Identification of an Industrial Process by Numerical Inversion of the Laplace Transform

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Abstract—A practical application of a novel parameter identification technique for industrial process control systems is presented. The solution algorithm is based on numerical inversion of the Laplace Transform, as proposed by Bellman, *et al.* Its application with the aid of a minicomputer (or a desk calculator) is fairly easy and fast. Using this method, process gain and time-constants of an electric furnace with on-off control are evaluated. Results of an experimental study are presented to validate the theoretical predictions. This identification technique is suitable for a wide range of industrial processes, and it can be applied to direct digital control systems.

NOMENCLATURE

a, a'	Constant parameters for process.
b, b'	Constant parameters for process.
C_0, C_1	Initial conditions for process differential equations.
F	Constant parameter for process.
f	Forcing function.
N	Order of Legendre-Gauss quadrature formula.
P_N	Legendre polynomial of order N.
$P_N{}^*$	Shifted Legendre polynomial of order N.
s	Laplace transform variable.
$T_1, T_1{}'$	Furnace time-constants.
T_2	Thermowell and thermocouple time-constant.
t	Time.
t_i	Prescribed instants of time.
w_i	Christoffel weights.
x_i	Roots of shifted Legendre polynomials.
β	Derivative coefficient.
ϵ	Weighted mean square error.
Θ	Laplace transform of temperature θ.
Φ	Estimated value of Θ from experimental data.
θ	Temperature.
θ_x	Temperature set point.
τ_d	Dead time.
ξ	Amplitude of oscillations of temperature around the set point.

INTRODUCTION

SATISFACTORY controller design for industrial processes often requires accurate parameter identification from experimental data. For multiple capacitance processes, these parameters are usually process gain, dead time, and dominant time-constants. Existing identification methods [1]-[4] may not be readily adaptable in many industrial systems because of computer size and speed requirements. On the other hand, graphical and other simpler techniques [5] do not provide sufficient accuracy. This paper presents practical applications of an analytical technique for identifying essential parameters from experimental data. Although the algorithm is based on advanced mathematical principles [6], [7], the required computation with the aid of a minicomputer (or a desk calculator) is fairly easy and fast. This identification technique is also applicable for on-line computer control for rapidly changing processes [7].

In the following sections, mathematical description and experimental verification of the method are presented. Design of an improved compensator for a simple temperature control process is given to illustrate how the method is applied.

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MATHEMATICAL DESCRIPTION

The algorithm is based on numerical inversion of the Laplace transform, as formulated by Bellman *et al.* [6], [7], and has been extensively reported in literature [8] and applied in many diverse fields [9]-[11]. However, its application to industrial process parameter identification has apparently not been attempted before.

The algorithm applies to finite-dimensional linear time-invariant systems [6]. The mathematical steps will be derived for a two-dimensional case for illustrative purposes, and there is no loss of generality.

Let a process with two capacitances and no dead time be expressed in differential equation form, following the notation of Bellman *et al.* [6], [7] as

$$\ddot{\theta} + a\dot{\theta} + b\theta = f(t) \tag{1}$$

with initial conditons $\theta(0) = C_0$ assumed known, and $\dot{\theta}(0) = C_1$ unknown.

Laplace transform of (1) yields

$$\Theta(s) = \frac{F(s) + (s + a)C_0 + C_1}{s^2 + as + b} \tag{2}$$

where

$$\Theta(s) = \int_0^\infty \theta(t) \exp(-st)\, dt, \qquad t > 0. \tag{3}$$

Substituting $t = -\ln x$, (3) may be expressed as

$$\Theta(s) = \int_0^1 \theta(-\ln x)x^{s-1}\,dx. \tag{4}$$

The novelty of the method lies in approximating the integral with Legendre-Gauss quadrature formula of order N. In the interval [0, 1], it has the form

$$\Theta(s) \cong \sum_{i=1}^{N} w_i\theta(t_i)x_i^{s-1}, \tag{5}$$

where x_i are roots of the shifted Legendre polynomial $P_N^*(x) = P_N(1-2x)$; w_i are corresponding Christoffel weights; and $\theta(t_i)$ are observed values of the process output $\theta(t)$ at instants t_i where $t_i = -ln\ x_i$. The values of x_i, w_i, and t_i are available in tabular form [6] for $N = 3, 4, \ldots, 15$.

Equation (5) allows determination of the approximate Laplace transform of $\theta(t)$ by observing the process at specified time intervals. Let this approximate Laplace transform be designated by $\Phi(s)$. It is important to note that $\Phi(s)$ is not identically equal to $\Theta(s)$, the exact Laplace transform of the process output $\theta(t)$ given by (2).

The main task in modeling the process is to choose parameters a and b that make $\Theta(s)$ as close to $\Phi(s)$ as possible. To accomplish this, mean square error between $\Theta(s)$ and $\Phi(s)$ is minimized. However, for computational convenience a weighted mean square error ϵ is considered.

$$\epsilon = \sum_{s=1}^{N} [(\Phi(s) - \Theta(s))(s^2 + as + b)]^2. \tag{6}$$

Substituting (2) yields

$$\epsilon = \sum_{s=1}^{N} [(s^2 + as + b)\phi(s) - (F(s) + C_0 s + C_1 + aC_0)]^2 \tag{7}$$

The problem is restricted at this stage to step inputs only. Let the step magnitude be F, an unknown constant. Hence, $F(s) = F/s$. ϵ is minimized over the four variables a, b, C_1, and F only if the following relationships are satisfied:

$$\frac{\partial\epsilon}{\partial a} = \frac{\partial\epsilon}{\partial b} = \frac{\partial\epsilon}{\partial C_1} = \frac{\partial\epsilon}{\partial F} = 0. \tag{8}$$

Solution of (8) and (7) yields the following relationship:

$$V = AU \tag{9}$$

where V and U are column vectors, and A is a square matrix defined as

$$V = \begin{bmatrix} -\sum_{s=1}^{N} s(s\Phi(s) - C_0)^2 \\ -\sum_{s=1}^{N} s(s\Phi(s) - C_0)\Phi(s) \\ -\sum_{s=1}^{N} s(s\Phi(s) - C_0) \\ -\sum_{s=1}^{N} (s\Phi(s) - C_0) \end{bmatrix}; U = \begin{bmatrix} a \\ b \\ C_1 \\ F \end{bmatrix};$$

$$A = \begin{bmatrix} \sum_{s=1}^{N} (s\Phi(s) - C_0)^2 & \sum_{s=1}^{N} (s\Phi(s) - C_0)\Phi(s) & -\sum_{s=1}^{N} (s\Phi(s) - C_0) & -\sum_{s=1}^{N} (s\Phi(s) - C_0)/s \\ \sum_{s=1}^{N} (s\Phi(s) - C_0)\Phi(s) & \sum_{s=1}^{N} (\Phi(s))^2 & -\sum_{s=1}^{N} \Phi(s) & -\sum_{s=1}^{N} (\Phi(s))/s \\ \sum_{s=1}^{N} (s\Phi(s) - C_0) & \sum_{s=1}^{N} \Phi(s) & -N & -\sum_{s=1}^{N} 1/s \\ \sum_{s=1}^{N} (s\Phi(s) - C_0)/s & \sum_{s=1}^{N} (\Phi(s))/s & -\sum_{s=1}^{N} 1/s & -\sum_{s=1}^{N} 1/s^2 \end{bmatrix}$$

The matrix A has, in general, an inverse [6]; hence, U can be found easily from the above equation, yielding values for a, b, C_1, and F.

MODELING OF AN ELECTRIC FURNACE

The technique described above is used to identify parameters of a small electric furnace regulated by an on-off controller. The furnace is resistance-heated and is typical of those used for heat treatment of metals [12], [13]. To evaluate the unknown parameters, the temperature transients were obtained experimentally and are given by curves 1 to 8 in Figs. 1 and 2.

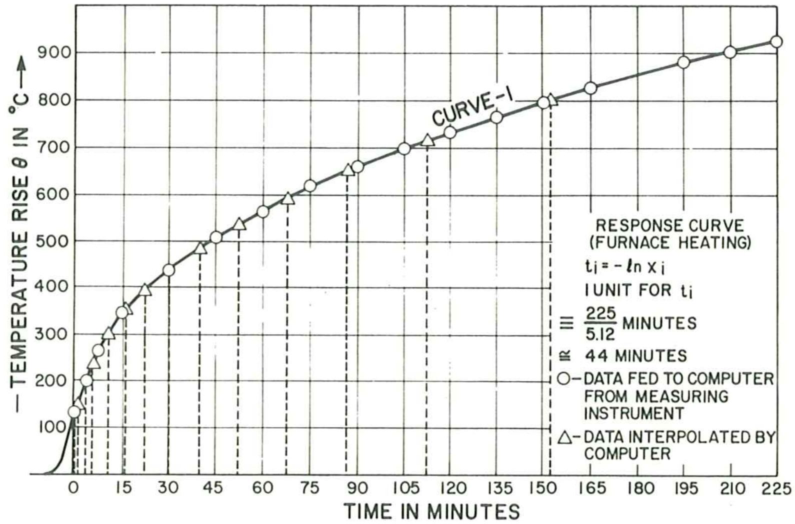

Fig. 1. Time response of furnace temperature in the active mode.

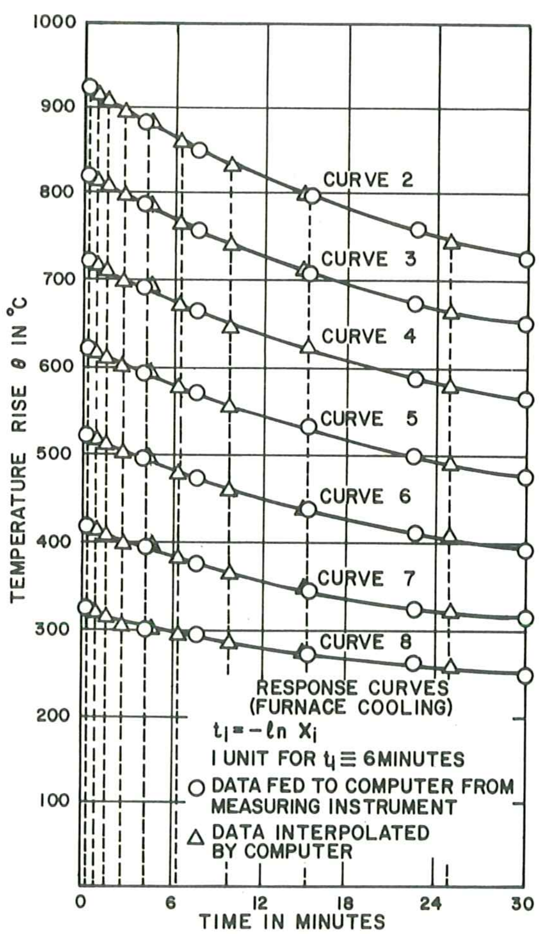

Fig. 2. Time response of furnace temperature in the passive mode.

Uncompensated on-off controlled processes exhibit cyclic operation. To minimize the amplitude of oscillations around a set point, a compensator was designed on the basis of time-constants associated with heating (active) and cooling (passive) modes. The active time-constant is different (usually smaller) from the passive time-constant. As the heat source is turned off, the furnace is cooled by radiation loss from its external surface. In the narrow operating range around a certain set point, the nonlinear cooling process is approximated by its locally linearized characteristics. Thus, the passive time-constant is strongly dependent on initial temperature.

In earlier studies [14]-[16], the temperature transients have been represented by a single time-constant. If this process could be described by a single time-constant, the response would appear as a sawtooth wave, rather than smoothly oscillating as commonly observed. Thus, the presence of at least one more significant time-constant cannot be ignored.

In the following analysis, the second time-constant is assumed mode-independent. Physically, this time-constant is related to the time-lag effect of thermowell and thermocouple and is unaffected by the laws of heating and cooling.

There is practically no dead time associated with cooling [13], [14]. In active mode, the dead time τ_d was evaluated from experimental response (Fig. 1) and was found to be negligible because the furnace under consideration has a small heating chamber. In general, τ_d is not negligible and must be determined separately. Further analytical work is needed to apply this technique for identification of dead time, which is beyond the scope of the present discussion.

The differential equations governing the furnace temperature are

$$\ddot{\theta} + a\dot{\theta} + b\theta = F, \qquad \text{in the active mode} \tag{10}$$

and

$$\ddot{\theta} + a'\dot{\theta} + b'\theta = 0, \qquad \text{in the passive mode} \tag{11}$$

where F, a, b, a', b', and initial slope $\dot{\theta}(0) = C_1$ are unknown. F is dependent on the rate of heat input, assumed constant in the active mode. The process gain and time-constants are evaluated from F, a, b, a', and b'.

T_1 and T_2 are time-constants in the active mode, and T_1' and T_2' in the passive mode. The dominant time-constants T_1 and T_1' are mode-dependent; the passive time-constant T_1' is a function of initial temperature C_0. The nondominant parameters T_2 and T_2' due to thermowell and thermocouple are very nearly equal.

Active mode parameters are evaluated from (9) for $N = 15$. $\Phi(s)$ were calculated from the experimental data (Fig. 1) using (5). In the passive mode, the forcing function $F = 0$ reduces the A matrix dimension to three, and in this case (9) was solved for $N = 9$ at several operating points (Fig. 2). The results are summarized below:

$T_1 = 143$ min; $T_2' \cong T_2 = 5.2$ min

C_0 (deg C)	920	820	720	620	520	420	320
T_1' (min)	147	150	155	170	200	240	300.

Final steady-state value of the furnace temperature rise above ambient (if allowed to attain) was determined to be 1064°C.

VERIFICATION OF RESULTS

To verify the accuracy of the parameters estimated, the process was simulated on an analog computer. The impulse response of the simulated process is

$$G_p(s) = \begin{cases} \dfrac{K}{(T_1 s + 1)(T_2 s + 1)} & \text{in the active mode} \\ \dfrac{C_0}{(T_1' s + 1)(T_2 s + 1)} & \text{in the passive mode,} \end{cases}$$

and dead time is assumed to be zero. Transition from one mode to another was simulated by a comparator and a relay.

Amplitudes of steady-state oscillation at several set points were measured and compared with those obtained from furnace temperature records. Fig. 3 shows that the oscillation amplitude ξ obtained from simulation results agrees closely with experimental data at higher values of set point θ_x and gradually diverges at lower values of θ_x. This is explained by the fact that ξ increases monotonically with decreasing θ_x, causing relatively larger errors in the linear representation of the nonlinear cooling process. It is possible to divide the range of oscillations (at a particular set point θ_x) into piecewise linear zones and obtain a better representation of the nonlinear process. However, in this particular application it is not necessary because the modeling error is not large (see Fig. 3).

COMPENSATOR DESIGN

Derivative action was selected for compensator design [15]. Intuitively, anticipatory cut-in and cut-off of power would tend to minimize the amplitude of oscillations around the set point. On the basis of the simulated process, the amplitude of oscillations ξ is plotted for different values of derivative time β and set point θ_x in Fig. 4. This family of design curves enables selection of β to achieve a specified ξ. However, any severe restrictions on ξ trade off with possible noise and high frequency oscillations.

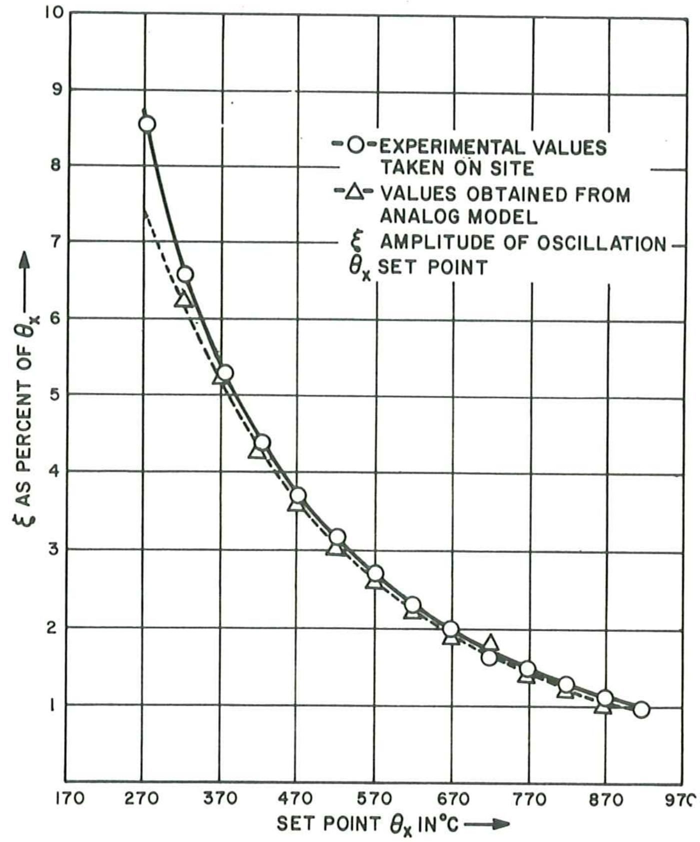

Fig. 3. Comparison of experimental and simulation results.

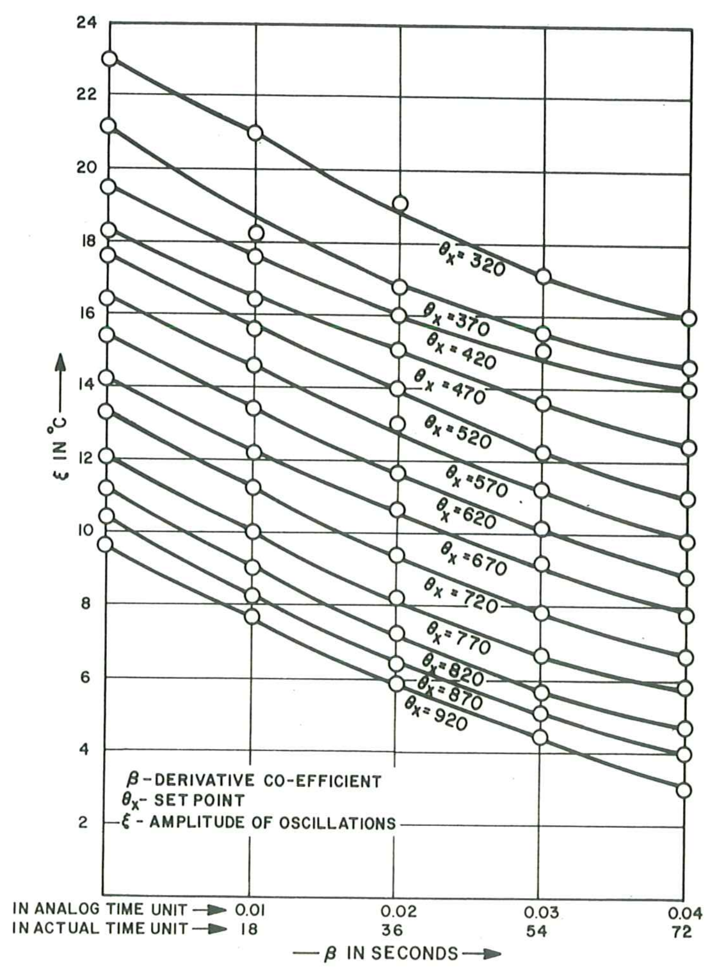

Fig. 4. Family of curves for temperature compensator.

CONCLUSIONS

A practical application of a relatively unfamiliar parameter identification technique is presented. The technique has been used to evaluate process gain and time-constants of an electric furnace. The results were experimentally verified and used for the design of an improved compensator.

The method of analysis is particularly suitable for processes with smooth and monotonic transients and may be used for parameter identification of a wide range of industrial applications. Due to its simplicity and speed, this method has potential for the implementation of control algorithms on small computers in direct digital applications.

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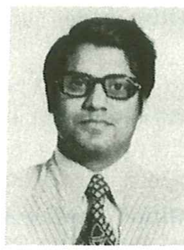

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