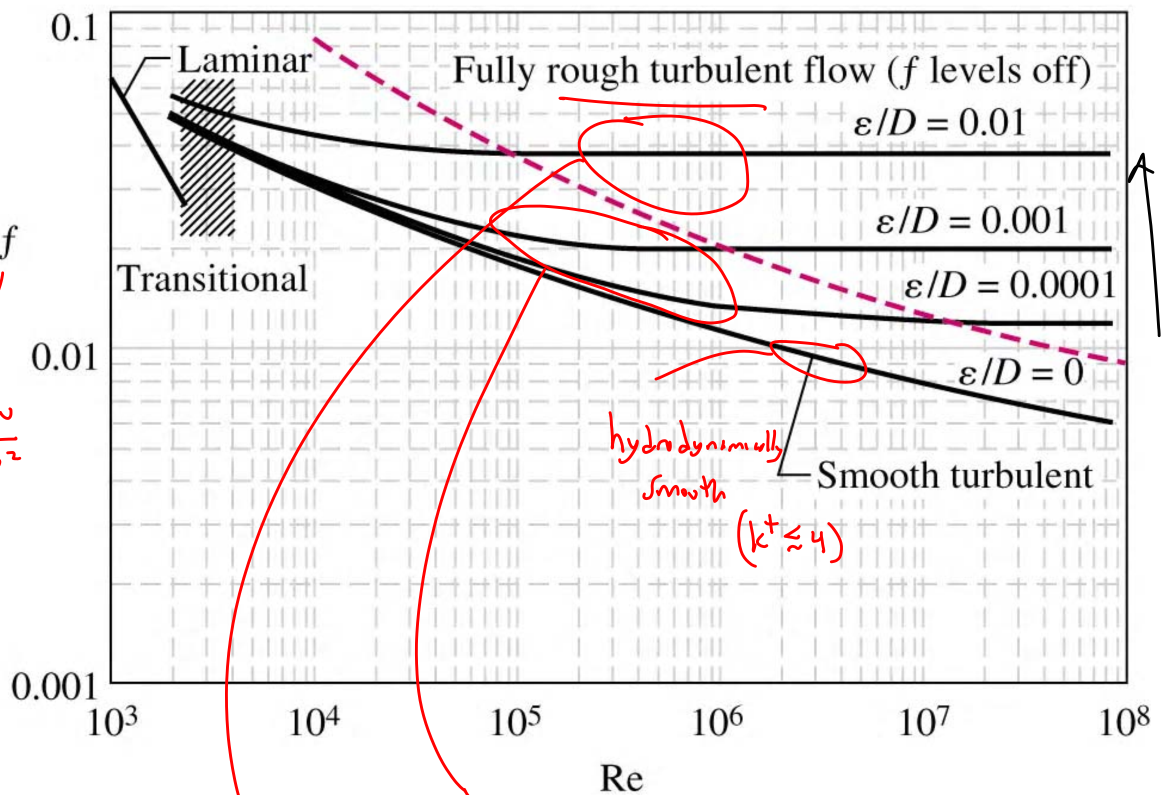
Today, we will:

- Continue to discuss wall roughness, and how it affects pipe flow pressure drop and drag on a flat plate
- Discuss wall functions, and how to incorporate the log law into CFD boundary conditions near the wall
- Do Candy Questions for Candy Friday

Wall roughness is the basis for the **Moody chart** for fully developed pipe flows:
Fig. 8-28, p. 342, Çengel and Cimbala, *Fluid Mechanics: Fundamentals and Applications*, McGraw-Hill, 2006.

$$f = \frac{8\tau_w}{\rho V^2}$$



Increasing roughness

hydrodynamically smooth $(k^+ \lesssim 4)$

transitionally rough $(4 \lesssim k^+ \lesssim 60)$

fully rough $(k^+ \gtrsim 60)$

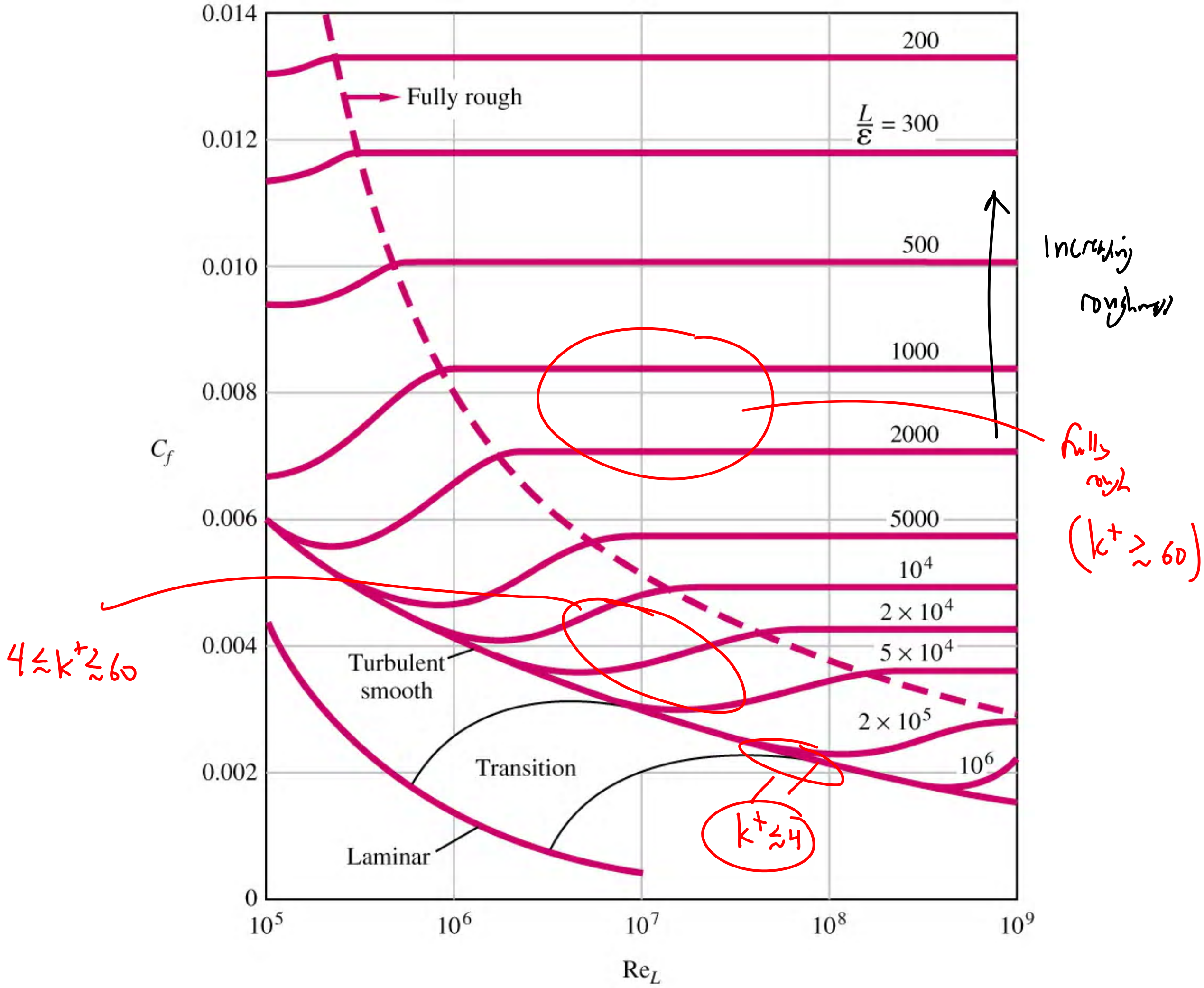
$$u^* = \sqrt{\frac{\tau_w}{\rho}} \rightarrow \boxed{\frac{f}{8} = \left(\frac{u^*}{V}\right)^2}$$

V = avg pipe velocity

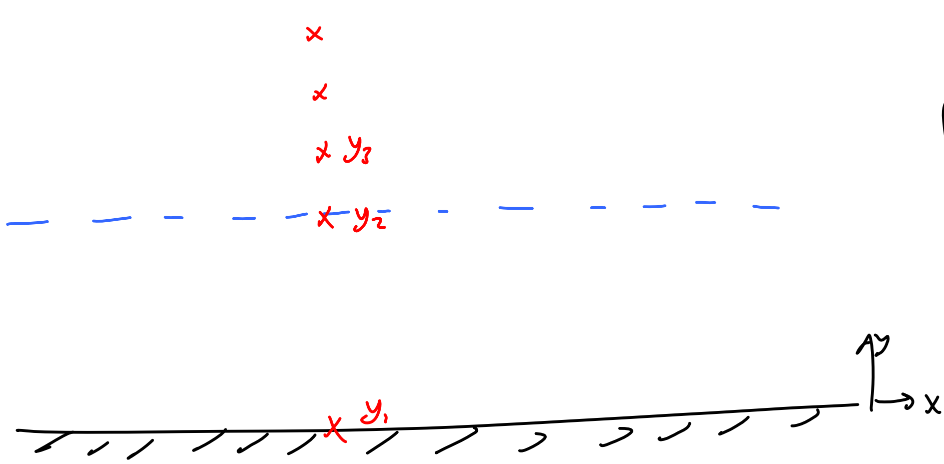
$f \propto u^{*2}$ for a given V

Analogous behavior for flat plate BLs

Wall roughness is also the basis for the **friction coefficient chart** for turbulent BLs:
Fig. 11-31, p. 583, Çengel and Cimbala, *Fluid Mechanics: Fundamentals and Applications*, McGraw-Hill, 2006.



C. <u>Wall functions</u> → take advantage of the near universality of the log law



[We y in BL form]

Assume y_2 is <u>in</u> the log law layer

Typ. $30 \lesssim y^+ \lesssim 300$ we are inside the log law region

[Fluent uses 50 to 500 as the default when grid adapting]

We know then, at point 2,
$$\boxed{U = u^*\left[\frac{1}{\kappa}\ln y^+ + B\right]} \quad \text{log law} \qquad (1)$$

x-mom eq in the log law layer in terms of wall units
$$\boxed{\frac{du^+}{dy^+} = 1 + \frac{\overline{uv}}{u^{*2}} - \cancel{\frac{y^+}{R^*}}} \quad \text{neglect}$$

where $R^* = \dfrac{u^* b}{\nu}$ or $\dfrac{u^* \delta}{\nu}$ $\qquad y^+ = \dfrac{u^* y}{\nu}$

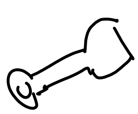
let y^+ be small compared to R^*

In physical variables, this is
$$0 = \frac{d}{dy}\left[\nu \frac{dU}{dy} - \overline{uv}\right]$$

Use an eddy viscosity, namely $-\overline{uv} = \nu_e \dfrac{dU}{dy}$

So,
$$\boxed{\frac{d}{dy}\left[(\nu + \nu_e)\frac{dU}{dy}\right] = 0}$$

x-mom eq. near the first grid pt. away from the wall — assumed to be in the log-law region



- Assume $\nu \ll \nu_e$ at this point

$\downarrow$

far enough away from wall that ν_e dominates over ν

but near enough that $\dfrac{y^+}{R^*} \ll 1$

$$\boxed{\frac{d}{dy}\left(\nu_e \frac{dU}{dy}\right) = 0} \quad (2)$$

k-ε model $\longrightarrow$ $$\boxed{\nu_e = C_\mu \frac{K^2}{\varepsilon}} \quad (3)$$

At this point, K & ε eq reduce to:

$$K: \quad \boxed{0 = \nu_e\left(\frac{dU}{dy}\right)^2 + \frac{d}{dy}\left[\left(\cancel{\nu} + \frac{\nu_e}{\sigma_k}\right)\frac{dK}{dy}\right] - \varepsilon} \quad (4)$$

neglect

$$\varepsilon: \quad \boxed{0 = C_{\varepsilon_1} C_\mu K\left(\frac{dU}{dy}\right)^2 - C_{\varepsilon_2}\frac{\varepsilon^2}{K} + \frac{d}{dy}\left[\left(\cancel{\nu} + \frac{\nu_e}{\sigma_\varepsilon}\right)\frac{d\varepsilon}{dy}\right]} \quad (5)$$

neglect

Use (1) [Log law] to calculate $\frac{dU}{dy}$: $\quad U = \frac{u^*}{K}\ln\left(\frac{y u^*}{\nu}\right) + u^* B$

[u^* = const w.r.t. y]

$$\frac{dU}{dy} = \frac{u^*}{K}\,\frac{\cancel{\nu}}{y\cancel{u^*}}\,\frac{\cancel{u^*}}{\cancel{\nu}} = \frac{u^*}{Ky}$$

$$\boxed{\frac{dU}{dy} = \frac{u^*}{Ky}} \quad (6)$$

Now we can solve for K & ε using (4) & (5)

Get $\boxed{K = \frac{u^{*2}}{\sqrt{C_\mu}}}$ = constant in the log layer

ε in terms of $u^*, K, \& \; y$

Plug in K

(4) becomes $\quad 0 = \nu_e\left(\frac{dU}{dy}\right)^2 - \varepsilon \longrightarrow \varepsilon = \nu_e\left(\frac{dU}{dy}\right)^2$

$$= C_\mu \frac{K^2}{\varepsilon} \quad \frac{u^{*2}}{K^2 y^2}$$

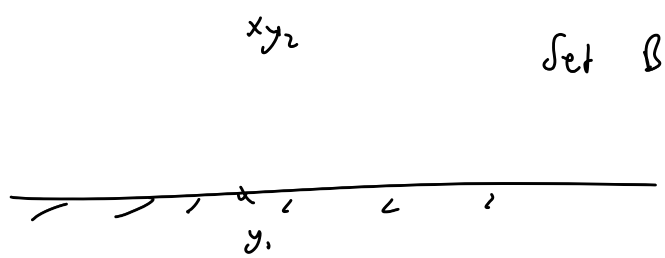
$$\varepsilon = C_\mu \frac{u^{*4}}{C_\mu \varepsilon}\,\frac{u^{*2}}{K^2 y^2}$$

$$\varepsilon^2 = \frac{u^{*6}}{K^2 y^2} \rightarrow \boxed{\varepsilon = \frac{u^{*3}}{Ky}}$$

Consistent with

$$\varepsilon \sim \frac{u'^3}{\ell}$$

[Ky is a "mixing length" in the log law layer]



Set BCs @ y_2

$$\boxed{K = \frac{u^{*2}}{\sqrt{C_\mu}}} \qquad \boxed{\varepsilon = \frac{u^{*3}}{Ky}}$$

Where do I get u^*? Use the log law!

$$\boxed{U = u^* \left[\frac{1}{K} \ln\left(\frac{u^* y}{\nu} \right) + B \right]} \rightarrow \text{solve for } u^*$$

U is one other unknown & we are iterating

must solve for u^* → Implicit eq. for u^* (use Newton's method, for example)

Comments:

- Solutions may be sensitive to the near wall grid
- Roughness is easy to incorporate → modify the constant B
- Wall funct. argument breaks down @ a separation point → $u^* \to 0$ at separation

[must be careful] $\quad (\tau_w \to 0)$

• Fluent's wall functions are similar to the above, but not identical

↳ [The above is based on Wilcox's book]

• Can modify the technique when y_2 is "too close" to the wall.

↓

2-layer model:

- use log law to find u^* if $y_2^+ >$ some cutoff (typ ≃ 10 or 11)

- use $u^+ = y^+$ if $y_2^+ <$ the cutoff

u^* in terms of U, ν, & y

$$u^+ = y^+ \qquad \frac{U}{u^*} = \frac{y u^*}{\nu} \Rightarrow y {u^*}^2 = U\nu$$

$$\boxed{u^* = \sqrt{\frac{U\nu}{y}}}$$

A Blended eq. can be used instead: Blends viscous sublayer & log law

Spalding

$$\boxed{y^+ = u^+ + e^{-KB}\left[e^{Ku^+} - 1 - Ku^+ - \frac{(Ku^+)^2}{2} - \frac{(Ku^+)^3}{6}\right]}$$