DIGITAL CONTROL OF POWER TRANSIENTS IN A NUCLEAR REACTOR

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Abstract

An integrated, closed-loop, control system for on-line operations in nuclear power plants has been developed and demonstrated with an LSI-11/23 micro-processor on the 5 MWt fission reactor (MITR-II) that is operated by the Massachusetts Institute of Technology. This control system has inherent capabilities to perform on-line fault diagnosis, information display, sensor calibration, and measurement estimation. Recently, its scope has been extended to include the direct digital control of power changes ranging from 20-80% of the reactor's licensed limit. This controller differs from most of those discussed in theoretical and simulation studies by recognizing the non-linearity of reactor dynamics, calculating reactivity on-line, and controlling the rate of change of power by restricting both period and reactivity. The controller functions accurately using rods of non-linear worth in the presence of non-linear feedback effects.

Introduction

The direct digital control of nuclear power plants has been the subject of many theoretical and simulation studies but little experimental work. Many such studies linearize the equations that describe the reactor's transient response and then apply advanced concepts, such as optimal control using state estimators, to the linearized system. This study differs by recognizing, from the outset, that reactor dynamics are non-linear. The controller, designed to be applicable over the reactor's power range, allows for the following:

(1) The rate of change of reactor power is, among other things, proportional to the product of reactivity and power.

(2) The reactivity is dependent on the reactor power through various power-dependent feedback mechanisms such as fission product poisons (xenon), primary coolant temperature, fuel depletion, etc.

(3) Control mechanisms have finite speeds as well as position-dependent, non-linear worths.

The controller has four unique features. First, it utilizes validated data from multiple sensors and incorporates sensor fault detection and identification (FDI) techniques [1-4]. Second, it determines reactivity on-line via either a balance or inverse kinetics. Third, it incorporates reactivity feedback. Fourth, overshoots are prevented by restricting reactivity so that reactor period can always be made infinite by reversing the direction of control rod motion. This is a vital safety feature since finite rod speeds and position-dependent rod worths may otherwise make it impossible to terminate a power transient without over- or undershoot. This controller is designated as the MIT-CSDL Non-Lincar Digital Controller (NLDC). Experimental comparisons of the NLDC's performance with that of algorithms that rely only on period measurements and standard P-I-D logic have shown that the latter may result in substantial overshoots while the NLDC exhibits little or no overshoot.

The objectives of this paper are to (1) review the reactor physics considerations relevant to the design of the NLDC, (2) describe the NLDC reactivity restriction, and (3) report the results of initial experimental testing of the NLDC.

Reactor Physics Considerations

Reactors such as the MITR-II, which have close-coupled cores, may be described by the space-independent point kinetics equations [5]. These are:

$$\frac{dn(t)}{dt} = \frac{(\rho(t) - \bar{\beta})}{\ell^*} n(t) + \sum_{i=1}^{N} \lambda_i C_i(t) \qquad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\bar{\beta}_i}{\ell^*} n(t) - \lambda_i C_i(t) \text{ for } i = 1, N \qquad (2)$$

where $n(t)$ is chosen to be the reactor power,

$\rho(t)$ is the reactivity,

$\bar{\beta}$ is the effective delayed neutron fraction,

λ_i is the decay constant for the ith precursor group,

$C_i(t)$ is the concentration of the ith precursor group normalized to the initial power,

ℓ^* is a measure of the prompt neutron lifetime,

$\bar{\beta}_i$ is the effective fractional yield of the ith group of delayed neutrons,

N is the number of groups of delayed neutrons.

Power increases are accomplished by inserting positive reactivity until a specified steady reactor period is attained. Power is then allowed to increase exponentially on this steady period with reactivity being changed as necessary to compensate for short-term feedback effects, principally temperature. Once a certain percent of full power is achieved, the excess reactivity is gradually reduced so that the power levels off without overshoot. Figure 1 depicts a computer simulation in which the reactor was modelled using the six-group point kinetics equations with a time step of 0.0001 second. A rod of linear differential worth was withdrawn for 20 seconds so as to add reactivity at the rate of 5 mbeta/second (0.004% $\Delta K/K$ per second), held constant for 40 seconds, inserted for 40 seconds at half-speed thereby removing reactivity at the rate of 2.5 mbeta/second, and then held constant. Feedback effects were omitted in order not to obscure the basic relationship between the rod motion and the reactor power. The figure shows the reactivity, startup rate

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(SUR), and power as functions of time. (Note: Start-up rate, which equals (26.06/period) and is measured in decades-per-minute (DPM), is a measure of the inverse of the reactor period. It, like the concept of period, applies not only to reactor startups but to power transients and steady-state operation as well. It is shown because its steady-state value is zero, not infinity.) Note the following:

(1) The SUR initially increases almost as a step and then rises smoothly. The reactor power rises smoothly at an ever-increasing rate.

(2) When the reactivity insertion is halted at 20 seconds, the SUR remains positive but drops, almost discontinuously, and then remains nearly constant. The reactor power continues to increase but there is a sharp decrease in slope corresponding to the decrease in SUR. Power then rises almost exponentially. The SUR (or period) present when the reactivity was being changed is referred to as 'dynamic' while that which exists when the reactivity is constant is called 'steady'.

(3) When the reactivity decrease is started at 60 seconds, the SUR drops sharply and then continues to decrease, eventually becoming negative. The reactor power continues to increase but at an ever-lessening rate until the SUR goes negative indicating a decreasing power. Note that this occurs when there is still positive reactivity in the core. This demonstrates the decoupling between the observed SUR (or period) and the actual reactivity due to the delayed neutron effect.

(4) Reactivity removal stops at 100 seconds when the reactivity is zero. The SUR increases rapidly and then approaches zero. The reactor power curve again exhibits a discontinuous change of slope and the power levels off.

Figure 2 shows a portion of the power trace recorded during an experimental run on the MITR-II in which reactivity was raised from 0.0 to 69 mbeta in 12.2 seconds, held essentially constant for 83 seconds and then reduced to 0.0 mbeta in 18.3 seconds. Shown is the discontinuous change in slope of the power increase when the reactivity decrease starts. Also, this figure dramatically shows that the power increase continues even though a reactivity reduction has been started, that the rate of power change momentarily goes to zero while there is still positive reactivity present, and that the power can substantially overshoot the intended steady-state value due to the reactivity present.

These curves, simulated and experimental, demonstrate that the rate of change of reactor power is a function of both the rate of change of reactivity and the value of the reactivity present. As noted earlier, this phenomenon is the result of the effect of delayed neutrons on the reactor dynamics. It occurs because the production of precursors, being an integral part of the fission process, is in equilibrium with the transient reactor power while, given that precursors have a finite lifetime, such is not the case with their decay. For example, considering a power increase, the precursor concentration will always be less than what its equilibrium value would be at the transient power. Hence, the contribution of the delayed neutrons will always be less than what it would be if the precursors were at equilibrium and the contribution of the prompt neutrons will be correspondingly greater. Thus, whenever the reactor is not at steady-state, the non-equilibrium condition of the delayed neutrons will cause a delayed time response that must be recognized and balanced by use of the reactivity control mechanisms. This can be quantified by rewriting equations (1) and (2) for steady-state while noting that $\bar{\beta} = \Sigma\bar{\beta}_i$ and recalling that the production of precursors is in equilibrium with the fission process:

$$0 = \rho(t)n(t) - \sum_{i=1}^{N} [\bar{\beta}_i n(t) - \ell^* \lambda_i C_i(t)] \tag{3}$$

$$0 = \bar{\beta}_i n(t) - \ell^* \lambda_i C_i^o(t) \quad \text{for } i = 1, N \tag{4}$$

where the superscript denotes the equilibrium value. Substituting (4) into (3) yields:

$$0 = \rho(t)n(t) - \ell^* \sum_{i=1}^{N} [\lambda_i C_i^o(t) - \lambda_i C_i(t)] \tag{5}$$

Equation (5) shows that, as a result of the mismatch between the equilibrium and the actual precursor concentrations that exists during a power change, the reactor power can only be kept constant by a time-dependent adjustment of the reactivity.

Controller Design

The fact that the reactor period is a function of both the rate of change of the reactivity and the value of the reactivity present places complex requirements on the control system since power changes must be accomplished without overshoot. Given the data shown in Figures 1 and 2, it is evident that, even with the direction of control mechanism motion reversed, reactor power could still increase if sufficient positive reactivity were present. Hence, some restrictions must be placed on reactivity in order to assure that control is always feasible with the specified control mechanism. The first step in the controller design was to obtain a relation between the reactivity, its rate of change, period, and power. Assuming both the prompt-jump approximation and a one-group delayed neutron model to be viable, these relations are given by the dynamic period equations which are:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{d\rho(t)/dt + \lambda_e(t)\rho(t)} \tag{6}$$

$$A(t) = \int_o^t d\sigma/\tau(\sigma) \tag{7}$$

$$P(t) = P_o\, e^{A(t)} \tag{8}$$

where $\tau(t)$ is the reactor period,

P_o is the initial reactor power,

$P(t)$ is the instantaneous reactor power.

Other symbols are as previously defined. The effective one-group decay constant is defined as:

$$\lambda_e(t) = \frac{\Sigma\, \lambda_i C_i(t)}{\Sigma\, C_i(t)} \quad \text{for } i = 1, N \tag{9}$$

The effective one-group decay constant is time-dependent because the ratio of short to long-lived precursors varies with the short-lived ones dominating during a power increase and long-lived ones during a decrease. Hence, the use of the term "one-group approximation" differs from its standard meaning which implies that the decay constant is fixed at some value typical of the transients being studied. A derivation of equation (6) is given in the Appendix.

Having established the dynamic period equations, the condition(s) under which it is feasible to transfer a reactor using a specified control mechanism from a given power level and period to a specified power level

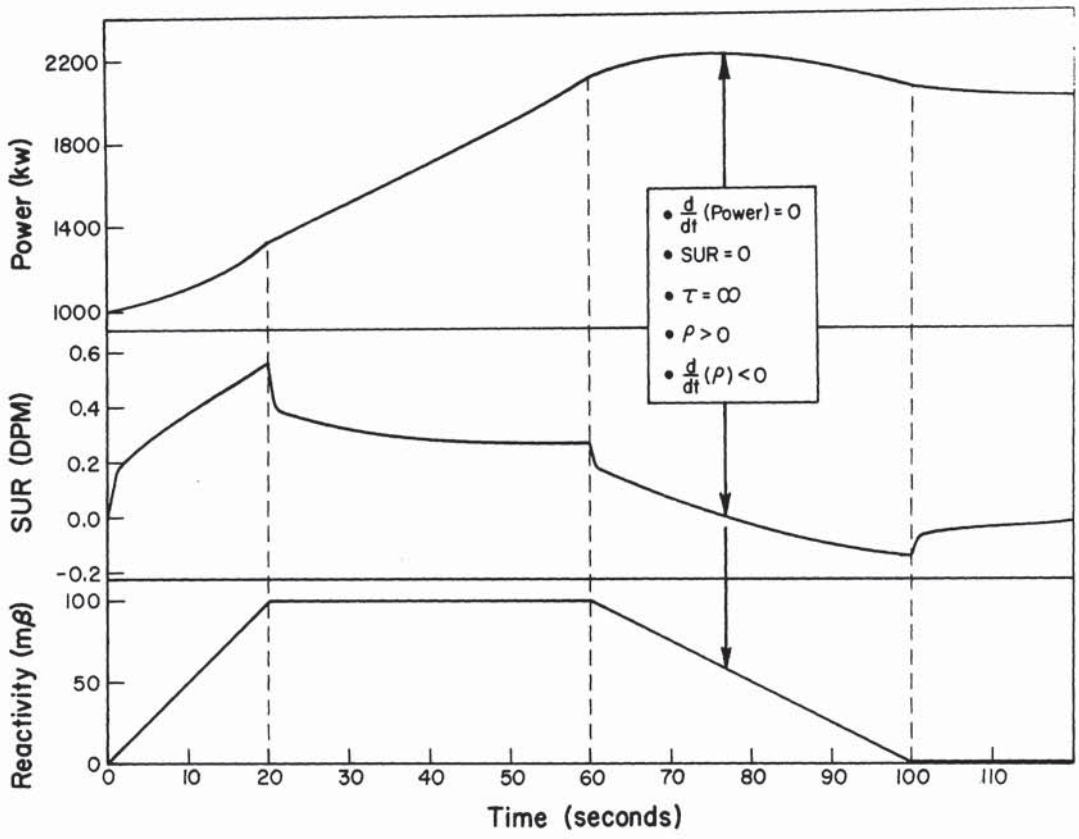

Figure One:

Reactor Power Transient - Computer Simulation

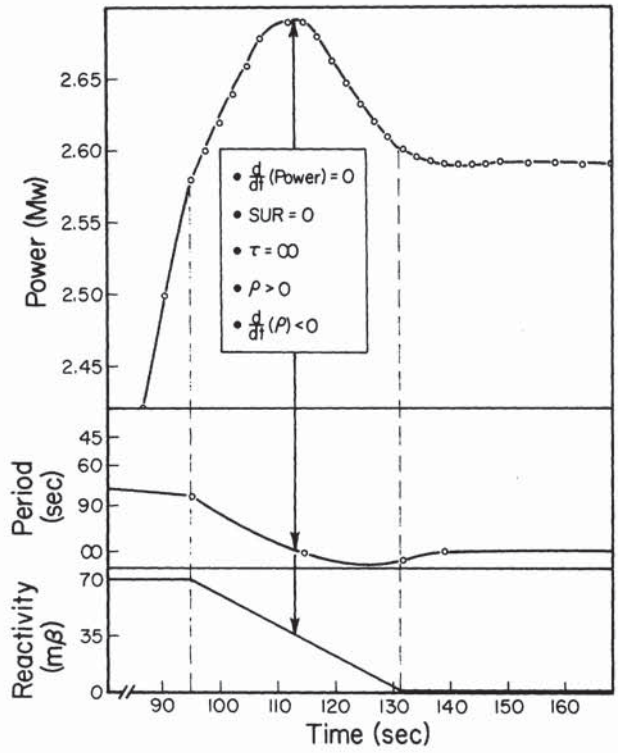

Figure Two:

Reactor Power Transient

- Experimental Data

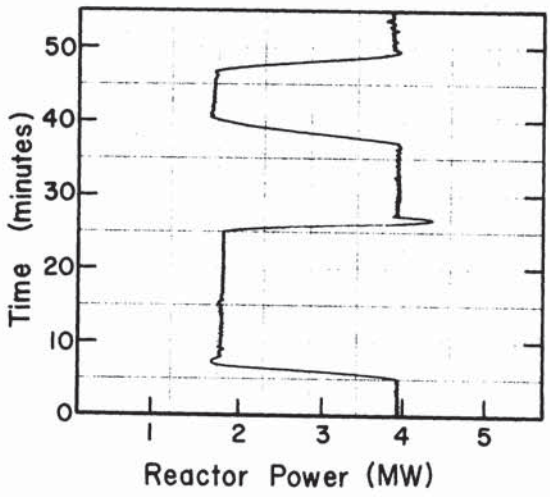

Figure Three: Effect of Reactivity Restriction on NLDC Performance

without overshoot can be determined. First, note that the reactor period must be made and held infinite if a transient is to be terminated and power kept at steady-state. Second, safety dictates that $(\bar{\beta}-\rho)$ be kept positive (i.e., ρ less than prompt critical). Hence, for the period to be infinite, the expression $(d\rho/dt + \lambda_e\rho)$ must be zero. Not all the parameters in this expression are subject to manipulation since once a control mechanism is selected, the maximum possible rate of change of reactivity $(d\rho/dt)$ is fixed by the reactor's design and safety considerations. Second, the effective one-group decay constant (λ_e), being a physical parameter, is not subject to direct control. Therefore, the only way that period can be rapidly made infinite is if the total reactivity, both that added directly by the control mechanisms and that present indirectly from feedback effects, is maintained less than the maximum available rate of change of reactivity divided by the effective one-group decay constant. That is:

$$-\left|\frac{d\rho}{dt}\right| \leq \lambda_e\rho \leq \left|\frac{d\rho}{dt}\right| \qquad (10)$$

where λ_e and ρ are the actual existing system parameters and $d\rho/dt$ is the maximum possible rate of reactivity change that could be obtained were the selected control mechanism to be moved. Physically, if the reactivity is so constrained, then, by reversal of the direction of motion of the control mechanism, it is possible to negate the effect of the reactivity present and make the period infinite. A control sequence fashioned on this relation would be needlessly conservative since power could be leveled at any time when all that is necessary is that it be possible to level power at the desired termination point. Recognizing this, the control objective can be achieved by constructing an algorithm that, while monotonically approaching the desired power, constrains reactivity according to the relation:

$$-\left(\frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}|\tau\ln(P_f/P)\right) \leq \rho \leq \left(\frac{|\dot{\rho}|}{\lambda_e} + |\dot{\rho}|\tau\ln(P_f/P)\right) \qquad (11)$$

More physical insight can be had by writing (11) for power increases and decreases respectively as:

$$\left.\begin{aligned}(\rho-|\dot{\rho}|/\lambda_e)/|\dot{\rho}| &\leq \tau\ln(P_f/P)\\ (\rho+|\dot{\rho}|/\lambda_e)/|\dot{\rho}| &\geq \tau\ln(P/P_f)\end{aligned}\right\} \qquad (12)$$

These equations compare the time remaining to attain the final power (P_f) with the time required to eliminate the additional reactivity present beyond the controllable value given by equation (10).

Implementation of a controller based on (12) is contingent upon both measuring the reactivity and calculating the effective one-group decay constant. The former is normally derived from a reactivity balance but may also be obtained via inverse kinetics. The latter is approximated in real time using the relation derived in section A.2 of the Appendix. Finally, it should be noted that, as a precaution against any potential control instabilities that could arise if the power were to somehow overshoot, an instruction is included in the implementing algorithm that sets the right side of (12) to zero should such a situation occur.

Experimental Evaluation

The NLDC is an algorithm which incorporates the reactivity constraints (12) as well as ones on power and period. It contains separate safety and instruction sections. The former ensures that the constraints are satisfied. The latter correlates predetermined control actions, which have been evaluated off-line for safety, with the reactor's power and period. Safety section signals supercede all others. Separate subroutines exist for raising and lowering power.

Subsequent to off-line testing via computer simulation and determination that it could control in real time, the NLDC was tested on-line in closed-loop form on the MIT Research Reactor. The NLDC was allowed to control the MITR's regulating rod which has integral and average differential worths of 0.17% $\Delta K/K$ and $1.1\cdot10^{-3}$% $\Delta K/K$ per second respectively. A detailed description of the experimental facility is given in (4). In general, the NLDC's period restrictions are limiting at the outset of a transient while the one on reactivity controls the transient's termination. Use of the NLDC normally results in the movement of the selected control rod until the period limit is attained. The rod's position is then kept constant while the power continues to change. Once the reactivity constraint is no longer met, rod insertion is begun and continued in a more-or-less stepwise manner until the rod is returned to its critical position. Figure 3 shows two experimental runs in which power was lowered from 4 to 1 MW, held constant for several minutes, and then returned to 4 MW. First, the response of a controller designed without the reactivity constraint is depicted. There is an undershoot at 1 MW and a substantial overshoot at 4 MW. Second, the response of the NLDC is shown. There is neither an undernor overshoot. Subsequently, tests were conducted in which the NLDC was directed to raise and lower power in steps of approximately 1 MW. The resulting control was both excellent and repeatable.

Having established that the NLDC was capable of preventing automated control actions from challenging the existing reactor safety system, it became possible to test other control strategies on the MIT Reactor. These studies have so far included controllers that simulate operator instructions, heuristic programming including both adaptive and learning theory routines, and predictive methods. The experimental results of these tests are described in [6].

Conclusions

A reactivity constraint that prevents power overshoots has been designed on the basis of the interrelation of reactor power, period, and reactivity that exists during a power transient. A closed-loop control algorithm that incorporates this constraint was designed and successfully tested on the 5 MWt MIT Research Reactor. The potential benefits of such a controller are that it reduces the possibility of power overshoots, provides early detection of rod withdrawal or reactivity insertion accidents, and permits the safety aspects of a reactivity change to be determined on-line in real time. It is hoped that, as part of this research program, the safety and advantages of the direct digital control approach can be realized by first demonstrating them on the MIT Research Reactor.

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Appendix

Derivation of Dynamic Period Equation (A.1)

Given that the prompt neutron lifetime is brief compared to the time domain of interest for light-water reactor control, it is viable to assume that the neutron population is in immediate equilibrium with the precursor population. Mathematically, this means neglecting the time-derivative of the neutron population in the first kinetics equation. Applying this assumption, the prompt-jump approximation, together with the one-group approximation, modified to allow for a time-dependent decay constant, and defining symbols as before, the point kinetics equations become:

$$0 = (\rho(t) - \bar{\beta})n(t) + \ell^* \lambda_e(t) C(t) \qquad \text{(A-1)}$$

$$\dot{C}(t) = (\bar{\beta}/\ell^*)n(t) - \lambda_e(t) C(t) \qquad \text{(A-2)}$$

$$\lambda_e(t) = \frac{\Sigma\, \lambda_i C_i(t)}{\Sigma\, C_i(t)} \quad \text{for } i = 1, N \qquad \text{(A-3)}$$

Differentiating (A-1) with respect to time:

$$0 = \dot{\rho}(t)n(t) + (\rho(t) - \bar{\beta})\dot{n}(t) + \ell^* \dot{\lambda}_e(t) C(t) + \ell^* \lambda_e(t) \dot{C}(t) \qquad \text{(A-4)}$$

Substituting (A-2) into (A-4) to eliminate $\dot{C}(t)$ yields:

$$0 = \dot{\rho}(t)n(t) + (\rho(t) - \bar{\beta})\dot{n}(t) + \ell^* \dot{\lambda}_e(t) C(t) + \ell^* \lambda_e(t) \left[(\bar{\beta}/\ell^*)n(t) - \lambda_e(t) C(t)\right] \qquad \text{(A-5)}$$

Substituting (A-1) into (A-5) to eliminate C(t) yields:

$$0 = \dot{\rho}(t)n(t) + (\rho(t) - \bar{\beta})\dot{n}(t) + \lambda_e(t)\rho(t)n(t) + \ell^* \dot{\lambda}_e(t) \left[(\bar{\beta} - \rho(t))/\ell^*)n(t)/\lambda_e(t)\right] \qquad \text{(A-6)}$$

Consolidating terms and dividing through by n(t) yields:

$$0 = [\dot{\rho}(t) + \lambda_e(t)\rho(t)] + (\rho(t) - \bar{\beta})\dot{n}(t)/n(t) + (\dot{\lambda}_e(t)/\lambda_e(t))(\bar{\beta} - \rho(t)) \qquad \text{(A-7)}$$

Substituting the definition of reactor period ($\tau \equiv n/\dot{n}$) into (A-7):

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta} - \rho(t))} \qquad \text{(A-8)}$$

If the values of $\lambda_e(t)$ and $\dot{\lambda}_e(t)$ are accurately known, then (A-8) in conjunction with equations (7) and (8) of the main text may be used to predict reactor power.

Computer simulation shows that this approach, with a 1.0 second time step, is as accurate as a six-group point kinetics solution using a 0.0001 second time step for the transients being studied. Unfortunately, there is no facile method to evaluate the effective one-group decay constant and its derivative. However, given that the objective in deriving (A-8) was to determine when rod withdrawal should be halted and when rod insertion should be initiated, several simplifying assumptions are possible. These are:

(1) The value of $\lambda_e(t)$ is taken as $\lambda_e^o(t)$, the equilibrium value of the effective one-group constant. This is a valid assumption since it is reasonably accurate for the relatively slow transients being studied. Also, it results in a conservative control action since $\lambda_e^o(t)$ exceeds $\lambda_e(t)$ for power increases and the larger the value of the decay constant the more restrictive the bound on the allowed reactivity as shown by equation (11) of the main text.

(2) The third term in the denominator of (A-8) is simply dropped. This appears non-conservative since all three denominator terms are of the same magnitude. However, for the transients being studied, this term's sign changes within 1-2 seconds of a sign change in the rate of change of reactivity and it then contributes to the lengthening of the reactor period. Hence neglecting it is justifiable. Additional work is being conducted on the evaluation of this term.

Applying these two simplifications to (A-8) yields:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e^o(t)\rho(t)} \qquad \text{(A-9)}$$

The restrictions imposed on the use of (A-9) by the prompt-jump approximation should be noted. The approximation's validity depends on the reactor's period being large relative to the prompt neutron lifetime. This implies that (1) the reactivity must be substantially less than the effective delayed neutron fraction and that (2) the rate of insertion of reactivity must be slow enough so that no appreciable change in reactivity occurs over the prompt neutron lifetime. Simulation studies show that both the reactivities and reactivity rates being used in this research are well within acceptable limits.

Estimation of One-Group Decay Constant (A.2)

The steady reactor period is related to reactivity by the In-hour equation:

$$\rho(t) = \frac{\ell^*}{\tau(t)} + \sum_{i=1}^{N} \frac{\bar{\beta}_i}{1 + \lambda_i \tau(t)} \qquad \text{(A-10)}$$

Rewriting it using the one-group and prompt-jump approximations yields:

$$\lambda_e^o(t) = (\bar{\beta} - \rho(t))/(\tau(t)\rho(t)) \qquad \text{(A-11)}$$

Given the period, equation (A-10) can be used to solve for the reactivity. Using the period and reactivity, equation (A-11) can be solved for the equilibrium one-group decay constant. A short computer code was written to accomplish this and the results fitted to an exponential curve. The relation, for the MITR's effective delayed neutron fraction of 0.00786, is:

$$\lambda_e^o(t) = 0.080264 * \mathrm{EXP}(2.384459 * \rho(t))$$

with $\rho(t)$ in units of $\%\Delta K/K$.