M E 522 Spring 2008 Professor John M. Cimbala Lecture 44 05/02/2008

Today, [LAST LECTURE ☺] we will:

- Finish discussing the 2-D turbulent far jet – solve the similarity equation
- Briefly discuss the axisymmetric turbulent far jet
- Briefly discuss turbulent mixing layers
- Do Candy Questions for Candy Friday

Recall: 2-D turbulent far jet similarity equation: $\boxed{F''' + 2FF'' + 2F'^2 = 0}$, where $\boxed{\eta = \sigma \frac{y}{x}}$,

$\boxed{F'(\eta) = \frac{U}{U_s} = \frac{U}{U_c}}$, U_c is the centerline velocity of the jet, and $\boxed{U_c = Ax^{-1/2}}$.

Solve: $$F''' + \underbrace{2FF'' + 2F'^2}_{} = 0$$

$$F''' + 2(FF')' = 0$$

Int. $$\underline{F'' + 2FF'} = C_1$$ ← arbitrary, set = 0

BC's → @ ℄ $\frac{\partial U}{\partial y} = 0$ → $F''(0) = 0$

F ∝ stream function → add any constant to F

⇓

set $F = 0$ @ $y = 0$

$$F'' + (F^2)' = 0$$

Int $$F' + F^2 = a$$ = another constant

$$\frac{dF}{a - F^2} = d\eta$$ ⇒ solve analytically → $\boxed{F = a\tanh(a\eta)}$

* exact soln

Find constant a

$$F = a\tanh(a\eta)$$
$$F' = a^2\,\text{sech}^2(a\eta)$$

@ $\eta = 0$ (℄), $F'(\eta) = 1$ $U = U_s = U_c$ @ $\eta = 0$

$\text{sech}^2(0) = 1$ ⇒ $a^2 = 1$ → $\boxed{a = 1}$

$$\boxed{\frac{U}{U_c} = F'(\eta) = \text{sech}^2(\eta)}$$

Mean velocity profile for a 2-D turbulent jet

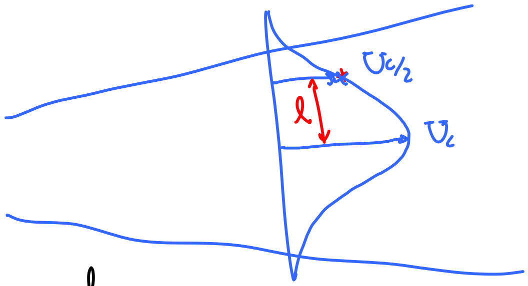
$$\left(\eta = \sigma \frac{y}{x}\right)$$

↓

Don't know σ !

σ is arbitrary because we can define the jet thickness any way we want

Let's use $\ell = \frac{1}{2}$-way velocity point

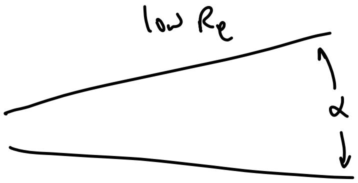


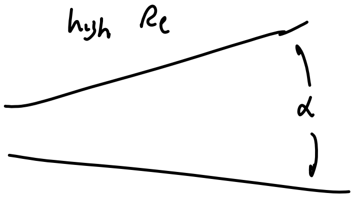
So find η where $\frac{U}{U_c} = \frac{1}{2}$

$$\text{sech}^2(\eta) = \frac{1}{2} \Rightarrow @\ \eta = 0.8814 = \sigma \frac{\ell}{x} \quad (@\ y = \ell)$$

- $\frac{\ell}{x}$ must ultimately come from experiment

All 2-D turbulent jets should grow at the same rate (angle)





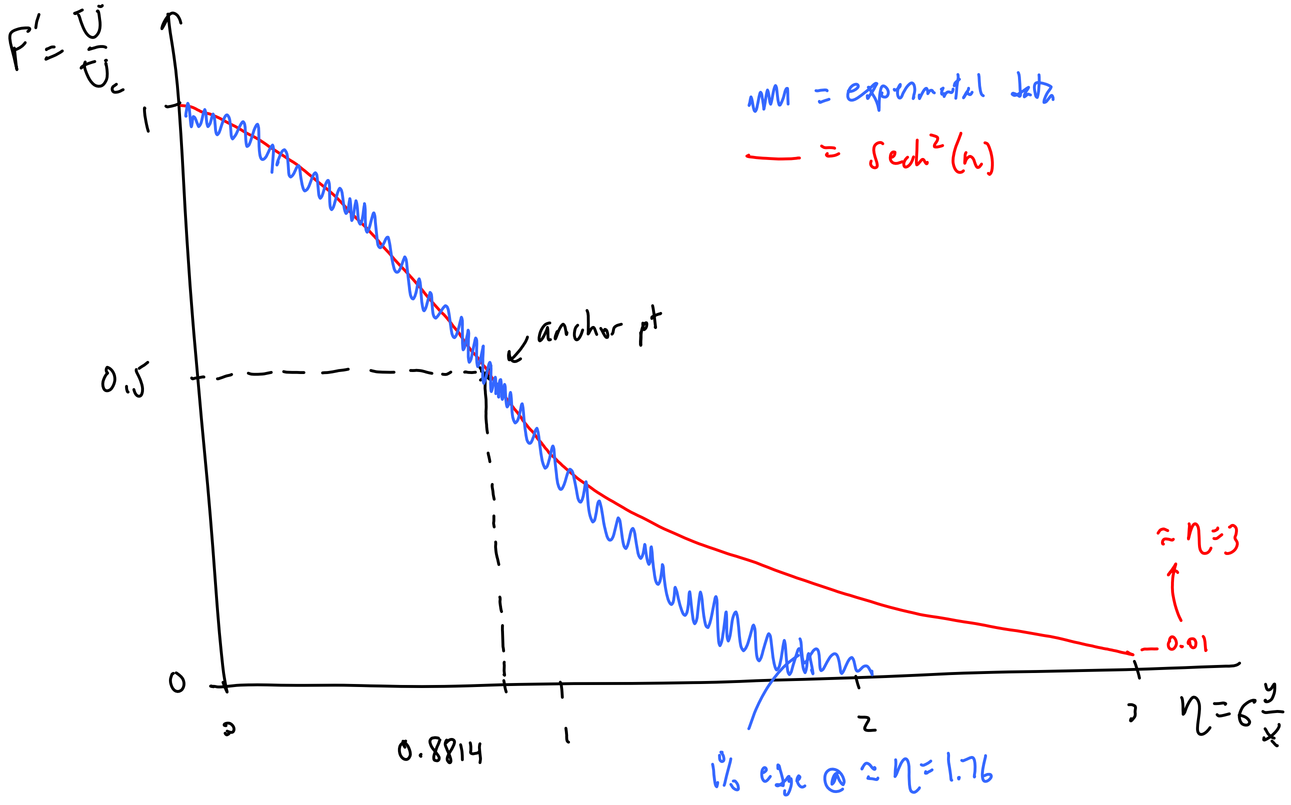
α should be the same

$$\boxed{\frac{\ell}{x} \approx 0.115}$$ from experiment

$\rightarrow$ So, $\sigma = \frac{0.8814}{0.115} = 7.67$

↓

Let's see if our predicted profile agrees with experiment



Disagreement is due primarily to intermittancy

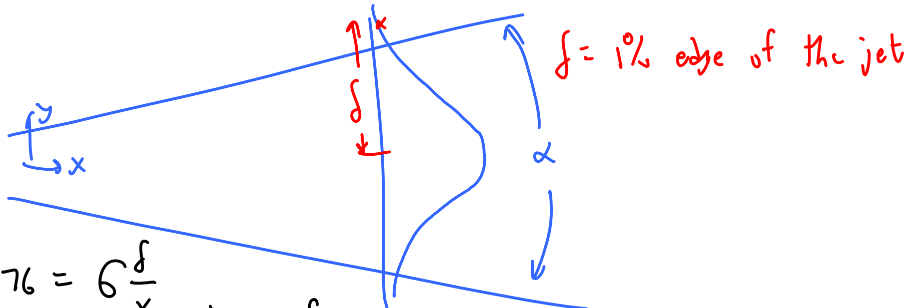
Recall ν_e = constant across the whole jet

in real life, ν_e drops off towards the edge due to intermittancy

[Can correct it by using $\nu_{e_{\text{corrected}}} = \gamma \, \nu_{e_{\text{const}}}$]

intermittancy

Spreading rate



Experiment → $\eta_{\text{edge}} = 1.76 = 6\frac{\delta}{x}$

$$\frac{\delta}{x} = \frac{1.76}{6} = 0.229$$

$\alpha/2 = \tan^{-1}\frac{\delta}{x} \simeq 13^\circ \longrightarrow$ $\alpha_{\text{total angle}} \simeq 26^\circ$

= spreading angle of <u>all</u> 2-D turb. jets.

AXISYMMETRIC JETS

similar analysis & experiment ↓

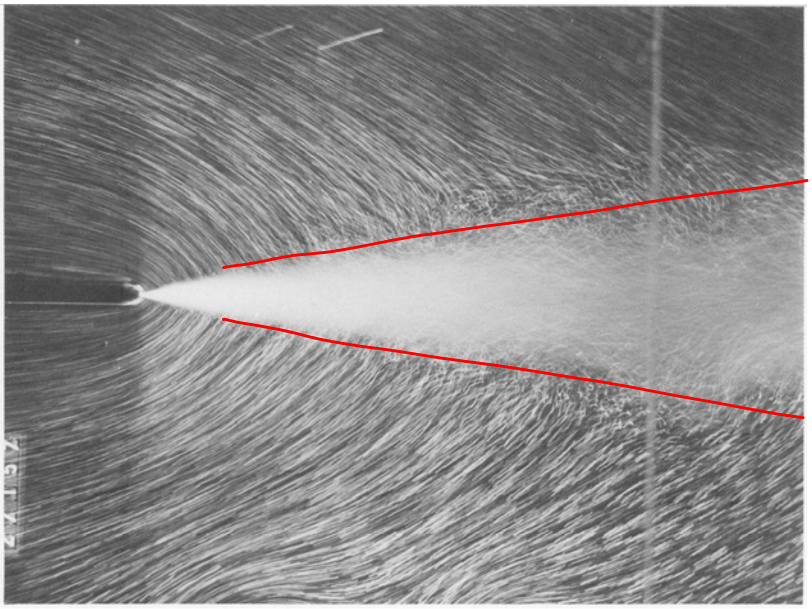
$\alpha_{\text{total angle}} \simeq 24^\circ$ = spreading angle for <u>all</u> axisymmetric turb. jets

See some pictures ↓

Turbulent Jets

Author: John M. Cimbala, Penn State University
Latest revision: 01 May 2008

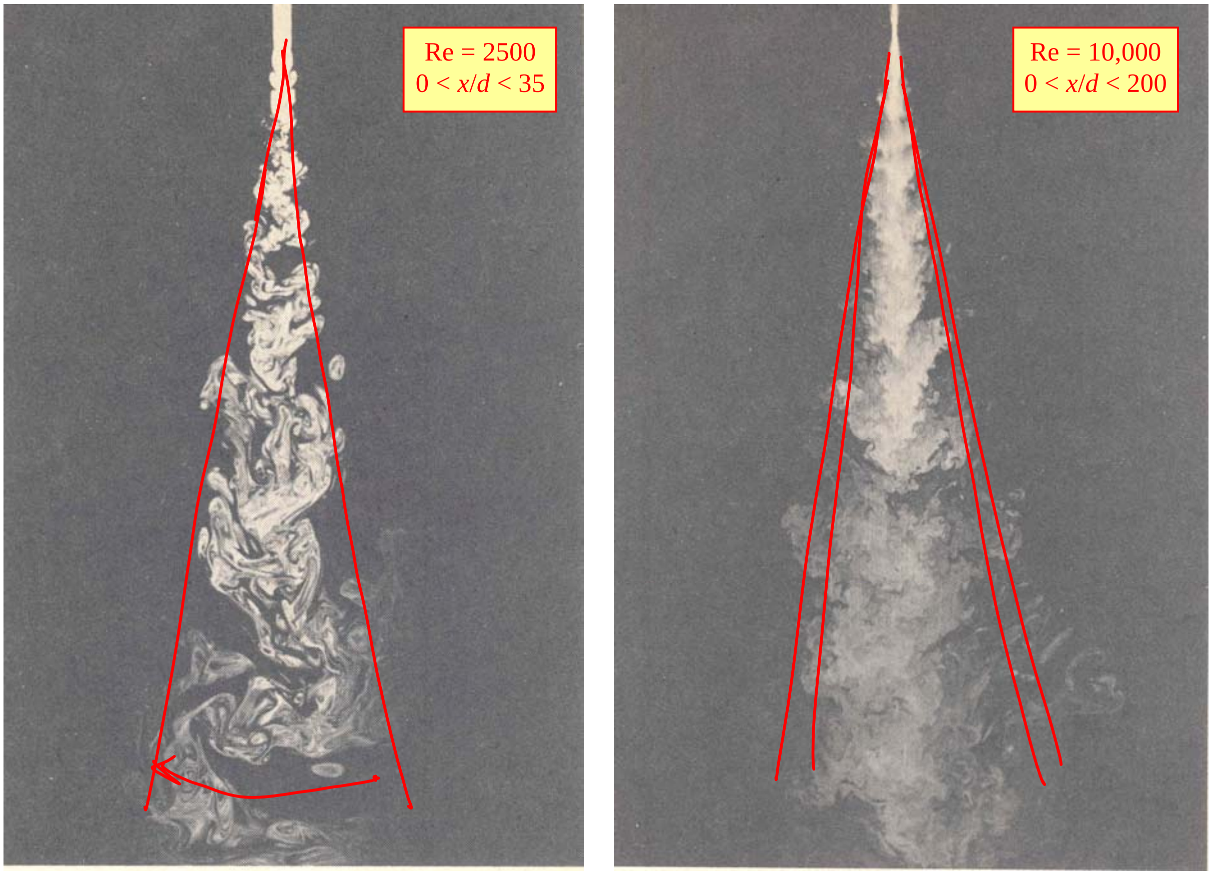
1. Two dimensional (plane) turbulent jet



169. Entrainment by a plane turbulent jet. A time exposure shows the mean flow of a plane jet of colored water issuing into ambient water at 100 cm/s. Tiny air bubbles mark the streamlines of the slow motion induced in the surrounding water. *ONERA photograph, Werlé 1974*

From: Van Dyke, M., *An Album of Fluid Motion*, Stanford, CA, The Parabolic Press, 1982, p. 99.

2. Axisymmetric (round) turbulent jet



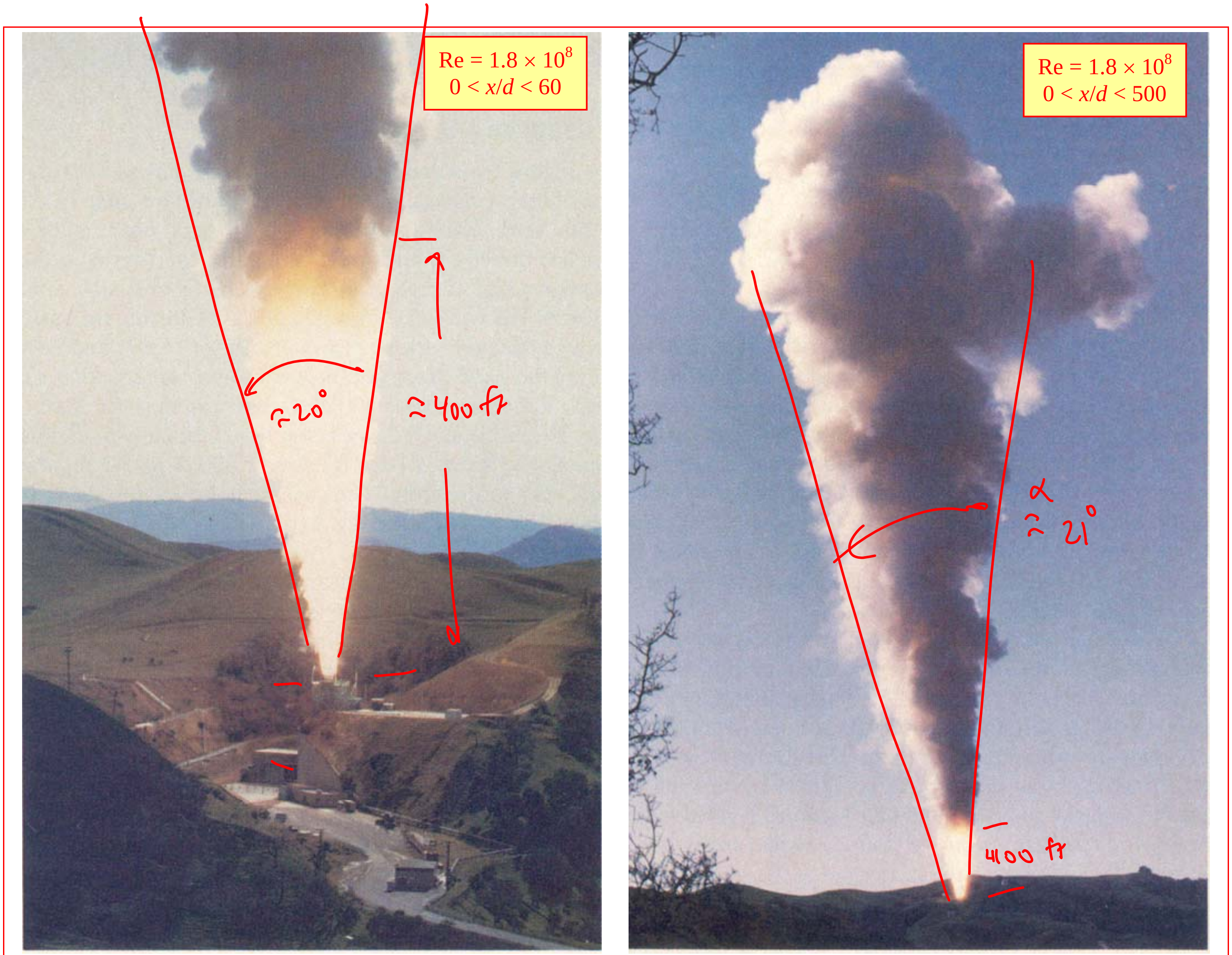
Re = 2500
0 < *x*/*d* < 35

Re = 10,000
0 < *x*/*d* < 200

α≈19°

α≈19°

From: Dimotakis, P. E., Miake-Lye, R. C., and Papantoniou, D. A., Structure and Dynamics of Round Turbulent Jets, *Physics of Fluids*, Volume **26**, No. 11, November 1983.

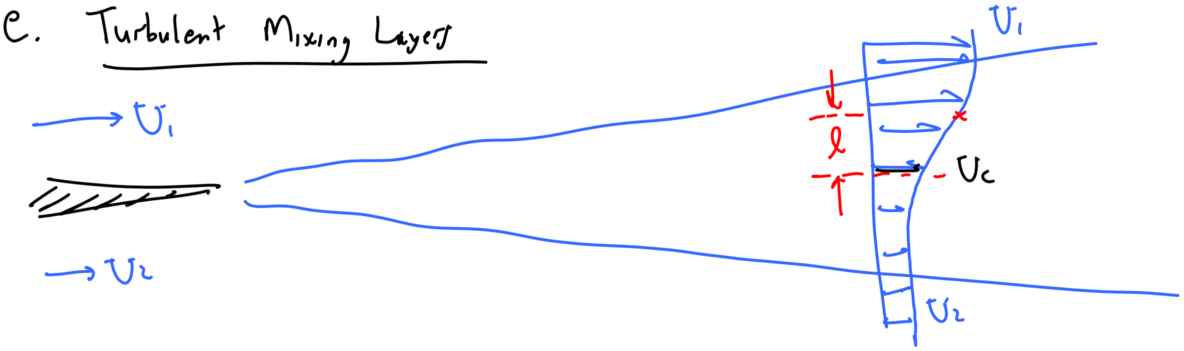


From: Mungal, M. G. and Hollingsworth, D. K., Organized Motion in a Very High Reynolds Number Jet, *Physics of Fluids A*, Volume **1**, No. 10, October 1989.

The above two pictures are of a TITAN IV rocket motor test. The thrust is 1.6×10^6 lbf, the jet exit diameter is 10 ft, and the Reynolds number is about 1.8×10^8. The rocket test stand is approximately 100 ft. high. The bright portion of the plume is about 400 ft high. The overall height of the plume in the left picture is about 600 ft, corresponding to $0 < x/d < 60$. The overall height of the plume in the right picture is about 5000 ft, corresponding to $0 < x/d < 500$.

d. Turbulent Far Wake → (see HW 12)

e. Turbulent Mixing Layers



Recall $U_s = U_1 - U_2 = \text{const}$

$$\tilde{U} = U_c = \text{const}$$

Eq's are the same as those of the 2-D jet

Cont $$\boxed{\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0} \qquad (1)$$

x-mom $$\boxed{U\frac{\partial U}{\partial x} + V\frac{\partial U}{\partial y} = \frac{\partial}{\partial y}\left(-\overline{uv}\right)} \qquad (2)$$

Recall, for a 2-D jet, we used integral control volume analysis

$$M = \int_{-\infty}^{\infty} \rho U^2 \, dy = \text{const}$$

M = an invariant in the problem

Here - U_s is known & is a constant → Don't need CV analysis

↘ U_s is already an invariant in the problem.

$$U_s = U_1 - U_2$$

Jet: ℓ & U_s are unknowns

ML: ℓ is unknown, but U_s is known = const.

↓ Use dim. anal. $\ell = \text{fnc.}\left(x, \rho, \underline{U_s}\right)$

use U_s instead of M

⇓

result of dim. anal:

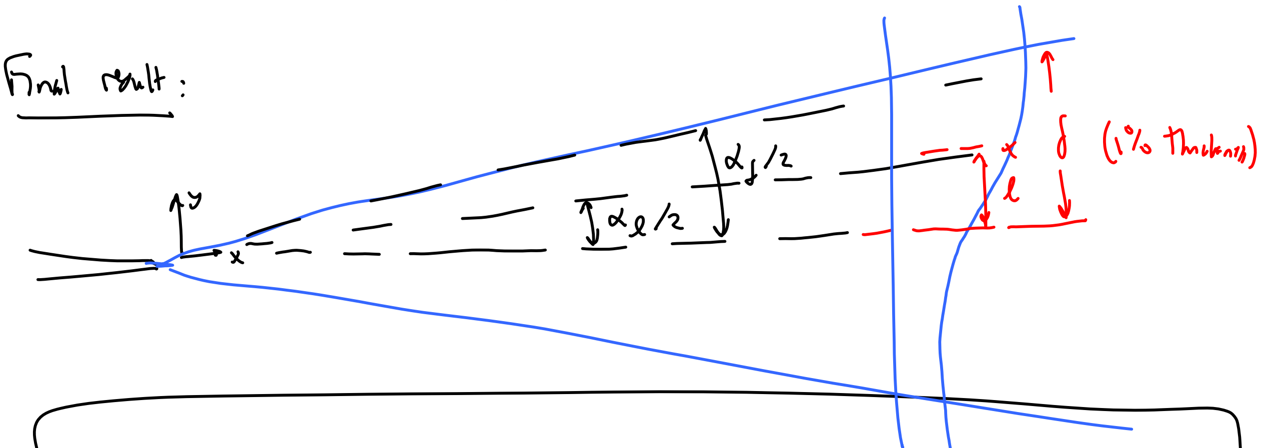
$$\Pi_1 = \frac{\ell}{x} = \text{const.}$$

Turbulent M.L. grows linearly with x

Similarity Soln → Do a similar analysis as with the jet

↓

Final result:



$\boxed{\alpha_\ell \text{ total spreading angle} \approx 13^\circ \text{ for any Turbulent Mixing Layer}}$

@ any Re.

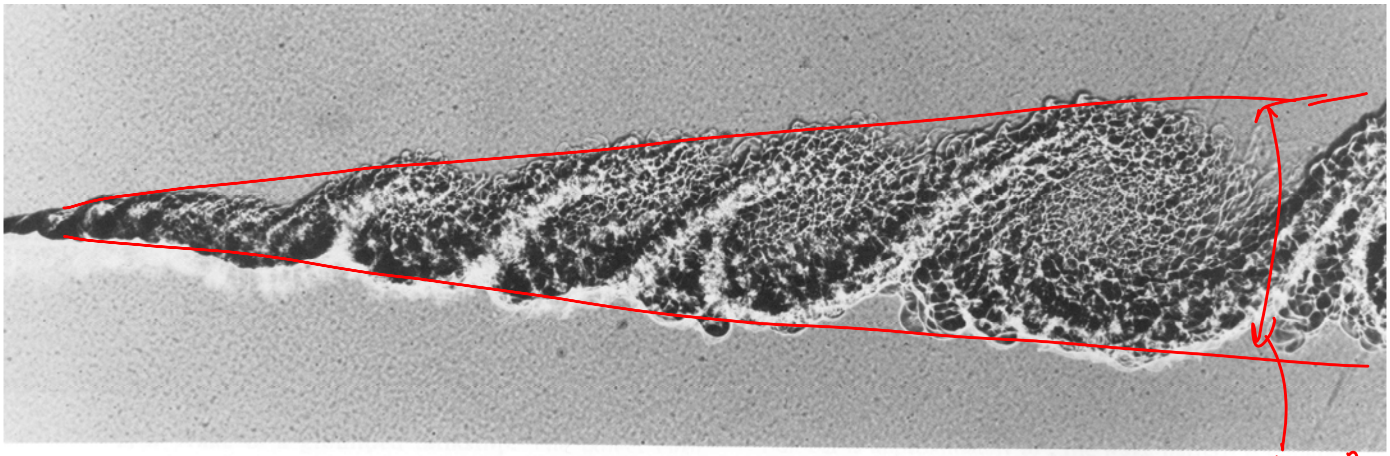
Turbulent Mixing Layers

Author: John M. Cimbala, Penn State University
Latest revision: 01 May 2008

176. Large-scale structure in a turbulent mixing layer. Nitrogen above flowing at 1000 cm/s mixes with a helium-argon mixture below at the same density flowing at 380 cm/s under a pressure of 4 atmospheres. Spark shadow photography shows simultaneous edge and plan views, demonstrating the spanwise organization of the large eddies. The streamwise streaks in the plan view (of which half the span is shown) correspond to a system of secondary vortex pairs oriented in the streamwise direction. Their spacing at the downstream side of the layer is larger than near the beginning. *Photograph by J. H. Konrad, Ph.D. thesis, Calif. Inst. of Tech.*, 1976.

Coherent structures

From: Van Dyke, M., *An Album of Fluid Motion*, Stanford, CA, The Parabolic Press, 1982, p.102. Re ≈ 4×10^5.



$\alpha \approx 13°$

177. Coherent structure at higher Reynolds number. This flow is as above but at twice the pressure. Doubling the Reynolds number has produced more small-scale structure without significantly altering the large-scale structure. *M. R. Rebollo, Ph.D. thesis, Calif. Inst. of Tech., 1976; Brown & Roshko 1974*

From: Van Dyke, M., *An Album of Fluid Motion*, Stanford, CA, The Parabolic Press, 1982, p.102. Re ≈ 8×10^5.

THE END !