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Dynamic modeling and simulation of a relief valve

by
Asok Ray
Mechanical Engineering Department
Carnegie-Mellon University
Pittsburgh, Pennsylvania 15213

ASOK RAY received his BE and ME degrees in electrical engineering from Calcutta University, India, and his MS in mathematics and PhD in mechanical engineering from Northeastern University in Boston, Massachusetts. For the last six years he has been engaged in dynamic modeling, simulation, and control system design of nuclear- and fossil-fueled power plants. His present academic interests include abstract and applied mathematics. Dr. Ray is a registered Professional Engineer in the Commonwealth of Massachusetts.

ABSTRACT

A nonlinear dynamic model of a relief valve is formulated in state-space form from fundamental principles of rigid-body motion and fluid dynamics. Model parameters are calculated from steady-state characteristics and the physical dimensions of the valve. The transient response of the nonlinear model (as well as system eigenvalues and the frequency response of a linearized model) is obtained by digital simulation. Results indicate that the opening time of the valve is linearly related to the dimensionless parameter given by the ratio of orifice length to its radius.

The analysis provides design information and performance evaluation for fluid systems incorporating relief valves. It is particularly suitable for relief valves for liquids and it can be adapted for other applications.

INTRODUCTION

A relief valve protects equipment against excessively high pressure.[1,3] It opens when vessel or pipeline pressure exceeds the allowable maximum value, releasing fluid to maintain pressure within design limits.

Hydraulic transients in relief valves are important in fluid system design. If the initial delay in opening the valve is comparable to the response time for fluid flow, system dynamics will be influenced by the location and characteristics of the valve. For example, tube rupture in a heat exchanger (containing liquid on the low-pressure side) may cause a pressure surge lasting a few milliseconds, whereas the opening time of the relief valve, on the low-pressure side, may be tens of milliseconds.[12] On the other hand, if the frequency of excursions in the inlet pressure of the valve matches its natural frequency, the valve may oscillate and chatter excessively.

Although fluid systems are often designed on the basis of steady-state valve performance,[1,3] some investigators have recognized the influence of relief-valve dynamics on the performance of high-pressure fluid systems.[5,12] To simulate relief-valve dynamics, Fowler *et al.*[5] considered only the mass-spring effect in studying pressure surges in a heat exchanger. In a similar study, Sumaria *et al.*[12] included fluid inertia effects in the valve orifice but did not arrive at an explicit model of the valve.

In this paper, a nonlinear time-invariant deterministic model of a relief valve is formulated in state-space form. Dynamic equations are derived from fundamental principles of rigid-body motion and fluid dynamics.[6,10] A semiempirical steady-state model is developed; it produces results that match the manufacturer's data.[3] Despite the empirical element in its design, the model uses the basic principles of fluid mechanics.[2,10]

Although the model is formulated for a relief valve for liquids, the approach can be adapted to other fluids. For example, Liao's[8] steady-state model of a relief-valve for vapor can be modified to incorporate dynamic effects.

The set of equations which form the mathematical model is given in the Appendix. Numerical results for the transient response of selected system variables were obtained from the nonlinear model using CSMP-III[7] on an IBM 370/158 computer. Eigenvalues and frequency response of linearized system models were generated from a FORTRAN version of the CSMP-III model used as a subroutine in general-purpose analytical programs.[4,11] Using a load module, the typical computer execution time for simulating the transients for a period of 1 second was about 5 CPU seconds on an IBM 370/158 (loading time excluded).

NOMENCLATURE

- a Coefficients
- A Effective valve disc area exposed to fluid pressure P_2
- A_m Maximum value of effective valve disc area
- A_o Orifice area
- B Body force
- cs Control surface
- cv Control volume
- C_d Discharge coefficient of valve
- f Coefficient for viscous damping for moving parts
- F Force acting on the entire control surface except the valve disc
- g Acceleration of gravity
- i Index
- j Index
- K_p Overpressure factor
- K_s Spring constant
- M Mass of moving parts
- n Unit normal vector
- P^* Set pressure in gauge
- P_1 Valve inlet pressure in gauge
- P_2 Pressure at valve seat in gauge
- r_o Orifice radius
- R Reference vector
- S Surface forces
- t Time
- u Fluid velocity
- V Fluid volume
- W Mass flow rate of fluid
- W_m Maximum mass flow rate of fluid
- z Valve lift
- z_m Maximum lift of valve
- z_o Orifice length
- z_s Initial deflection of spring
- α Normalized overpressure
- α_m Maximum value of normalized overpressure
- ρ Fluid density
- ξ Normalized valve lift (z/z_m)

DESCRIPTION AND OPERATING PRINCIPLE

The relief valve considered in this analysis (see Figure 1) is typical of those used in process and power industries for liquid service. It is a continuous-action device which functions as a spring-loaded pressure regulator. As the driving force (fluid pressure) overcomes the spring force, the valve starts to allow the liquid to escape. Further increases in system pressure lift the valve disc from its seat, and the flow rate increases. The flow is diverted downwards by the cup-shaped disc, and its direction is reversed. This creates reaction forces that lift the disc further. The valve discharges to atmospheric pressure.

ASSUMPTIONS

This study assumes that the infinite-dimensional distributed-parameter process can be represented by a finite-dimensional lumped-parameter model. This approach has been experimentally verified in dynamic modeling of electro-hydraulic servovalves.[9] Other pertinent assumptions are as follows:

(1) The pressure drop in the vertical portion of the valve orifice is caused by fluid inertia alone.

(2) The valve spring is linear.

(3) Coulomb friction in the moving parts is negligible.

(4) Changes in fluid density are negligible.

(5) The pressure difference between valve opening and closing (hysteresis) is not significant.

The following parameters were evaluated and found to be negligible:

(1) Pressure-drop caused by gravity inside the valve

(2) Frictional pressure drop in the vertical portion of the valve orifice

(3) Fluid mass inside the valve compared to the mass of the moving parts.

DEVELOPMENT OF MODEL EQUATIONS

A valve schematic showing the fluid control volume and moving parts is given in Figure 1. Conservation of linear momentum[6,10] in Cartesian tensor notation yields

$$S_i + \iiint_{cv} (B_i - \ddot{R}_i)\rho dV = \frac{\partial}{\partial t} \iiint_{cv} u_i \rho dV + \iint_{cs} u_i \rho u_j n_j dA \quad (1)$$

On a lumped average basis, the z-components in (1) for a homogeneous incompressible fluid can be approximated as

$$S_z - (g + \ddot{z})A_o z_o \rho = \frac{d}{dt}\left[W(z_o + z)\right] - (u_1 + u_3)W \quad (2)$$

where mass flow rate $W = \rho A_1 u_1 = \rho A_2 u_2$. The surface forces S_z in the z-direction can be split into two parts: the force exerted by the valve disc on the fluid, and the z-component F_z of the forces acting on the entire control surface except the valve disc.

$$S_z = -M(g + \ddot{z}) - f\dot{z} - K_s(z_s + z) + F_z \quad (3)$$

Substituting (3) in (2), and neglecting the term $A_o z_o \rho$ (because $A_o z_o \rho << M$) yields

$$\frac{d}{dt}[W(z_o + z)] = -M(g + \ddot{z}) - f\dot{z} - K_s(z_s + z) + F_z + (u_1 + u_3)W \quad (4)$$

Steady-state model

The steady-state form of (4) is

$$F_z = Mg + K_s(z_s + z) - (u_1 + u_3)W \tag{5}$$

The left-hand part of the above equation depends on line pressure P_1 and flow W, and an explicit analytical relationship is difficult to obtain. This problem is circumvented by constructing a semiempirical relationship in which parameters are adjusted so that the steady-state model results match the manufacturer's data.[3] This approach simplifies the model structure and thus facilitates simulation of the overall fluid system. It proceeds as follows:

Define a variable A (call it the effective area of the valve disc) such that

$$P_1A = F_z + (u_1 + u_3)W \tag{6}$$

When the valve is about to open at the set value P^* of line pressure P_1, Equation 5 reduces to

$$P^*A_o = K_sz_s + Mg \tag{7}$$

For steady-state conditions with valve lift z ($0 \le z \le z_m$) and normalized overpressure α (i.e., $P_1 = (1+\alpha)P^*$), Equations 5, 6, and 7 yield

$$A = [A_o + K_sz/P^*]/(1+\alpha) \tag{8}$$

At zero overpressure ($\alpha = 0$), the valve is closed (i.e., $z = 0$) and $A = A_o$. A is a strictly monotonically increasing function of z, i.e., the effective area of the valve disc increases as the valve rises. At maximum overpressure α_m, the valve is fully open ($z = z_m$) and the annular opening area for fluid flow is equal to the orifice area,[2] i.e., $2\pi r_oz_m = \pi r_o^2$, i.e., $z_m = r_o/2$. In this state, A also attains its maximum value and can be evaluated as

$$A_m = [A_o + K_sz_m/P^*]/(1+\alpha_m) \tag{9}$$

Using (9) in (8), A is obtained in terms of normalized valve lift ($\xi = z/z_m$) and overpressure α.

$$A = [(1+\alpha_m)/(1+\alpha)]\xi A_m + [(1-\xi)/(1+\alpha)]A_o \tag{10}$$

For incompressible flow, the mass flow rate W is approximated by the orifice equation[6,7]

$$W = C_d A_o \xi [2P^*(1+\alpha)\rho]^{\frac{1}{2}} \tag{11}$$

neglecting the gravity head of fluid since $z_og\rho << P^*(1+\alpha)$. For the fully open condition ($\xi_m = 1$), the maximum flow rate is

$$W_m = C_dA_o[2P^*(1+\alpha_m)\rho]^{\frac{1}{2}} \tag{12}$$

The overpressure factor K_p[1,3] can be found from (11) and (12):

$$K_p = W/W_m = \xi[(1+\alpha)/(1+\alpha_m)]^{\frac{1}{2}} \tag{13}$$

The manufacturer provides steady-state data (Table 1) in the form of the overpressure factor K_p as a func-

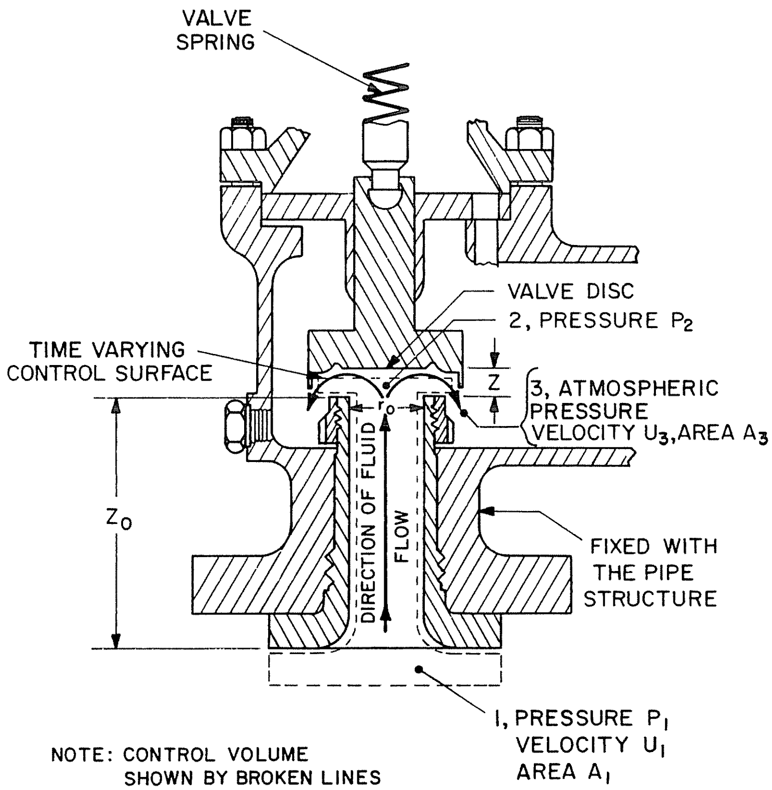

Figure 1 - Simplified valve schematic. The numbers 1, 2, 3 refer to the pipe carrying the fluid, the valve disc, and the relief outlet to atmospheric pressure, respectively.

Table 1

Steady-state valve characteristics

Normalized overpressure*	α	0.0	0.05	0.1	0.25
Overpressure factor*	K_p	0.0	0.34	0.6	1.0
Normalized valve lift	ξ	0.0	0.371	0.639 6	1.0
Effective valve seat area	A	A_o	$0.441\,6\,A_m + 0.599\,A_o$	$0.726\,8\,A_m + 0.327\,6\,A_o$	A_m

* Available directly from the manufacturer's data

tion of normalized overpressure α.[3] Using these data in Equation 13, ξ is obtained as a single-valued function of α, which is substituted in (10) to obtain effective valve area A as a function of normalized valve lift. A (ξ) is approximated by a third-order polynomial in the range of interest.

$$A = \sum_{i=0}^{3} a_i \xi^i \qquad (14)$$

The coefficients a_0, a_1, a_2, and a_3 should be evaluated from the manufacturer's data.

Dynamic equations

The dynamics of the valve is given by

$$M(\ddot{z} + g) + f\dot{z} + K_s(z_s + z) = P_2 A \qquad (15)$$

Equation 15 is combined with Equation 7 to give

$$\ddot{z} = (P_2 A - P^* A_o - f\dot{z} - K_s z) / M \qquad (16)$$

The pressure drop from location '1' to location '3' in Figure 1 is primarily caused by flow resistance and inertia. The stagnation pressures at locations '1' and '2' are almost equal in the steady state but may differ significantly under transient conditions because of fluid inertia in the orifice. Then, the pressure drop from location '2' to location '3' can be treated as the effect of flow resistance only and, in a form similar to (11), can be expressed as

$$W = C_d A_o \xi (2P_2 \rho)^{\frac{1}{2}} \qquad (17)$$

The distributed fluid-flow process has been approximated by a lumped model in which the pressure drop from '1' to '3' has been split into two parts: the transient component between '1' and '2' and the steady-state component between '2' and '3'.

Combining (6) and (15) with (4), and setting $P_1 = (1+\alpha)P^*$ yields

$$\frac{d}{dt}[W(z_o + z)] = [P^*(1+\alpha) - P_2]A \qquad (18)$$

Substituting (17) in (18), setting $\xi = z/z_m$, and rearranging yields

$$\frac{dP_2}{dt} = 2\left[\frac{[P^*(1+\alpha) - P_2]Az_m}{(2\rho/P_2)^{\frac{1}{2}} C_d A_o} - (z_o + 2z)P_2\dot{z}\right] / [z(z_o + z)] \qquad (19)$$

Equations 16 and 19 can be arranged in state-space form with z, $\dot{z}$, and P_2 as the state variables. The model equations and physical constraints are given in the appendix.

SIMULATION RESULTS AND DISCUSSION

The physical dimensions and operating conditions for the relief valve are given in Table 2. The simulation results are presented in Figures 2 to 4. The transient responses of the model were observed for a 10-percent step increase in valve inlet pressure from a steady-state condition P^* = 100 psig ($0.6894 \times 10^6 N/m^2 g$). Initial (state variable) conditions were

$$z(0) = 0 \quad ; \quad \dot{z}(0) = 0 \quad ; \quad P_2(0) = P^*$$

The nonlinear model was linearized at equilibrium condition corresponding to 10-percent overpressure. Frequency responses were obtained for this linear model.

Figure 2 shows the dynamics of valve lift caused by a 10-percent step increase in overpressure in the fluid line. After an initial delay, the position of the valve stem approaches a steady-state value asymptotically. The dynamic response of the valve depends on physical dimensions. Figure 3 shows the (almost) linear dependence of the opening time of the valve on the dimensionless number z_o/r_o, the ratio of orifice length to orifice radius; the opening time is defined as the time required for the valve to reach 95 percent of its final steady-state position following a step disturbance of the pressure in the fluid line. Hence, it is possible to estimate the opening time of a valve from the known data for another valve of similar type.

Table 2

Valve parameters and physical dimensions

$K_b = 6.042 \times 10^4$ N/m (345 lbf/in)
$R_o = 4.750 \times 10^{-3}$ m (0.187 in)
$C_d = 0.640\,5$
$f \approx 0$
M = 0.680 kg (1.5 lbm)
$P^* = 6.894 \times 10^5$ N/m² guage (100 psig)
W_m = 1.1296 kg/s (2.488 8 lbm/s)
$z_o = 4.445 \times 10^{-2}$ m (1.75 in)
$\alpha_m = 0.25$
$\rho = 9.994 \times 10^2$ kg/m³ (62.4 lbm/ft³)

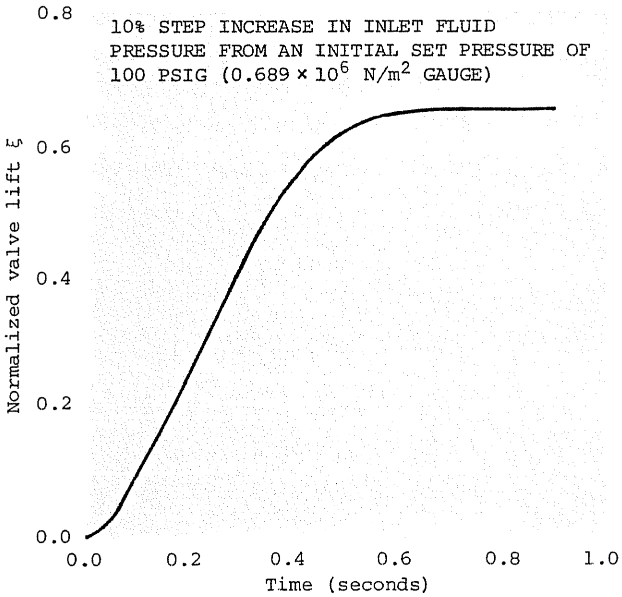

Figure 2 - Dynamic response of valve lift

The system matrix and eigenvalues for the linearized model are given in Table 3. The real negative eigenvalue (which is approximately 11 second^{-1}) causes the sluggish valve response (see Figure 2). The pair of complex eigenvalues indicates the possibility of high-frequency oscillations. The frequency response of normalized valve lift with respect to normalized overpressure is shown in Figure 4. The corner frequency in the vicinity of 10 radians per second is related to fluid inertia in the orifice, and the pair of corner frequencies at approximately 800 radians per second results from the mass-spring combination. Following a pressure surge, acoustic waves and periodic pressure variations are expected at the valve inlet. If the frequency of pressure waves is close

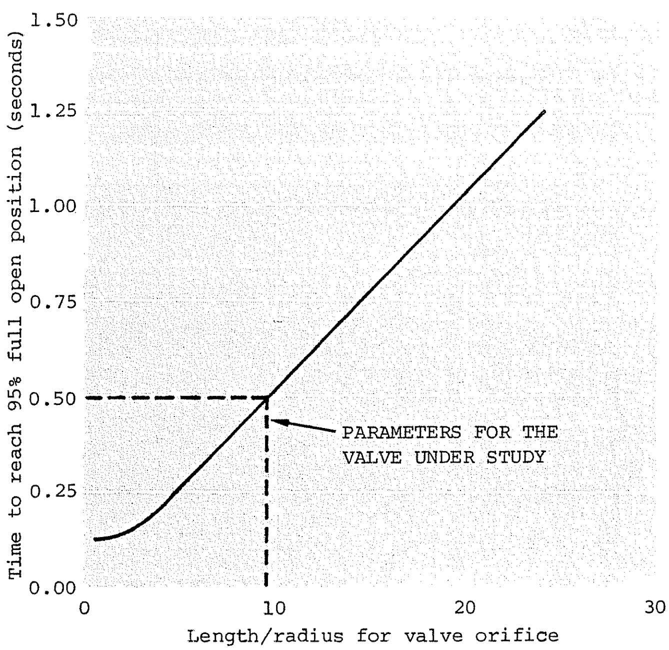

Figure 3 - Parametric evaluation of relief valve response

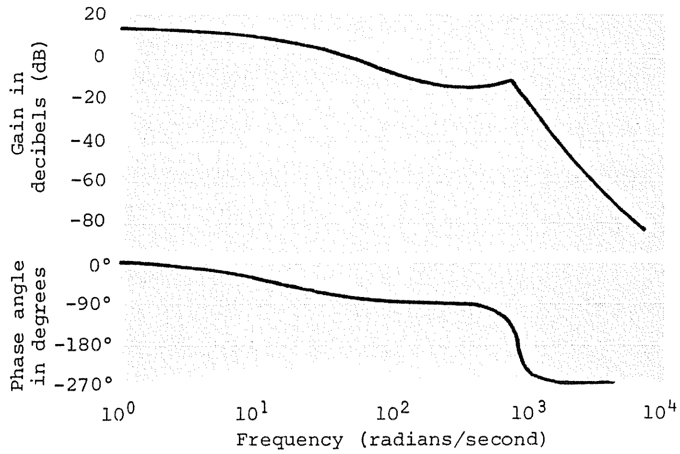

Figure 4 - Frequency response for normalized valve lift versus normalized overpressure

Table 3

System matrix and eigenvalues (model linearized at equilibrium point with 10-percent overpressure)

System matrix

	z	$\dot{z}$	P_2
$\frac{d}{dt}(z)$	0.0	1.0	0.0
$\frac{d}{dt}(\dot{z})$	-4.439×10^4	0.0	13.03
$\frac{d}{dt}(P_2)$	2.741×10^3	-4.537×10^4	-1.681×10^2

System eigenvalues (second^{-1})

Real part	Imaginary part
-11.675	0.0
-78.230	793.8
-78.230	-793.8

to the natural frequency of the valve, the valve may experience excessive oscillations and prolonged chattering.

CONCLUSIONS

A nonlinear dynamic model of a relief valve has been formulated in state-space form from fundamental principles of rigid-body motion and fluid dynamics. The model provides useful information for transient analysis and design of fluid systems incorporating relief valves.

Simulation results indicate that fluid inertia in the valve orifice produces a damping effect on valve motion resulting in an initial delay. The opening time of the valve is approximately proportional to the ratio of orifice length to radius. The mass-spring combination of moving parts may generate oscillations and chattering as a result of high-frequency pressure variations at the inlet of the valve.

The analytical technique presented here is particularly suited for dynamic modeling of relief valves for liquids, and it can be adapted for other applications as well.

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Appendix

Summary of model equations

The differential equations for the selected state variables z, $\dot{z}$, and P_2, are:

$$\frac{d}{dt}(z) = \dot{z} \qquad \text{(A-1)}$$

$$\frac{d}{dt}(\dot{z}) = (P_2 A - P^* A_o - f\dot{z} - K_s z) / M \qquad \text{(A-2)}$$

$$\frac{d}{dt}(P_2) = \frac{2\left[\dfrac{[P^*(1+\alpha) - P_2]Az_m}{(2\rho/P_2)^{\frac{1}{2}} C_d A_o} - (z_o + 2z)P_2\dot{z}\right]}{[(z_o + z)\Gamma]} \qquad \text{(A-3)}$$

where $\Gamma = \max(\delta z_m, z)$ and δ is a parameter chosen by the user (a typical value of δ is 0.001).

Equation A-3 is a modified form of (21), where the Γ factor is introduced to avoid division by zero when the valve lift is zero. Physically, an infinitely large pressure derivative is only possible for a truly incompressible fluid. No real fluid is absolutely incompressible, and the Γ factor circumvents the numerical difficulties.

Supporting algebraic equations are:

$$A_m = [K_s z_m / P^* + A_o] / (1 + \alpha_m) \qquad \text{(A-4)}$$

$$z_m = r_o / 2 \qquad \text{(A-5)}$$

$$A_o = \pi r_o^2 \qquad \text{(A-6)}$$

$$A = \sum_{j=0}^{3} a_j (z/z_m)^j \qquad \text{(A-7)}$$

The coefficients a_j were evaluated from manufacturers' data.[3]

$$a_0 = A_o$$

$$a_1 = 1.197\,92\,A_m - 1.102\,88\,A_o$$

$$a_2 = 0.084\,11\,A_m + 0.060\,53\,A_o$$

$$a_3 = -0.282\,08\,A_m + 0.049\,76\,A_o$$

Physical constraints are

$$z = 0 \quad \text{and} \quad \dot{z} = \max(0, \dot{z}) \quad \text{if} \quad z < 0 \qquad \text{(A-8)}$$

and

$$z = z_m \quad \text{and} \quad \dot{z} = \min(0, \dot{z}) \quad \text{if} \quad z > z_m \qquad \text{(A-9)}$$

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