Today, we will:

- Discuss turbulent free shear flows in general – equations of motion
- Begin an example problem – the turbulent jet

F. Turbulent Flow Solutions (cont)

1. Turbulent channel flow + BLs

2. <u>Turbulent Free Shear Flows</u>

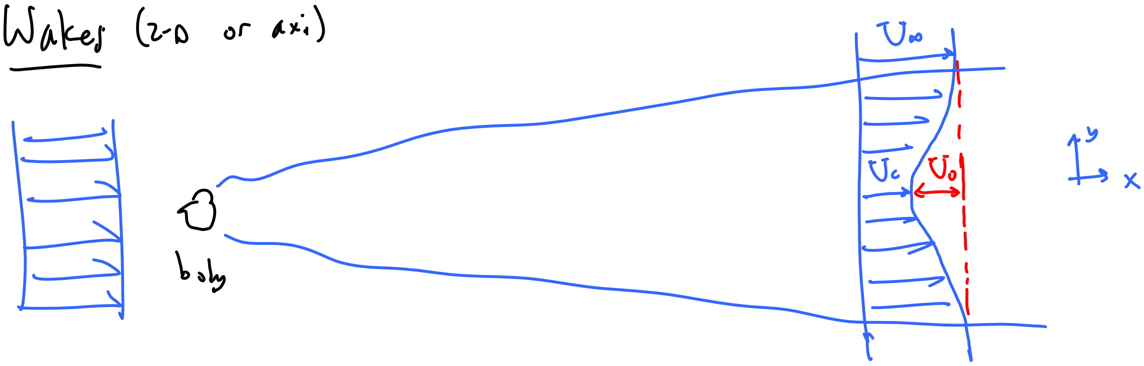
a. <u>Intro</u>

- Defn → no walls bounding the flow = free shear layer
- Simpler than wall-bounded flows → use high Re form of eq's (turb. models)

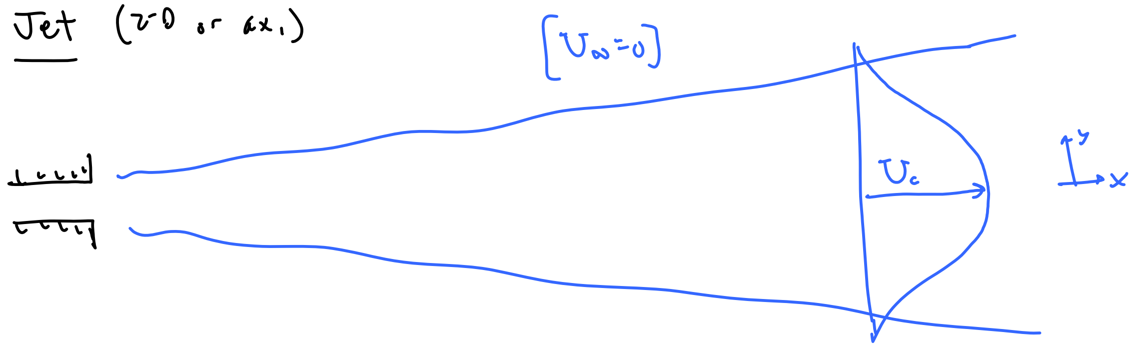
  — No wall functions, etc.

- <u>Three types</u> – concerned with the far field, not the near field

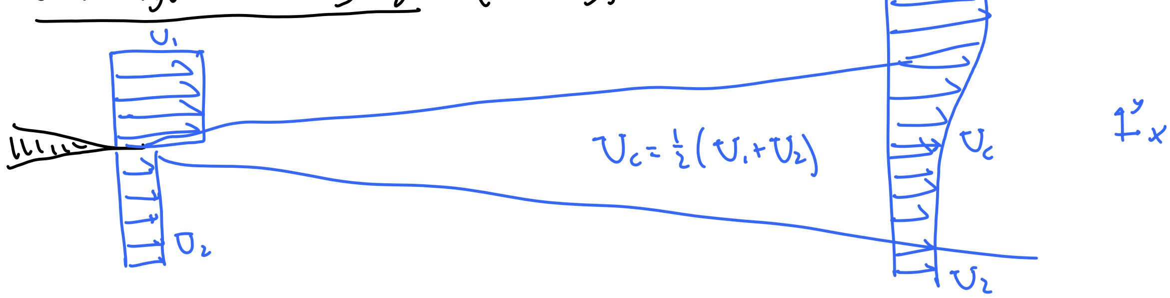
- <u>Wakes</u> (2-D or axi)



- <u>Jet</u> (2-D or axi)



- <u>Shear layer or mixing layer</u> (2-D only)



· Define $U_s \equiv$ mean shear velocity scale = difference between max U & min U at some x location

$$\boxed{\begin{array}{ll} \underline{\text{Wake}}: & U_s = U_\infty - U_c \approx U_0 \\ \underline{\text{Jet}}: & U_s = U_c \\ \underline{\text{M.L.}}: & U_s = U_1 - U_2 \end{array}}$$

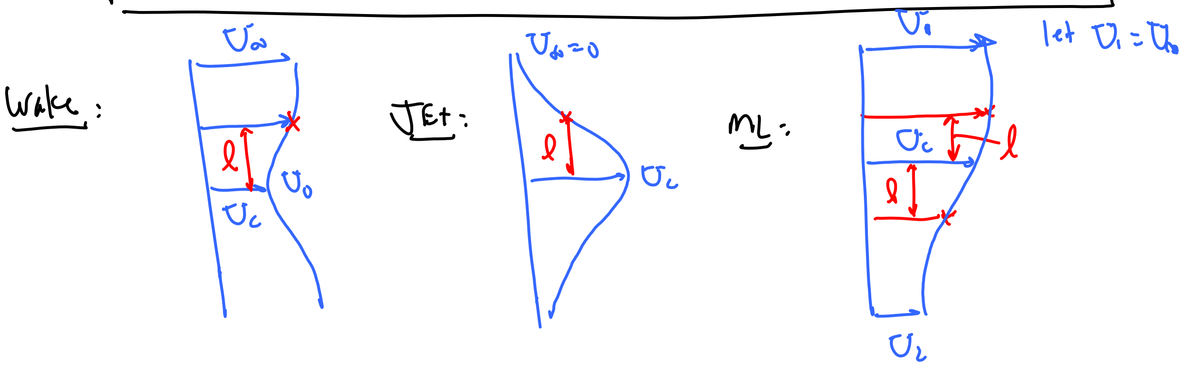
(decays with x) $U_s = U_s(x)$

(" ") "

(constant with x) $U_s \neq fnc(x)$

· Define $\ell \equiv$ characteristic cross-stream length scale

let $\boxed{\ell = \text{distance from centerline where } U - U_c = \tfrac{1}{2}(U_\infty - U_c)}$

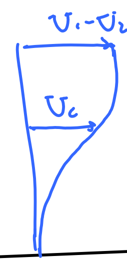


· Define $\tilde{U}$ = characteristic streamwise velocity scale [not same as $\tilde{u}$]

Wake: $\boxed{\tilde{U} = U_\infty}$ Jet: $\boxed{\tilde{U} = U_s}$

ML: depends on your frame of reference:

[Move the observer @ speed U_2] → [pretend $U_2 = 0$]



Let $\boxed{\tilde{U} = U_c}$

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b. Eq.'s of Motion

Assump./Approx:

1) stationary, $\frac{\partial}{\partial t}(\overline{\ \ }) = 0$

2) 2-D in the mean $\frac{\partial}{\partial z}(\overline{\ \ }) = 0$, $W = 0$

3) Use the BL approx $\frac{\partial}{\partial x}(\overline{\phi}) \ll \frac{\partial}{\partial y}(\overline{\phi})$

$\therefore \quad \ell \ll x$

4) No buoyancy or gravity

5) No forces pressure gradient (all these flows are exposed to ambient pressure P_a @ all x)

ORDER-OF-MAG. ANALYSIS

Rules: · When U appears by itself, use $\underline{U \sim \tilde{U}}$

· When U appears in a derivative, we $\underline{U \sim U_s}$

· turbulence quantities $\sim u'$ & ℓ as previously

· CONTINUITY

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \cancel{\frac{\partial W}{\partial z}}_{2\text{-}D} = 0$$

$$\begin{matrix} \wr & & \wr \\ \frac{U_s}{x} & & \frac{V}{\ell} \end{matrix} \Rightarrow \therefore \boxed{V \sim \frac{U_s \ell}{x}}$$

· Y-MOM

$$\overset{1}{U\frac{\partial V}{\partial x}} + \overset{2}{V\frac{\partial V}{\partial y}} = \overset{3}{-\frac{1}{\rho}\frac{\partial P}{\partial y}} + \frac{\partial}{\partial x}\left[\overset{4}{\nu\frac{\partial V}{\partial x}} - \overset{5}{\overline{uv}}\right] + \frac{\partial}{\partial y}\left[\overset{6}{\nu\frac{\partial V}{\partial y}} - \overset{7}{\overline{v^2}}\right]$$

$$\begin{matrix} \wr & \wr & \wr & \wr & \wr & \wr & \wr \\ \tilde{U}\frac{U_s\ell}{x^2} & \frac{U_s\ell}{x}\frac{U_s\cancel{\ell}}{x\cancel{\ell}} & ? & \frac{\nu}{x}\frac{U_s\ell}{x^2} & \frac{u'^2}{x} & \frac{\nu}{\ell}\frac{U_s\ell}{x\ell} & \frac{u'^2}{\ell} \\ \downarrow & \downarrow & & \downarrow & \downarrow & \downarrow & \downarrow \\ \frac{\tilde{U}}{u'}\frac{U_s}{u'}\left(\frac{\ell}{x}\right)^2 & \left(\frac{U_s}{u'}\right)^2\left(\frac{\ell}{x}\right)^2 & ? & \left(\frac{\nu}{u'\ell}\right)\frac{U_s}{u'}\left(\frac{\ell}{x}\right)^3 & \frac{\ell}{x} & \frac{\nu}{u'\ell}\frac{U_s}{u'}\left(\frac{\ell}{x}\right) & 1 \end{matrix}$$

Mult by $\frac{\ell}{u'^2}$

Define $\boxed{R_\ell \equiv \frac{u'\ell}{\nu}}$ Turbulent Reynolds # $[R_\ell \gg 1]$

Term ①: $\left(\frac{\tilde{U}}{u'}\right)\left(\frac{U_s}{u'}\right)\left(\frac{\ell}{x}\right)^2$

Wake: $\tilde{U} \sim U_\infty$

Jet, ML $\tilde{U} \sim U_s$

Could be large for a wake

But ~ 10 for a jet or ML

~ 10

$(\ll 1)^2$

Term 1 is negligible compared to unity

Term ②: $\left(\frac{U_s}{u'}\right)^2\left(\frac{\ell}{x}\right)^2$ → Term 2 is either same o.o.m. or smaller than Term 1

Term 2 is negligible

Term ③: $-\frac{1}{\rho}\frac{\partial P}{\partial y}$ term → Let's keep it.

Term ④: $\left(\frac{\nu}{u'\ell}\right)\left(\frac{U_s}{u'}\right)\left(\frac{\ell}{x}\right)^3$

$\frac{1}{R_\ell}$ small

~ 10

$(\ll 1)^3$

Term 4 is negligible

Term ⑤: $\frac{\ell}{x}$ → $\frac{\ell}{x} \ll 1$ → Term 5 is negligible

Term ⑥: $\left(\frac{\nu}{u'\ell}\right)\left(\frac{U_s}{u'}\right)\left(\frac{\ell}{x}\right)$

$\frac{1}{R_\ell}$ small

~ 10

$\ll 1$

Term 6 is negligible

Term ⑦: Must keep. → o.o.m. 1

$\boxed{\text{Terms ③ \& ⑦ must balance}}$

y-mom eq. $\rightarrow$ $$\boxed{\frac{1}{\rho}\frac{\partial P}{\partial y} \simeq -\frac{\partial}{\partial y}\overline{v^2}}$$

Integrate: $$\frac{P}{\rho} = -\overline{v^2} + f(x)$$

BC @ $y \to \infty$, $P \to P_a$ = constant $\neq$ fnc. of x & $\overline{v^2} \to 0$ $\Big\}$ $f(x) = \frac{P_a}{\rho}$

$$\boxed{P = -\rho\overline{v^2} + P_a} \qquad (1)$$

· X-mom eq: Do O.O.M. anal.:

$$\boxed{U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}(-\overline{uv})} \qquad (2)$$

Comment $\rightarrow$ Notice No Reynolds #, No viscosity!

[High Re version of turbulence models is consistent with this]

Viscous diffusion $\ll$ turbulent diffusion ✓

$\therefore$ All turbulent (MLs, wakes, jets), for example, are similar — indep. of Re in the mean flow!

C. Turbulent Jet — we will discuss in detail