M E 522 Spring 2008 Professor John M. Cimbala Lecture 38 04/18/2008

Today, we will:

- Finish discussing Reynolds stress turbulence models
- Look at the summary chart for turbulence models
- Start solving some turbulent flow problems! – Start with channel flow.
- Do Candy Questions for Candy Friday

RSM (comments)

· K appears in several terms $-\rho\overline{u_i u_j} = \tau_{ij\,turb}$

$K = \frac{1}{2}\overline{u_i u_i} \rightarrow \boxed{K = \frac{-1}{2\rho}\tau_{ii\,turb}}$ (Trace)

Fluent → 2-D RSM → Solves P, U, V + $\overline{uu}, \overline{vv}, \overline{ww}, \overline{uv}, \varepsilon, K$

3-D P, U, V, W + $\overline{uu}\ \overline{vv}\ \overline{ww}\ \overline{uv}\ \overline{vw}\ \overline{uw}, \varepsilon, K$

ε eq. → Not same as ε eq in K-ε model

RSM - no eddy viscosity

eddy viscosity appears in the eq.

C. SUMMARY OF RSM

Advant: · Generally perform better than 2-eq. models: (not always)

· more physics

· transport eqs for all 6 components of R.S.

· especially good for flows with large anisotropy e.g. curved ducts, cyclones, separated flows

rotating flows, flows with secondary flows

<u>Drawbacks</u>:

RSM 4 eqs + 7 new transport eqs [8 in Fluent] } 11 trans. eq. [12 in Fluent]

Compared to

k-ε 4 eqs + 2 new ones } 6 transport eq. [6 in Fluent]

- Eqs get <u>stiffer</u> — converges slower

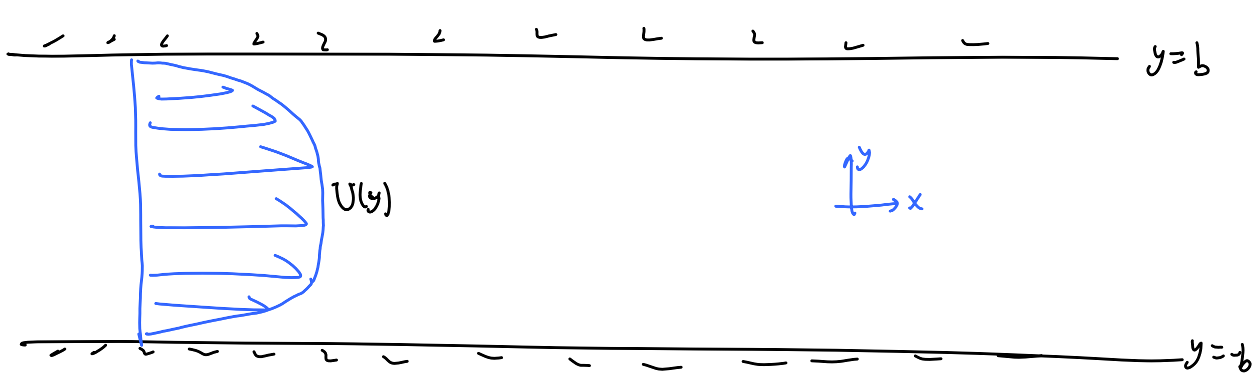
- <u>STILL NOT UNIVERSAL</u>

Look at the <u>summary chart</u>

F. <u>TURBULENT FLOW SOLUTIONS</u>

1. <u>Turbulent Channel Flow</u> (Fully developed)

a. <u>Analytical analysis</u> (as far as we can go)



Approx / Assumptions

(1) infinite, parallel walls

(2) 2-D $\quad \frac{\partial}{\partial z}\left(\overline{\ \ }\right) = 0$

(3) Incompressible & neglect gravity

(4) Fully developed $\rightarrow \frac{\partial}{\partial x}\left(\overline{\ \ }\right) = 0$

except $\frac{\partial P}{\partial x} \neq 0$ (Pressure drives the flow)

(5) stationary $\quad \frac{\partial}{\partial t}\left(\overline{\ \ }\right) = 0$

(6) parallel flow $\rightarrow \quad V = W = 0 \quad [U \neq 0]$

Mean Eqs:

Cont:
$$\frac{\partial U_i}{\partial x_i} = 0 \quad \rightarrow \quad \underbrace{\cancel{\frac{\partial U}{\partial x}}}_{(4)} + \underbrace{\cancel{\frac{\partial V}{\partial y}}}_{(6)} + \underbrace{\cancel{\frac{\partial W}{\partial z}}}_{(6)} = 0$$

Mom:
$$\rho\left[\underbrace{\cancel{\frac{\partial U_i}{\partial t}}}_{(5)} + U_j \frac{\partial U_i}{\partial x_j}\right] = -\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left[\mu \frac{\partial U_i}{\partial x_j} - \rho \overline{u_i u_j}\right]$$

3 components:

X-mom:
$$\rho U \underbrace{\cancel{\frac{\partial U}{\partial x}}}_{(4)} + \underbrace{\cancel{\rho V}\frac{\partial U}{\partial y}}_{(6)} + \underbrace{\rho \cancel{W} \frac{\partial U}{\partial z}}_{(6)} = \underbrace{-\frac{\partial P}{\partial x}}_{\text{keep}} + \underbrace{\cancel{\frac{\partial}{\partial x}\left[\mu \frac{\partial U}{\partial x} - \rho \overline{u^2}\right]}}_{(4)}$$
$$+ \underbrace{\frac{\partial}{\partial y}\left[\mu \frac{\partial U}{\partial y} - \rho \overline{uv}\right]}_{\text{keep}} + \underbrace{\cancel{\frac{\partial}{\partial z}}\left[\mu \frac{\partial U}{\partial z} - \rho \overline{uw}\right]}_{(2)}$$

$\overline{u^2} \neq 0$

$$\boxed{0 = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left[\mu \frac{\partial U}{\partial y} - \rho \overline{uv}\right]} \qquad (1)$$

Use Boussinesq eddy viscosity model to estimate (approximate) $\overline{vw}$ for this flow

$$\tau_{ij_{turb}} = -\rho\overline{u_i u_j} = -\frac{2}{3}\rho k\,\delta_{ij} + 2\mu_e E_{ij}$$

for $\overline{vw}$ $\quad i=2, \; j=3 \quad \rightarrow \; -\frac{2}{3}\rho k\,\cancel{\delta_{23}}_{\,0} + 2\mu_e\,\frac{1}{2}\left(\frac{\partial U_2}{\partial x_3} + \frac{\partial U_3}{\partial x_2}\right)$

$$\frac{\partial U_2}{\partial x_3} \rightarrow \cancel{\frac{\partial V}{\partial z}} \qquad \frac{\partial U_3}{\partial x_2} \rightarrow \cancel{\frac{\partial W}{\partial y}}$$

parallel flow

$\therefore$ $\boxed{\overline{vw} = 0 \text{ in this flow}}$ (if Boussinesq is applied)

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y-mom:

$$\rho\cancel{\frac{DV}{Dt}} = -\frac{\partial P}{\partial y} + \cancel{\frac{\partial}{\partial x}}[\;] + \frac{\partial}{\partial y}\left[\mu\cancel{\frac{\partial V}{\partial y}} - \rho\overline{v^2}\right] + \cancel{\frac{\partial}{\partial z}}[\;]$$

(6) $\quad$ fully dev. $\quad$ (6) $\quad$ (2)

$$\boxed{\frac{\partial P}{\partial y} = -\rho\frac{d}{dy}\left(\overline{v^2}\right)} \qquad (2)$$

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z-mom:

$$\cancel{\frac{DW}{Dt}} = \cancel{-\frac{\partial P}{\partial z}} + \cancel{\frac{\partial}{\partial x}[\;]} + \frac{\partial}{\partial y}\left[\cancel{\mu\frac{\partial W}{\partial y}} - \rho\overline{vw}\right] + \cancel{\frac{\partial}{\partial z}}[\;]$$

$$0 = \frac{d}{dy}(\overline{vw}) \longrightarrow \text{integrate: } \overline{vw} = \text{constant everywhere}$$

@ wall, $\overline{vw} = 0$ $\therefore$ const $= 0$ $\quad\therefore\quad$ $\boxed{\overline{vw} = 0 \text{ everywhere}}$

{Agrees with Boussinesq.}

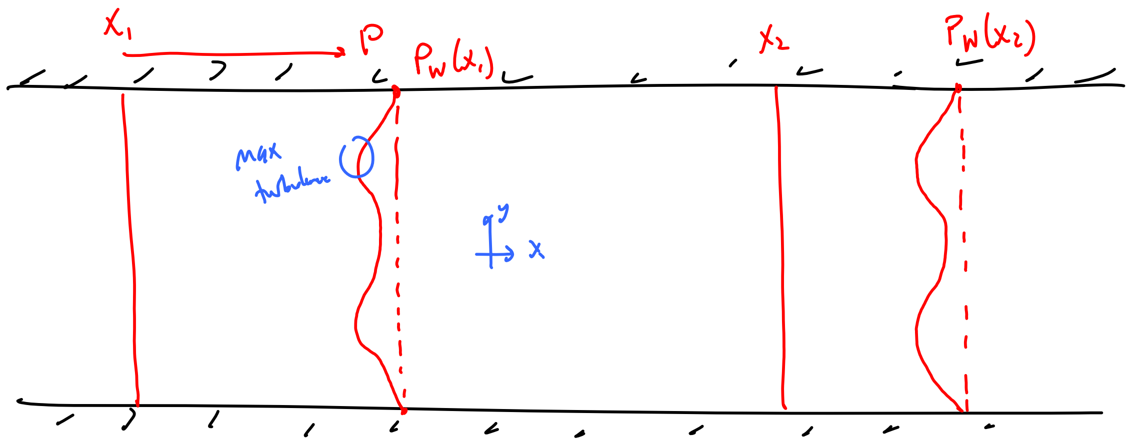
· Integrate (2) $\underline{P = -\rho\overline{v^2} + f(x)}$

Specify $P = P_w$ at either wall, at $y = \pm b$, $\underline{\underline{P = P_w(x)}}$

↑ fnc. of x

& $\overline{v^2} = 0$ @ $y = \pm b$

$$\therefore \boxed{P(x,y) = P_w(x) - \rho\overline{v^2}(y)}$$



Same P profile — not changing with x

except that P_w decreases with x