to deal with these extra terms, $\overline{p u_j}$, $\overline{u_i^2 u_j}$, $\overline{u_i e_{ij}}$, ε

We must model these!

First, compare o.o.m. of terms IV & VII:

$$\frac{\partial}{\partial x_j}\left(\nu \overline{u_i e_{ij}}\right) \quad | \quad \varepsilon$$

$$\frac{1}{\ell}\nu\frac{u'u'}{\ell} \quad | \quad \frac{u'^3}{\ell}$$

$$\frac{\nu}{u'\ell}\frac{u'^3}{\ell} \quad | \quad \frac{u'^3}{\ell}$$

$\frac{\nu}{u'\ell} = \frac{1}{Re}$ for large turb. eddies is v. big $Re >> 1$

$\therefore$ Term IV is negligible except very close to the wall

the:

$$\boxed{\underbrace{\frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j}}_{\text{LHS - does not need modeled}} = \underbrace{-\frac{\partial}{\partial x_j}\left(\frac{1}{\rho}\overline{p u_j} + \frac{1}{2}\overline{u_i^2 u_j}\right)}_{\text{Diffusion or transport}} \underbrace{- \overline{u_i u_j}\frac{\partial U_i}{\partial x_j}}_{\text{Production term}} - \varepsilon}$$

away from walls

Production term

$$-\overline{u_i u_j}\,\frac{\partial U_i}{\partial x_j}$$

use Boussinesq eq. $\quad -\overline{u_i u_j} = \nu_e\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right) - \frac{2}{3}K\delta_{ij}$

$$\text{Prod.} = \nu_e\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j} + -\frac{2}{3}K\underbrace{\delta_{ij}\frac{\partial U_i}{\partial x_j}}_{\cancel{\frac{\partial U_i}{\partial x_i}} \to 0 \text{ (cont)}}$$

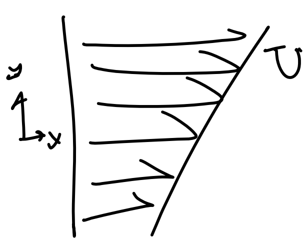
$$\boxed{\text{Prod. term} = \nu_e\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}$$

Diffusion terms

Use ☆ Gradient Diffusion Model (a cornerstone of turb. modeling)

$$\frac{\partial}{\partial x_j}\left(\sim\sim\sim \right)$$

In 2-D flow, recall x-mom BL eq.



$$U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 U}{\partial y^2} + \frac{\partial}{\partial y}\left(-\overline{uv}\right)$$

$\|$

Model as

$$\boxed{\frac{\partial}{\partial y}\left(\nu_e\frac{\partial U}{\partial y}\right)}$$

We diffuse U ; argue that this term is proportional to the gradient of U, $\frac{\partial U}{\partial y}$

Extend this to any turbulence property Φ Φ can be K, ε, $\overline{u_i u_j}$, etc.

transport eq. for Φ

$$\frac{D\Phi}{Dt} = \sim\sim\sim + \underbrace{\frac{\partial}{\partial x_j}\left(\text{some ugly terms}\right)}_{\text{transport or diffusion}} + \sim\sim$$

Gradient Diffusion Model

$$\boxed{\frac{\partial}{\partial x_j}\left(\nu_\Phi \frac{\partial \Phi}{\partial x_j}\right)}$$

ν_Φ = turbulent transport coeff. for Φ

Here, for the eq., let $\Phi = K$

$\therefore$ we model $\boxed{\underbrace{\frac{\partial}{\partial x_j}\left(\sim\sim\right)}_{\text{TERMS II + III}} \simeq \frac{\partial}{\partial x_j}\left(\nu_K \frac{\partial K}{\partial x_j}\right)}$

$\boxed{\nu_K = \text{turbulent transport coeff. for t.k.e. } (K)}$

let $\boxed{\sigma_K \equiv \frac{\nu_e}{\nu_K}}$ = ratio of eddy viscosity to the transport coeffs.

Physics $\rightarrow$

Argue that the large eddies transport momentum in the same way as they transport the (or any other Φ)

$\boxed{\text{Expect } \therefore \text{ that } \sigma_K = 1}$