Dynamic modelling of power plant turbines for controller design

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A nonlinear dynamic model of a steam turbine is formulated from approximation of fundamental equations, and does not rely on empirical relations. The model can be used as a part of an integrated power system model for dynamic simulation and control system design.

Introduction

Mathematical modelling and simulation have proved to be useful, analytical tools for the investigation of potential operational and control problems in electric power plants, as well as for controller design.[1,2] However, the turbine models are often oversimplified with respect to the overall boiler-turbine-generator system simulation. In some cases, the main steam turbines have been represented by simple transfer functions[3] or by empirical relationships.[2] On the other hand, detailed turbine models contain many equations which require complex numerical algorithms (iteration and/or integration for example) to be solved. Although these models are valuable for turbine design and investigation of specific problems, they cannot be used as a part of an overall power system model, because computer simulation cost may be excessive and the task of controller design very difficult.

In this paper, a deductive approach has been used for steam turbine modelling. Instead of relying on empirical relations, model equations were derived by approximating fundamental equations.[4] The model can be used as a part of an integrated power system model for dynamic simulation and control system design. Steady-state performance of the model was verified with heat balance data for a commercial scale turbine, and the model results were found to be suitable for control system design.

Model development

The steady-state model is presented first, and then the dynamic model is developed. Simplifying assumptions are explained when model equations are derived.

Steady-state model

For impulse stages, flow through the nozzles may be subsonic or sonic depending on the pressure ratio across the nozzles. Treating superheated steam as a perfect gas, flow equations were derived; details are available in the standard literature[4] and the final form is:

$$F = C_d A \sqrt{k\left(\frac{2k}{k+1}\right)^{(k+1)/(k-1)} p_0 \rho_0} \qquad \phi(p_0, p_i, k) \tag{1}$$

where

$$\phi = \begin{cases} \left(\frac{k+1}{2}\right)^{1/k-1}\left(\frac{p_i}{p_0}\right)^{1/k}\sqrt{1-\left(\frac{p_i}{p_0}\right)^{(k-1)/k}} & \text{for } \left(\frac{p_i}{p_0}\right) > \left(\frac{2}{k+1}\right)^{k/k-1} \\ 1 & \text{otherwise} \end{cases}$$

C_d is discharge coefficient, and p_0 and p_i are stagnation pressure at nozzle inlet and static pressure at nozzle exhaust, respectively. A typical value of k is 1.3 for superheated steam at standard throttle steam conditions. For impulse stages, pressure drop occurs primarily in the nozzles; it is negligible across the moving blades.[5,6] The 'impulse stage pressure', measured after the moving blades, is assumed equal to nozzle exhaust pressure p_i.

For reaction stages, pressure drop occurs across both moving and stationary rows.[5,6] Steam expanding through a reaction stage experiences much smaller enthalpy and pressure changes than in an impulse stage nozzle. Flow equations for reaction stages are derived as follows.

For frictional flow through a turbine stage, actual enthalpy differential[7] is given by:

$$\mathrm{d}h = \frac{\eta}{\rho}\,\mathrm{d}p \tag{2}$$

where η is turbine stage efficiency ($0 < \eta < 1$) which includes effects of friction losses in blades; a brief discussion

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on η is given in Appendix I. Using the perfect gas relations ($p = R\rho T$, $C_p - C_v = R$), equation (2) yields:

$$\rho T^{-\nu} = \text{constant} \tag{3}$$

where $\nu = [(1-\eta)k + \eta]/[\eta(k-1)]$ and $k = C_p/C_v$.

Neglecting heat transfer and gravitational potential, steady flow through the stationary guide vanes of a reaction stage is approximately:

$$v_d^2 = 2C_p(T_u - T_d)/(1 - \beta^2) \tag{4}$$

where subscripts u and d refer to upstream and downstream of the guide vanes and velocity ratio $\beta = v_u/v_d$. Since the stationary and moving vanes in a reaction stage have identical mechanical structures,[5,6] an expression similar to equation (4) can be obtained for the moving vanes in terms of relative velocities.

Analogous to the common assumption that the departure flow angle relative to the blade remains unchanged, velocity ratio β is assumed invariant for both stationary and moving vanes under design and off-design conditions, i.e., $\beta = \beta^*$. Then, the temperature differences are:

$$\Delta T^* = \frac{v_d^{*2}}{2C_p}(1-\beta^2) \quad \text{at design condition}$$
$$\Delta T = \frac{v_d^{2}}{2C_p}(1-\beta^2) \quad \text{at off-design condition} \tag{5}$$

Steady-state mass flow rates through the turbine stages are $F^* = A\rho^* v_d^*$ and $F = A\rho v_d$ for design and off-design conditions, respectively. Therefore, equation (5) yields:

$$\left(\frac{\rho}{F}\right)^2 \Delta T = \left(\frac{\rho^*}{F^*}\right)^2 \Delta T^* \tag{6}$$

Applying the thermodynamic relation (3) in equation (6):

$$\frac{T^{2\nu}}{F^2}\Delta T = \frac{T^{*2\nu}}{F^{*2}}\Delta T^* \tag{7}$$

Equation (7) is valid for stationary and moving vanes of one turbine stage. Temperature drops across a large number of stages can be obtained as the sum of temperature differences across consecutive turbine stages. Since ΔT across stationary and moving vanes of each single stage is small, these sums are approximated by integrals with the limits i and e signifying inlet and exhaust of the reaction turbine, respectively:

$$\int_i^e \frac{T^{2\nu}}{F^2}\,dT = \int_i^e \frac{T^{*2\nu}}{F^{*2}}\,dT^* \tag{8}$$

Since steady-state mass flow rate through the turbine stages is constant for a given operating condition, F and F^* are independent of T and T^*, respectively. Moreover, the turbine stage efficiency η is approximately constant for all (reaction) stages and does not change significantly for small deviations from design conditions (see Appendix I). Therefore, the index ν (which is a function of η) is treated as a constant. Then, equation (8) yields:

$$(T_e^{2\nu+1} - T_i^{2\nu+1})/F^2 = (T_e^{*2\nu+1} - T_i^{*2\nu+1})/F^{*2} \tag{9}$$

Assuming an identical law of expansion for both design and off-design conditions, equation (3) and the perfect gas law, when applied to equation (9), yields:

$$F = \frac{F^*}{\sqrt{p_i^{*\mu} - p_e^{*\mu}}}\sqrt{p_i^{\mu} - p_e^{\mu}} \tag{10}$$

where $\mu = [k(2-\eta) + \eta]/k$. This equation has a close similarity to the empirical relation suggested by Stodola.[6] For low pressure turbines, p_i is the pressure at turbine inlet. For high pressure turbines, p_i is the impulse stage pressure which is also the pressure at inlet of reaction stages.

Equation (10) was verified for flow through reaction stages of the high pressure turbine in a typical 600 MWe generating unit. *Table 1* shows heat balance data for impulse stage and exhaust pressures at several steam flows and constant throttle pressure. Power plant design assumes that high pressure turbine exhaust pressure p_e to be proportional to steam flow F, i.e. plant load. If $\mu = 2$, equation (10) provides a linear relationship between steam flow F and impulse stage pressure p_i. Assuming $k = 1.3$ for superheated steam and $\eta = 0.87$, the index μ is equal to 1.8. For different steam flow rates (under sequential opening of the eight governor valves), corresponding values of exhaust pressure p_e were interpolated from the heat balance data in *Table 1*. Using different sets of values of F and p_e, the corresponding values of p_i were calculated from equation (10). *Figure 1* shows

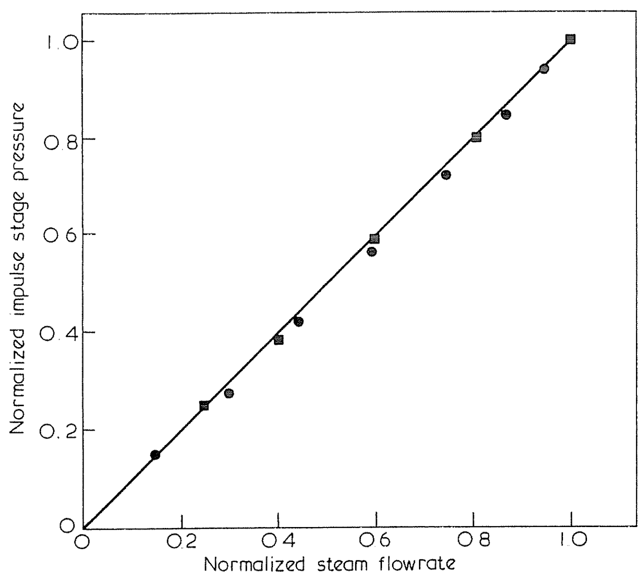

Figure 1 Comparison of model results with turbine heat balance data. (•), model results; (▪), heat balance data

Table 1 High pressure turbine heat balance data. Mode of operation: partial arc (PA). Throttle pressure = 2415 psia (16.651 x 10^6 N/m^2)

Steam flow	10^6 lbm/h	4.1940*	3.3552	2.5164	1.6776	1.0485
	kg/s	528.34	422.68	317.01	211.34	132.09
Exhaust pressure	psia	1000*	800	600	400	250
	10^6 N/m^2	6.8948	5.5158	4.1369	2.7579	1.7237
Impulse stage pressure	psia	1840*	1470	1100	730	460
	10^6 N/m^2	12.686	10.135	7.5842	5.0331	3.1716

* Indicates rated design conditions

profiles of impulse stage pressure p_i/p_i^* versus steam flow F/F^*, both normalized with respect to rated design values. For power plant controller design, impulse stage pressure is often used as an index for plant load. In this perspective, agreement between steady-state model results and heat balance data can be considered satisfactory.

Dynamic model

The lumped parameter approximation of fundamental conservation equations has been experimentally verified for modelling different types of steam power plants.[8–10] In a turbine model, lumped nodes can be selected at the impulse chamber and at any extraction point. With steam density as a state variable, governing equations at each node can be approximated from conservation of mass:

$$\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}t} = (F_1 - \Sigma F_2)/V \tag{11}$$

ΣF_2 represents the sum of all flows leaving the node, into subsequent stages and extraction, if any, and V is the ratio of lumped steam mass to the steady-state value of average density $\bar{\rho}$ at the node. Chamber volume and steady-state values of steam pressure and enthalpy at a number of turbine stages are usually available from manufacturers. Using these data, the lumped volume V around a node can be evaluated. A typical example is given in Appendix 2.

Equation (11) can be modified so that pressure p is the state variable instead of density $\bar{\rho}$. For an isentropic expansion ($\eta = 1$), $\bar{\rho}$ is a function of $\bar{\rho}$ with constant inlet entropy. On the other hand, for a throttling process ($\eta = 0$), $\bar{\rho}$ is a function of $\bar{\rho}$ with constant inlet enthalpy. Therefore, the derivative $\mathrm{d}\bar{\rho}/\mathrm{d}t$ is approximated by the weighted average of isentropic and isenthalpic conditions:

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \Gamma\frac{\mathrm{d}\bar{\rho}}{\mathrm{d}t} = (F_1 - \Sigma F_2)/(V/\Gamma) \tag{12}$$

where $\Gamma = \eta(\partial p/\partial \rho)_{s^*} + (1-\eta)(\partial p/\partial \rho)_{h^*}$, and s^* and h^* are inlet entropy and enthalpy, respectively, at the design condition. For high pressure turbines, inlet conditions are held constant by a controller; for low pressure turbines, inlet temperature is maintained constant but pressure varies. Under these circumstances, Γ equation (12) does not change significantly for design and off-design conditions. A typical example is given in Appendix 2.

Pressure difference between two consecutive lumped nodes is a combination of nozzle action, frictional forces, and dynamic inertial effects of steam flow. The inertial effects give rise to very fast transients that will be attenuated in the steam chamber that acts as a low pass filter. Hence, steam flow between two consecutive nodes is obtained from steady-state relations in terms of time-dependent pressures.

Steam enthalpy at each node is calculated from inlet steam conditions and the node pressure as illustrated in Appendix 1. Shaft power for both high and low pressure turbines is computed as the sum of the products of steam flow between adjacent nodes and respective enthalpy differences:

$$W_t = \sum_j F_j \Delta h_j \tag{13}$$

W_t is time-dependent, and dynamically balanced by electrical generator power W_g and losses W_l due to generator winding resistance, hysteresis and friction. Turbine-generator shaft speed dynamics are given by torque balance:

$$\frac{\mathrm{d}}{\mathrm{d}t}(\tfrac{1}{2} IN^2) = (W_t - W_g - W_l)$$

or

$$\frac{\mathrm{d}N}{\mathrm{d}t} = (W_t - W_g - W_l)/(IN\ ) \tag{14}$$

W_g and W_l, computed in the electrical generator model, are inputs to the turbine model.

A more exact approach than the one described would require information on process characteristics (turbine blade configuration, for example) that may not be readily available; in addition, the model may grow too large and complex for power system simulation. This turbine model has been incorporated in an integrated system model of an 1160 MWe HTGR power plant;[11] the integrated model was used for simulation of plant transient performance, as well as for multivariable control system design.

Conclusions

The steam turbine model, presented here, is more complete than the models customarily used in power systems analysis but still simple enough to be incorporated in the overall system model. The modelling approach is deductive, not empirical. The state-space structure of the model is suitable for digital simulation and control system design.

The analytical technique can be extended to modelling other gas and vapour cycle turbines.

References

1. McDonald, J. P. and Kwatny, H. G. *IEEE Tran.* 1973, **AC-18** (3), 202
2. Berkowitz, D. A. ed., *Proc. the Seminar Boiler Modeling,* The Mitre Corporation, Bedford, MA, November 1974
3. Laubli, F. and Fenton, F. H. *IEEE Trans.* 1971, **PAS-90**, 4, July/August
4. Shames, I. H. 'Mechanics of fluids', McGraw-Hill, New York, 1962
5. Potter, P. J. 'Power plant theory and design', Ronald, New York, 1959
6. Stodola, A. 'Steam and gas turbines', (translated by L. C. Lowestein), Vol. I and II, Peter Smith, New York, 1945
7. Callen, H. B. 'Thermodynamics', John Wiley, New York, 1960
8. Adams, J. *et al. IEEE Trans.* 1965, PAS-84 (2), 146
9. Kwatny, H. G. *et al. Proc. Joint Automatic Contr. Conf.,* 1971, pp. 227–236
10. McNamara, R. W. *et al. Proc. Joint Automatic Contr. Conf.* 1977, pp. 345–350
11. Ray, A. and Bowman, H. F. *IEEE Trans. Nucl. Sci.* 1978, **NS-25** (4), 1068
12. Baumeister, T. ed., 'Standard handbook for mechanical engineers' 7th edition, McGraw-Hill, New York, 1967

Nomenclature

A	Area
C	Specific heat
F	Mass flowrate
h	Specific enthalpy
I	Moment of inertia of turbine-generator assembly
k	Ratio of specific heats

N Angular speed of turbine shaft
p Pressure
s Specific entropy
T Absolute temperature
t Time
V Lumped control volume
v Velocity
W Power
β Velocity ratio
Δ Difference operator
η Turbine stage efficiency
μ Thermodynamic index
ν Thermodynamic index
ρ Density

Superscripts

— Lumped average
* Design condition

Subscripts

d Downstream
e Reaction turbine exhaust
i Impulse stage/reaction turbine inlet
o Stagnation condition at nozzle inlet
p Constant pressure
s Isentropic
u Upstream
v Constant volume
1 Inlet condition
2 Outlet condition

Appendix I

Turbine stage efficiency and outlet enthalpy

Turbine stage efficiency is the ratio of actual enthalpy difference across the stage to the isentropic enthalpy change for the same inlet and outlet pressures.[6] It is influenced by many design factors that include blade construction and operating mode. Theoretical and practical criteria for blade design, which determine stage efficiency are available.[6] Turbine stage efficiency can be expressed semi-empirically as a function of the ratio of blade tip velocity to theoretical steam velocity. Blade tip velocity is proportional to turbine shaft speed, and theoretical steam velocity to the square root of isentropic enthalpy drop across the stage. Therefore:

$$\eta = \eta^* - \alpha\left[\frac{N/\sqrt{\Delta h_s}}{N^*/\sqrt{h_s^*}} - 1\right]^2$$

where α is a positive constant. For main steam turbine reaction stages, typical values of η^* and α are 0.87 and 2.0 respectively.[12]

Turbine speed in a power plant is closely controlled to the design value N^*. However, Δh_s varies with plant load. Typical values of Δh_s for the high pressure turbines of a 600 MWe generating unit are 105 BTU/lbm and 117 BTU/lbm at full and half loads, respectively. Therefore, for constant speed operation ($N = N^*$), typical values of turbine efficiency are:

$$\eta_{\text{full load}} = \eta^* = 0.87$$

$$\eta_{\text{half load}} = \eta^* - 2\left(\frac{N/\sqrt{\Delta h_s}}{N^*/\sqrt{\Delta h_s^*}} - 1\right)^2 = 0.87 - 2\left(\sqrt{\frac{105}{117}} - 1\right)^2 = 0.854$$

Taking $k = 1.3$, $\mu = [k(2 - \eta) + \eta]/k$ becomes:

$$\eta_{\text{full load}} = 1.7992 \text{ and } \mu_{\text{half load}} = 1.8006$$

Therefore, in this analysis, turbine efficiency has been assumed constant for design and off-design conditions.

Inlet steam entropy s_1 is calculated from pressure p_1 and enthalpy h_1. For isentropic expansion to the outlet pressure p_2, outlet enthalpy is obtained from thermodynamic parameters p_2 and s_1, and isentropic enthalpy drop $\Delta h_s = h_1 - h_s$ across the stages. Actual enthalpy drop $\Delta h = \eta \Delta h_s$, i.e., actual enthalpy outlet $h_2 = h_1 - \Delta h$.

Appendix 2

Model parameter evaluation

Parameter Γ in equation (12)

For a typical 600 MWe generating unit, reheat steam conditions are 600 psia and 1000°F at full load, and 300 psia and 1000°F at half load. At full load, $(\partial p/\partial \rho)_s = 1058$ psi/(lbm/ft^3), $(\partial p/\partial \rho)_h = 842$ psi/(lbm/ft^3) and $\eta = 0.87$. Then, $\Gamma_{\text{full load}} = 1030$ psi/(lbm/ft^3). At half load, $(\partial p/\partial \rho)_s = 1072$ psi/(lbm/ft^3), $(\partial p/\partial \rho)_h = 856$ psi/(lbm/ft^3) and $\eta = 0.864$. Then, $\Gamma_{\text{half load}} = 1041$ psi/(lbm/ft^3). Variation in Γ from full load to half load is approximately 0.1%, and does not affect the steady-state results.

Parameters V in equation (11) and V/Γ in equation (12)

In a high pressure turbine, the impulse chamber and a part of the following reaction stages may be represented as a lumped node. For a typical 600 MWe generating unit, the lumped volume V may be in the order of 1250 ft^3. For throttle steam conditions at 2415 psia and 950 F, the partial derivatives $(\partial p/\partial \rho)_s = 927$ psi/(lbm/ft^3) and $(\partial p/\partial \rho)_h = 718$ psi/(lbm/ft^3); assuming $\eta^* = 0.87$, equation (12) yields $\Gamma = 900$ psi/(lbm/ft^3). Therefore, $V/\Gamma = 1.39$ psi/lbm. For low pressure turbines, larger values of V may be selected.