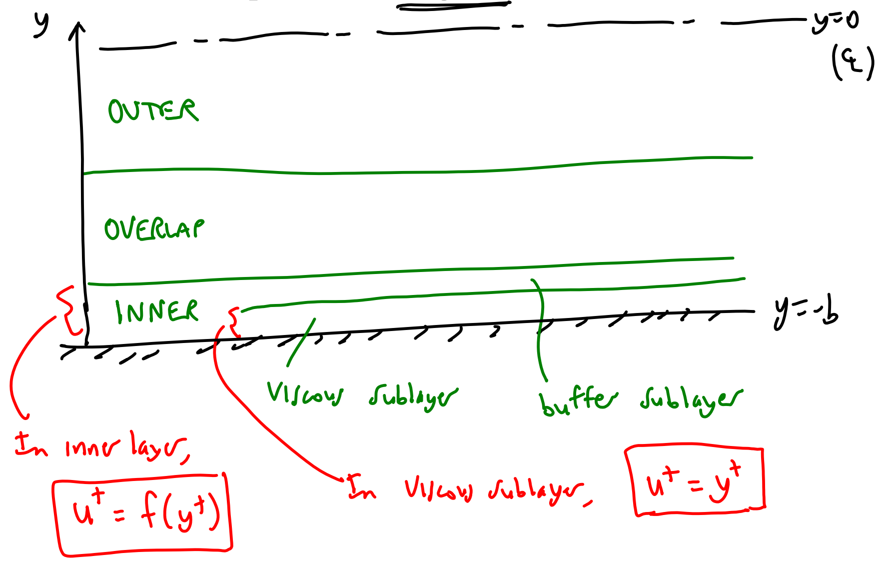
Today, we will:

- Continue to discuss turbulent channel flow – the velocity defect law, the log law, effect of wall roughness
- If time, discuss wall functions – how to incorporate the log law into CFD codes

- Recall, 3 layers in a BL or one half of a channel flow:



- Now let's consider the Outer Layer

Argument of von Karman → U itself is not an important parameter, rather $U_{℄} - U$ or [$U_e - U$ for a BL]

→ Viscosity is not an important parameter

Dim. anal. → $\boxed{U - U_{℄} = \text{fnc}(\tau_w, \rho, y+b, b)}$

($y+b$:) if a BL, we y itself

(b:) if a BL, we δ

get $\boxed{\dfrac{U - U_{℄}}{u^*} = F(\xi)}$ where $\boxed{\xi = \dfrac{y+b}{b}}$ (or $\dfrac{y}{\delta}$ for a BL)

[u^* = friction velocity = $\sqrt{\dfrac{\tau_w}{\rho}}$ as previously]

→ OUTER VARIABLES

→ VELOCITY DEFECT LAW

Now consider the overlap layer → asymptotic matching problem

$y^+ \to \infty$ AND $\xi \to 0$

✠ [The proper way to match these two is to equate the slopes (gradients) of U instead of U itself.]

· Match slope of U w.r.t. y @ some fixed x location

Inner layer

$$U = u^* f(y^+)$$

$$\left\{ y^+ = \frac{(y+b)u^*}{\nu} \qquad \frac{dy^+}{dy} = \frac{u^*}{\nu} \right.$$

$$\frac{dU}{dy} = u^* \frac{df}{dy^+}\frac{dy^+}{dy}$$

$$\boxed{\frac{dU}{dy} = \frac{u^{*2}}{\nu}\frac{df}{dy^+}}$$

Outer layer

$$U = u^* F(\xi) + U_c$$

$$\left\{ \xi = \frac{y+b}{b} \qquad \frac{d\xi}{dy} = \frac{1}{b} \right.$$

$$\frac{dU}{dy} = u^* \frac{dF}{d\xi}\frac{d\xi}{dy}$$

$$\boxed{\frac{dU}{dy} = \frac{u^*}{b}\frac{dF}{d\xi}}$$

Equate these slopes in the overlap layer

$$\frac{u^*}{\nu}\frac{df}{dy^+} = \frac{1}{b}\frac{dF}{d\xi}$$

mult by $(y+b)$

$$\boxed{y^+ \frac{df}{dy^+} = \xi \frac{dF}{d\xi}}$$

a fnc. of y^+ only a fnc. of ξ only $\therefore$ = constant

So,

$$\boxed{y^+ \frac{df}{dy^+} = \xi \frac{dF}{d\xi} = \frac{1}{K} = \text{constant}}$$

✠ K = von Kármán constant

Integrate both sides separately

LHS (inner layer) → $\boxed{u^+ = \frac{U}{u^*} = f(y^+) = \frac{1}{K}\ln y^+ + a}$

★ LOG LAW in terms of inner variables

some authors use B instead of a

RHS (outer layer) → $\boxed{\frac{U - U_e}{u^*} = F(\xi) = \frac{1}{K}\ln\xi + b}$

★ LOG LAW in terms of outer variables

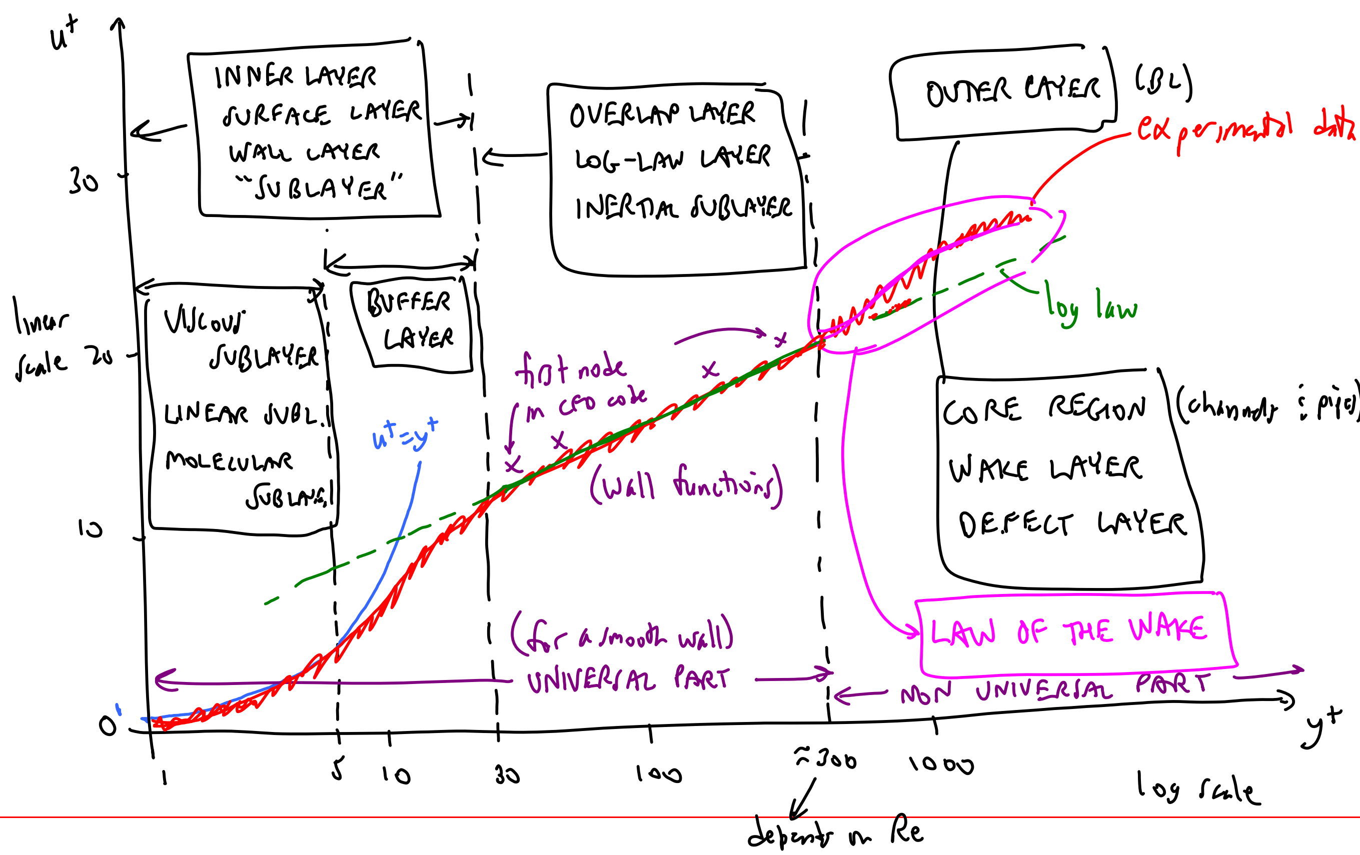
From experiments → $\boxed{\begin{matrix} K = 0.40 - 0.41 \\ a \text{ or } B \simeq 5.0 \text{ to } 5.5 \end{matrix}}$ "UNIVERSAL" for any turbulent flow over a smooth wall

b is not universal even for a smooth wall

$b \simeq -1.0$ for pipes & channels

$b \simeq -2.5$ for flat plate BL

Summary plot of Layered approach for turbulent flow near a wall



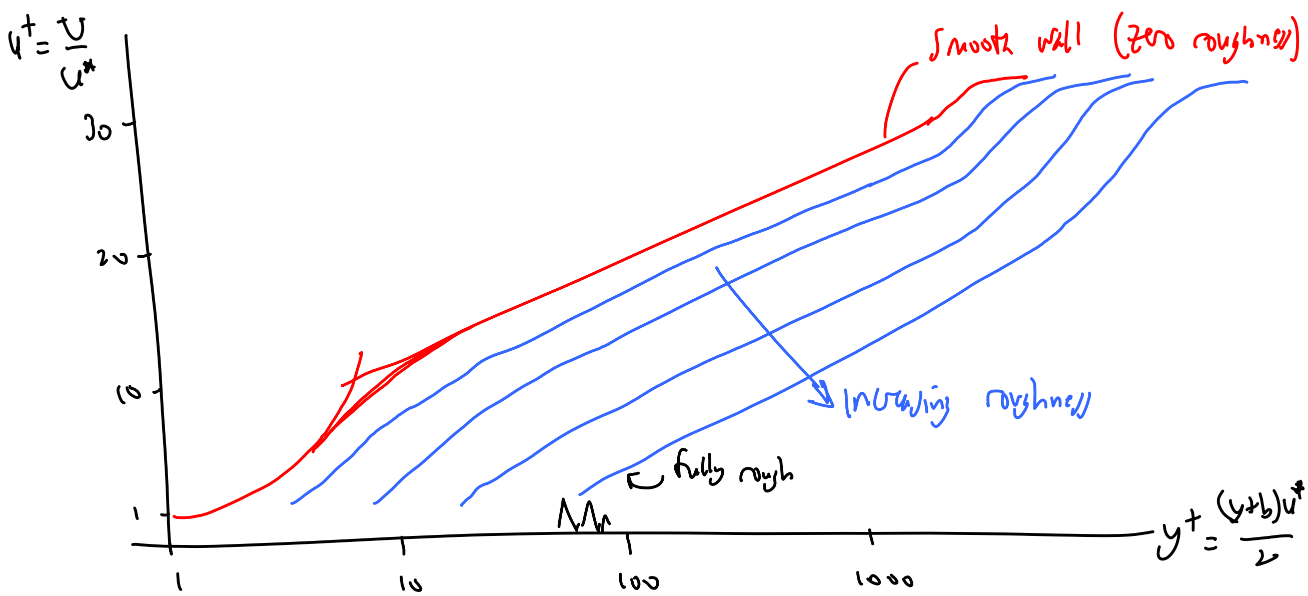
Effect of wall roughness

Recall $u^* = \sqrt{\frac{\tau_w}{\rho}}$

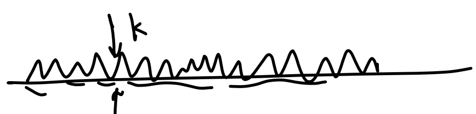
roughness increases τ_w significantly

$\therefore$ u^* increases significantly

Log law still holds with the same slope $\left(\frac{1}{\kappa}\right)$, but it shifts to the right & down



Quantitatively

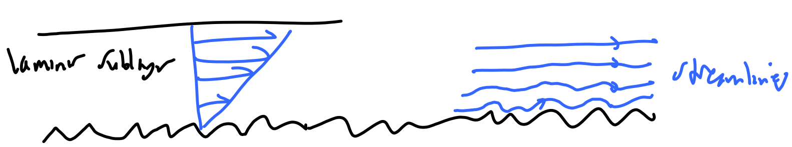


k = avg roughness height

Define $\boxed{k^+ = \frac{k u^*}{\nu}}$

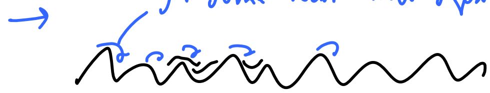
3 regimes of roughness:

$k^+ \lesssim 4$ → Roughness lies within viscous sublayer
Roughness has negligible effect



★ HYDAULICALLY SMOOTH

$4 \lesssim k^+ \lesssim 60$ → "TRANSITIONALLY ROUGH" → Roughness is mostly in the buffer layer get some local flow separation & increased drag

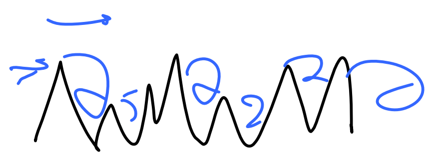


→ causes a shift in the log law

Constant a (or B) gets smaller

$k^+ \gtrsim 60$ → "fully rough" → Roughness so big that there is no viscous sublayer at all

Wall frictional drag is due almost entirely to flow separation on the roughness elements



- log law is still there, but independent of viscosity!

∴ " " Re

same slope $\left(\frac{1}{K}\right)$

I added this after class:

For a given value of roughness height k, (or a)

the log law becomes $u^+ = \frac{1}{K} \ln y^+ + B - \frac{1}{K} \ln\left(1 + 0.3\, k^+\right)$

THE DOWNWARD SHIFT IN THE LOG LAW DUE TO ROUGHNESS