Today, we will:

- Continue to discuss the turbulent jet
- SRTEs to be conducted during the last 10 minutes of class today

\[ \text{C. Turbulent Jets} \]

Recall, \( \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 \) \hspace{1cm} (1)

\[ \text{x-mom} \quad U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \frac{\partial P}{\partial y} (-U) \] \hspace{1cm} (2)

\[ \text{Integral Analysis} \quad 2-D \rightarrow M = \text{mass flux} = \int_0^\infty p U^2 dy = \text{const} \] \hspace{1cm} (2)

\[ \text{a} \text{ } \text{M does not vary with } x \]

\[ \text{axi} \rightarrow M = \text{mom flux} = \int_0^\infty p U^2 2\pi r dr = \text{const} \] \hspace{1cm} (4)

\[ \text{Similarity Solution in the Far Jet} \]

Far enough away, the jet "forgets" everything except \( M \)

No length scale in the problem -> expect similarity solution
MAT: combine \( x, y \) into one variable \( \eta \)

Dim. And. help to define \( \eta \)

we expect \( l = \text{func}(x, \rho, M) \)

Dim. and. \( \xi = \frac{l}{x} \) = only \( \rho \); \( \xi \) = cont.

So, we conclude \( l = \text{const.} x \)

\[ \therefore \text{ALL TURBULENT FAR JETS GROW LINEARLY WITH } x \] (both 2-D, i.e., axi)

Let's define \( \eta = \frac{y}{x} \) for 2-D or \( \frac{r}{x} \) for axi

\[ l = \text{const.} x \]

So, let \( \eta = \frac{y}{x} \) \( (2-D) \) or \( \eta = \frac{r}{x} \) \( (a-x) \)

\[ \left( \xi = \text{some constant} \right) \]

Consider \( U: \rightarrow \) non-dimensionalize \( U \)

we set \( \frac{U}{U_s} = \text{func}(\eta) \)

better: \( \rightarrow \) set \( F(\eta) = \frac{U}{U_s} \)

For 2-D case: \( M = \int_0^\infty \varphi U^2 dy \)

Assume \( U_s = A x^a \) \( A, a \) are constants
\[ U = U_s F' = A \lambda^4 F'(\lambda) \]
\[ \eta = 6 \frac{y}{x} \Rightarrow dy = \frac{x}{6} d\eta \]
\[ M = \rho \int_{-\infty}^{\infty} U^2 dy 
= \rho \int_{-\infty}^{\infty} A^2 x F^2 \left( \frac{x}{6} d\eta \right) = \text{cont.} \]
\[ M = \rho \int_{-\infty}^{\infty} A^2 x F^2 \left( \frac{x}{6} d\eta \right) = \text{cont.} \]
\[ \therefore 2a + 1 = 0 \Rightarrow a = -\frac{1}{2} \]

Thus, \[ U_s = A x^{-\frac{1}{2}} \]
for a 2-D turb. jet.

\[ U_s = U_c = \text{catalytic velocity} \]

Self-similar 2-D turbulent jet

We expect for complete self-similarity, both the mean and the turbulent quantities are similar — collapse into same plot in similarity variables.
Terminology for complete similarity → “self-similar”
“self-preservation”
“equilibrium state”
“invariance”

“Canonical” → the jet forgets its origin

Defined only on M

Experiments → mean profiles become similar quickly by \( \left( \frac{x}{d} \approx 50 \right) \)

turb → “farther downstream by \( \left( \frac{x}{d} \approx 70 \right) \)

Turbulence has a “memory” from upstream

Reynolds x-mom eq:

\[

U \frac{dU}{dx} + V \frac{dU}{dy} = \frac{2}{\nu} \left( -\overline{uv} \right)

\]

\( \nu = \frac{y}{x} \) \( (y = \text{const}) \)

\( U_j = A x^{-1/2} \)

\( F'(x) = \frac{U}{U_j} = \frac{U}{A \frac{x}{\sqrt{x}} \text{SN}} \)

Plus, we need some model for \( -\overline{uv} \) → can we zero eq.

\( \text{one eq.} \)

\( \text{two eq.} \)

\( \text{three eq.} \)

E.g. Use a simple eddy viscosity-mixing length model (zero eq model)

\[

-\overline{uv} = \nu \frac{dU}{dy}

\]

[PL form of the Bowin\( u' \) eddy viscosity]

Also, for free shear flow,

\( \nu = \text{const} \cdot \chi \)

\( \frac{l}{\chi} \approx \frac{1}{l} = \text{const} \cdot x \)
\[ U_e = B \sqrt{x} \] 
where \( B \) is some new constant.

\[ U_e = \text{constant across the jet (not a func. of } y) \]

but varies with \( x \)

\[ -\overline{uv} = B \sqrt{x} \frac{dU}{dy} \]

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2-D turb. jet. similarity soln for the far field:

\[ \frac{2U}{dx} + \frac{2V}{dy} = 0 \]  
(1)

\[ \frac{U}{dx} + V \frac{dU}{dy} = 2 \frac{dy}{dx} (-\overline{uv}) \]  
(2)

Control vol. and \( \rightarrow U_f = A x^{-\frac{1}{2}} \)

Similarity approx.:

\[ \eta = \frac{x}{y} \]

\[ F'(\eta) = \frac{U}{U_f} \]

**Hey, wind is back!**

\[ \overline{uv} = B \sqrt{x} \]

\[ F'^2 + FF'' + \frac{2B \delta^2}{A} F'' = 0 \]

\[ F'''' + 2FF'' + 2F'^2 = 0 \]

* Final similarity eq. = constant (it is arbitrary)

Set the const. = \( \frac{1}{2} \)