The Standard \( K-\varepsilon \) Two-Equation Turbulence Model (High Reynolds number form)

Two-equation turbulence models still employ the eddy viscosity assumption. However, the mixing length is removed since both \( u' \) and \( \ell_m \) are specified: \( u' = \sqrt{K} \) (same as one-equation models), and

\[
\ell_m \approx \text{constant} \cdot u'^3 = \text{constant} \cdot \frac{K^\frac{3}{2}}{\varepsilon}
\]

Plugging these into the definition of eddy viscosity, \( \nu_c \approx \text{constant} \cdot \ell_m \cdot u' \) yields \( \nu_c = C_\mu \frac{K^2}{\varepsilon} \), where \( C_\mu \) is a new (combined) constant. Since \( K \) and \( \varepsilon \) are additional unknowns, two additional transport equations must be solved: (1) the same \( K \) equation we used for one-equation models, and (2) a modeled transport equation for \( \varepsilon \),

\[
\frac{DK}{Dt} = \frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu_c \frac{\partial K}{\sigma_k \partial x_j} \right) + \nu_c \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} - \varepsilon
\]

Total rate of change = Diffusion (transport) + Production – Dissipation

\[
\frac{D\varepsilon}{Dt} = \frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu_c \frac{\partial \varepsilon}{\sigma_\varepsilon \partial x_j} \right) + \frac{C_{\mu} \nu_c \varepsilon}{K} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} - C_{\varepsilon} \frac{\varepsilon^2}{K}
\]

Total rate of change = Diffusion (transport) + Production – Dissipation

The standard constants are \( \sigma_k = 1.0 \), \( \sigma_\varepsilon = 1.3 \), \( C_\mu = 0.09 \), \( C_{\varepsilon_1} = 1.44 \), and \( C_{\varepsilon_2} = 1.92 \).

Alternate Version of \( K-\varepsilon \) Transport Equations: Note: Wilcox (and some other authors) define both dissipation and molecular transport differently. With his definitions, the \( K-\varepsilon \) transport equations are given as

\[
\frac{DK}{Dt} = \frac{\partial K}{\partial t} + U_j \frac{\partial K}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial K}{\sigma_k \partial x_j} \right) + \nu \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} - \varepsilon
\]

Total rate of change = Diffusion (transport) + Production – Dissipation

\[
\frac{D\varepsilon}{Dt} = \frac{\partial \varepsilon}{\partial t} + U_j \frac{\partial \varepsilon}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \nu \frac{\partial \varepsilon}{\sigma_\varepsilon \partial x_j} \right) + \frac{C_{\mu} \nu \varepsilon}{K} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) \frac{\partial U_i}{\partial x_j} - C_{\varepsilon} \frac{\varepsilon^2}{K} - R
\]

where \( R = f(\zeta) \cdot P_k \cdot \varepsilon \)

and

\[
\zeta = (2E_i E_j) \frac{K}{\varepsilon}, \quad \text{where} \quad E_j = \frac{1}{2} \left( \frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right)
\]

The RNG constants are \( \sigma_k = 0.719 \), \( \sigma_\varepsilon = 0.719 \), \( C_{\mu} = 0.085 \), \( C_{\varepsilon_1} = 1.42 \), and \( C_{\varepsilon_2} = 1.91 \).

Studies have shown that this extra term generally leads to better solutions, especially in flows with swirl.