M E 522 Spring 2008 Professor John M. Cimbala Lecture 37 04/16/2008

Today, we will:

- Discuss low-Reynolds number closure for two-equation turbulence models
- Begin to discuss Reynolds stress turbulence models – no eddy viscosity!

K-ε model (continued)

d. The near-wall region (low Reynolds number closure)

The "high Re" forms are not valid as we approach a wall

Why not?

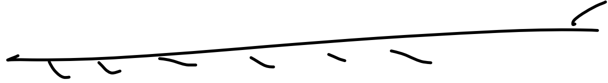
1) We neglected viscous or molecular transport terms

ν becomes important near a wall

ν " dominant over ν_e near a wall

2) The constants are calibrated for turbulence far from a wall

3) Turbulence is damped as you approach the wall



@ wall $u=v=w=0$ (no slip)

fluctuating velocity component

$\tilde{u} = \tilde{v} = \tilde{w} = 0$

But, ε does not necessarily go to zero → must treat ε separately

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What do we do?

(1) Use "wall functions" — [Fluent's default]

Assumes the mean velocity obeys the log law near the wall

[will discuss later]

(2) Use "damping functions"

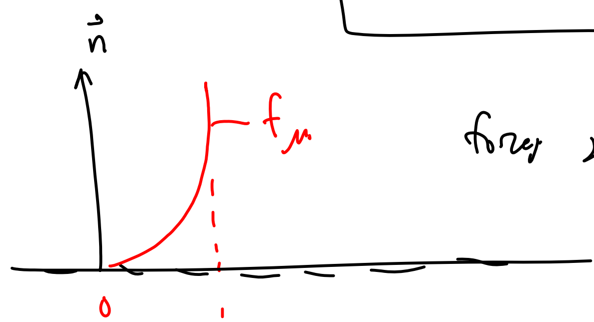
Modifies the turbulence model itself so that turbulence quantities are "damped" as we approach the wall.

Terminology → "near wall modeling"
"near wall closure"
"Low Reynolds number modeling"

e.g., recall $\boxed{\nu_e = C_\mu \dfrac{K^2}{\varepsilon}}$ (high Re model)

low Re model, let $\boxed{\nu_e = C_\mu f_\mu \dfrac{K^2}{\varepsilon}}$ f_μ = <u>damping function</u>

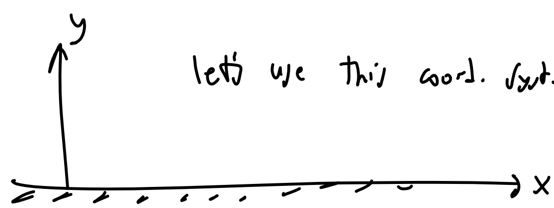
typically



forces $\nu_e \to 0$ as we approach the wall

Wall BCs for K & ε (K-ε model) [we are using damping functions not wall functions]

At the wall, $K = 0$
what about ε?



let's use this coord. syst.

- Use Wilcox's form of the K & ε eqs. (handout)

$$\frac{DK}{Dt} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_e}{\sigma_K}\right)\frac{\partial K}{\partial x_j}\right] + \underbrace{\nu_e\left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i}\right)\frac{\partial U_i}{\partial x_j}}_{\text{production}} - \underbrace{\varepsilon}_{\text{dissipation}} \qquad (*)$$

molecular diffusion (ν), turbulent diffusion ($\frac{\nu_e}{\sigma_K}$)

- At the wall, $K = 0$ → LHS goes to zero

recall, $\nu_e = C_\mu f_\mu \dfrac{K^2}{\varepsilon} \to 0$ ε does <u>not</u> go to zero but K does

$\therefore \left.\dfrac{\partial K}{\partial z}\right)_{wall} = \left.\dfrac{\partial K}{\partial x}\right)_{wall}$ also must → 0 ⟶ $\dfrac{\partial K}{\partial y} \neq 0$

At the wall, (*) reduces to $\boxed{\dfrac{d}{dy}\left(\nu \dfrac{dK}{dy}\right) - \varepsilon = 0}$

So, $\varepsilon = \dfrac{d}{dy}\left(\nu \dfrac{dK}{dy}\right)$ at wall

so we get $\boxed{\varepsilon_{wall} = \nu \left.\dfrac{d^2K}{dy^2}\right)_{y=0}}$

How to apply this BC in a CFD code

$$\varepsilon_{wall} = \nu \frac{d^2K}{dy^2} \quad \text{at } y=0$$

$$= \nu \frac{d}{dy}\left(\frac{d}{dy}(\sqrt{K})^2\right)$$

Differentiate by parts:

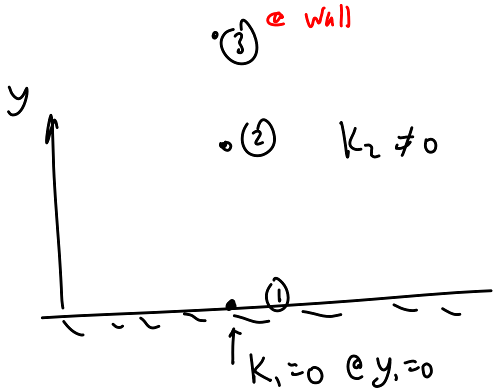
$$= \nu \frac{d}{dy}\left(\sqrt{K}\frac{d\sqrt{K}}{dy} + \sqrt{K}\frac{d\sqrt{K}}{dy}\right) = 2\nu \frac{d}{dy}\left(\sqrt{K}\frac{d\sqrt{K}}{dy}\right)$$

Differentiate again

$$= 2\nu\left[\cancel{\sqrt{K}}\frac{d^2\sqrt{K}}{dy^2} + \frac{d\sqrt{K}}{dy}\frac{d\sqrt{K}}{dy}\right]$$

But K=0 @ wall

$$\boxed{\varepsilon_{wall} = 2\nu\left(\frac{d\sqrt{K}}{dy}\right)^2}$$

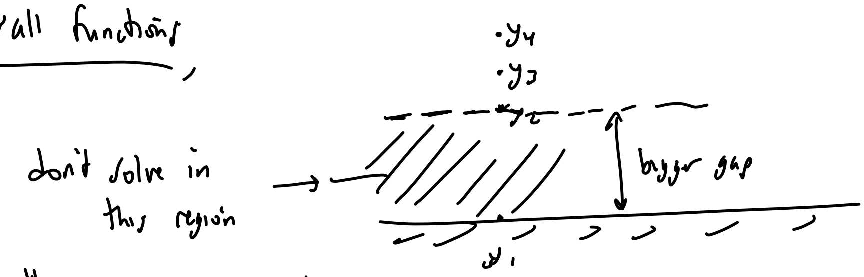


Numerically, $\dfrac{d\sqrt{K}}{dy} \simeq \dfrac{\sqrt{K_2} - \cancel{\sqrt{K_1}}}{y_2 - \cancel{y_1}} = \dfrac{\sqrt{K_2}}{y_2}$

So, $\varepsilon_{wall} = 2\nu\left(\dfrac{d\sqrt{K}}{dy}\right)^2 = 2\nu\left(\dfrac{\sqrt{K_2}}{y_2}\right)^2 \Rightarrow \boxed{\varepsilon_{wall} = 2\nu \dfrac{K_2}{y_2^2}}$

"Integrating to the wall" $\rightarrow$ $\therefore$ need BC for ε @ wall.

If we wall functions,



We don't "integrate to the wall"

Instead, we assume that we know the shape of the BL near the wall (log-law)

$\therefore$ Do not need BCs for K & ε at the wall!

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6. Closure for Reynolds Stress Turbulence Models

a. Intro $\rightarrow$ So far all models we've discussed (zero-, one-, & two-eq. models)

use the eddy viscosity approximation

$\rightarrow$ model $-\rho\overline{u_i u_j}$ by some eddy viscosity x gradient of mean velocity (see Eq. (3) of handout)

- Full RSM does not use an eddy viscosity ✱

Instead - write transport eqs for the Reynolds stress tensor itself!

b. Equations (see pdf file on website)

Comments:

- $\tau_{ij_{turb}}$ has 9 components, but only 6 indep. components

so need 6 additional transport eqs

- Term VI → viscous dissipation term $\boxed{\mathcal{E}_{ij} = 2\mu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_j}{\partial x_k}}}$

Isotropic assumption is usually employed

$$\mathcal{E}_{ij} = \begin{bmatrix} \mathcal{E}_{11} & 0 & 0 \\ 0 & \mathcal{E}_{22} & 0 \\ 0 & 0 & \mathcal{E}_{33} \end{bmatrix} = \frac{2}{3}\rho\mathcal{E}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This means that $\overline{\frac{\partial u_i}{\partial x_j}\frac{\partial u_j}{\partial x_i}} = 0$ when $i \neq j$

[this is not necessarily the case!]

$\boxed{\mathcal{E} = \nu \overline{\frac{\partial u_i}{\partial x_k}\frac{\partial u_i}{\partial x_k}}}$ is the isotropic dissipation rate

↓

[not the same as Kundu's $\mathcal{E} = 2\nu\overline{e_{ij}e_{ij}}$]

- Anisotropic Dissipation Rate Models (ADRM)

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typically use algebraic expressions to yield different values for each component of $\mathcal{E}_{ij}$