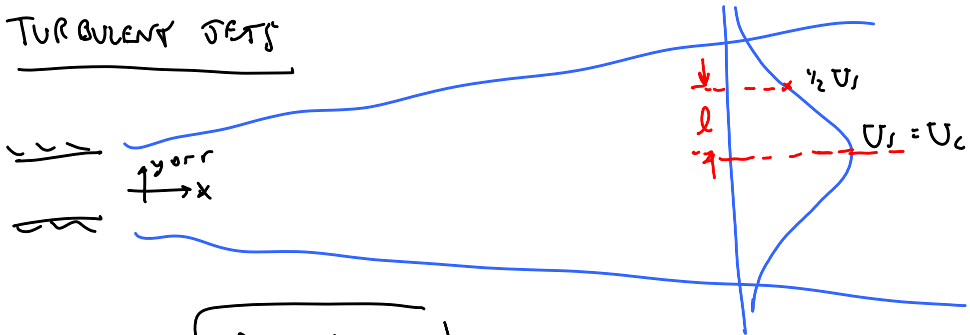
Today, we will:

- Continue to discuss the turbulent jet
- SRTEs to be conducted during the last 10 minutes of class today

C. Turbulent Jets



Recall,

cont.: $$\boxed{\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0} \quad (1)$$

x-mom: $$\boxed{U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(-\overline{uv}\right)} \quad (2)$$

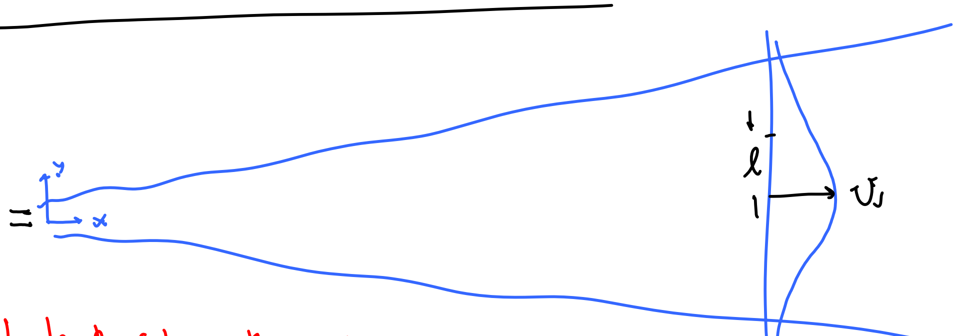
Integral Analysis

2-D → $\boxed{M = \text{momentum flux per unit depth into page @ some } x \text{ location} = \int_{-\infty}^{\infty} \rho U^2 \, dy \approx \text{const}}$ (3)

✲ M does not vary with x

axi → $\boxed{M = \text{mom. flux} = \int_0^{\infty} \rho U^2 \, 2\pi r \, dr \approx \text{const}}$ (4)

· Similarity Solution in the Far Jet



Far enough away, the jet "forgets" everything except M

No length scale in the problem → ∴ expect similarity sol'n

Math → combine x & y into one variable η

Dim. Anal. helps to define η

we expect $\ell = \text{fnc}(x, \rho, M)$

Dim. anal. $\Pi_1 = \frac{\ell}{x}$ = only P_i ∴ $\Pi_1 = \text{const.}$

So, we conclude $\boxed{\ell = \text{const.}\, x}$

∴ ALL TURBULENT FAR JETS GROW LINEARLY WITH X ☆☆

(both 2-D & axi)

Let's define $\eta = \frac{y}{\ell}$ for 2-D or $\frac{r}{\ell}$ for axi

$\ell = \text{const.}\, x$

So let $\boxed{\eta = \sigma \frac{y}{x}}$ (2-D) or $\boxed{\eta = \sigma \frac{r}{x}}$ (axi)

[σ = some constant]

Consider U: → nondimensionalize U

we set $\frac{U}{U_s} = \text{fnc}(\eta)$

better: → set $\boxed{F'(\eta) = \frac{U}{U_s}}$

For 2-D case:

$$M = \int_{-\infty}^{\infty} \rho U^2 \, dy$$

Assume $\boxed{U_s = A x^a}$ A & a are constants.

$$U = U_s F' = A x^a F'(\eta)$$
$$\eta = 6\frac{y}{x} \Rightarrow dy = \frac{x}{6} d\eta$$

$$M = \rho \int_{-\infty}^{\infty} U^2 \, dy$$

$$M = \rho \int_{-\infty}^{\infty} A^2 x^{2a} F'^2 \frac{x}{6} d\eta = \text{const.}$$

$$M = \underbrace{\frac{\rho A^2}{6}}_{\text{const}} \, x^{2a+1} \underbrace{\int_{-\infty}^{\infty} F'^2 \, d\eta}_{\text{const}} = \text{const}$$

$\therefore \; x^{2a+1} = \text{const} \Rightarrow 2a+1 = 0$

$\therefore \; \boxed{a = -\frac{1}{2}}$

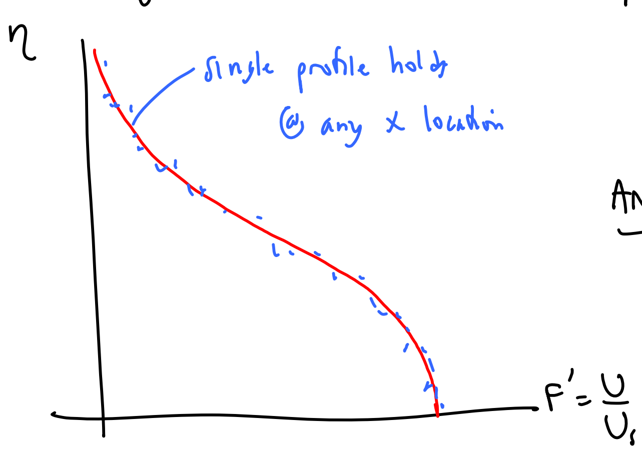
Thus, $\boxed{U_s = A x^{-1/2}}$ for a 2-D turb. jet

$U_s = U_c =$ centerline velocity

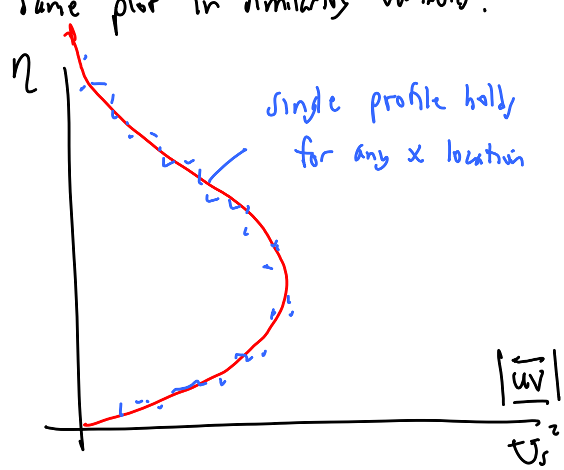
$$\star \; \boxed{\begin{array}{l} \ell \propto x \\ \therefore \; U_c \propto x^{-1/2} \quad \text{in a 2-D jet} \end{array}}$$

Self-similar 2-D turbulent jet

We expect for complete self-similarity, both the mean and the turbulent quantities are similar — collapse onto same plot in similarity variables.



AND



Terminology for complete similarity → "self-similar"
"self-preservation"
"equilibrium state"
"invariance"

"Canonical" → the jet forgets its origins
↘ Depends Only on M

Experiments → confirm   mean profiles become similar quickly by $\left(\frac{x}{d} \approx 50\right)$
turb. " " " farther downstream by $\left(\frac{x}{d} \approx 75\right)$

☆ turbulence has a "memory" from upstream

Recall, x-mom eq:
$$\boxed{U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(-\overline{uv}\right)} \qquad (2)$$

we have
$$\eta = \sigma \frac{y}{x} \qquad [\sigma = \text{const}]$$
$$U_s = A x^{-1/2}$$
$$F'(\eta) = \frac{U}{U_s} = \frac{U}{A}\sqrt{x}$$

Plus, we need some model for $-\overline{uv}$ ⇒ can use zero-eq.
one-eq
2-eq
RSM ...

e.g. Use a simple eddy viscosity-mixing length model (zero-eq model)

$$\boxed{-\overline{uv} = \nu_e \frac{\partial U}{\partial y}}$$
[BL form of the Boussinesq eddy viscosity]

Also, for free shear flows,
$$\boxed{\nu_e = \text{const}\cdot l_m \cdot u'}$$

set $u' = \text{const}\cdot U_s = \text{const}\cdot x^{-1/2}$

set $l_m \approx l = \text{const}\cdot x$

$\therefore \boxed{\nu_e = B\sqrt{x}}$ where B = some new constant

ν_e = constant across the jet (not a fnc. of y)

but varies with x

$$\therefore \boxed{-\overline{uv} \approx B\sqrt{x}\,\frac{\partial U}{\partial y}}$$

2-D turb. jet. similarity soln for the far field:

Cont $\boxed{\dfrac{\partial U}{\partial x} + \dfrac{\partial V}{\partial y} = 0}$ (1)

x-mom $\boxed{U\dfrac{\partial U}{\partial x} + V\dfrac{\partial U}{\partial y} = \dfrac{\partial}{\partial y}\left(-\overline{uv}\right)}$ (2)

Control vol. anal $\rightarrow$ $\boxed{U_s = A x^{-1/2}}$

Similarity approx: $\boxed{\eta = \sigma\dfrac{y}{x} \quad F'(\eta) = \dfrac{U}{U_s}}$ or $\dfrac{U}{U_c}$

Eddy viscosity model $\boxed{-\overline{uv} = B\sqrt{x}}$

algebra

$$\boxed{F'^2 + FF'' + \frac{2B\sigma^2}{A}F''' = 0}$$ Similarity eq.

= constant (it is arbitrary)

set the const = $\frac{1}{2}$

$$\boxed{F''' + 2FF'' + 2F'^2 = 0} \quad (6)$$ FINAL SIMILARITY EQ.