

USE OF REACTIVITY CONSTRAINTS FOR THE AUTOMATIC
CONTROL OF REACTOR POWER

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Abstract

A theoretical framework for the automatic control of reactor power has been developed and experimentally evaluated on the 5 Mwt Research Reactor that is operated by the Massachusetts Institute of Technology. The controller functions by restricting the net reactivity so that it is always possible to make the reactor period infinite at the desired termination point of a transient by reversing the direction of motion of whatever control mechanism is associated with the controller. This capability is formally designated as "feasibility of control". It has been shown experimentally that maintenance of feasibility of control is a sufficient condition for the automatic control of reactor power. This research should be of value in the design of closed-loop controllers, in the creation of reactivity displays, in the provision of guidance to operators regarding the timing of reactivity changes, and as an experimental envelope within which alternate control strategies can be evaluated.

Introduction

The Massachusetts Institute of Technology and the Charles Stark Draper Laboratory have undertaken a systematic program of theoretical and experimental research regarding the application of advanced control and instrumentation to nuclear reactors. One objective of this program has been the development of closed-loop control systems. It is believed that the use of such systems would improve safety and efficiency by enabling licensed operators to monitor a plant without having to simultaneously manipulate it. The design objectives adopted for this controller were that its actions should never result in a challenge to the reactor's safety system, that it should be separate from the safety system, and that its performance should be at least comparable to that attainable by licensed operators. Accomplishment of these objectives was contingent upon designing the controller to recognize that reactor dynamics are non-linear, to allow for the effect of various power-dependent feedback mechanisms (e.g., temperature, xenon, fuel depletion), and to recognize that control mechanisms have both finite speeds and non-linear worths. A controller that fulfills these objectives has been designed and demonstrated on the MIT Research Reactor. Designated as the MIT-CSDL Non-Linear Digital Controller or NLDC, it has been shown to be capable of both raising and lowering the reactor power in an efficient manner while using a rod of non-linear differential worth in the presence of non-linear feedback effects. The NLDC has thus far been used over the range of 20-95% of the reactor's rated power.

The objectives of this paper are to (1) review the theoretical relations that form the basis of the MIT-CSDL Non-Linear Digital Controller and (2) report the results of experimental tests germane to those theoretical relations.

Design Philosophy

The major design goal adopted by the MIT-CSDL research team was that the closed-loop control system should never present a challenge to the reactor's existing safety system. There were two possible approaches whereby this goal could be attained. The first was to perform a thorough off-line safety analysis of the behavior of a given controller acting within the envelope of the boundary conditions established by a particular plant under both normal and abnormal operating conditions. Aside from being plant-specific, this approach might be deficient in that there could be some unforeseen combination of unusual conditions that were not included in the analysis. The second approach was to build into the controller the capability to evaluate the safety of its own actions on-line and in real-time. The MIT-CSDL philosophy has been to pursue the second of these two approaches since it should lead to a universally useful controller of safe design.

Definition of Feasibility of Control

A reactor together with a specified control mechanism is defined as constituting a system that is "feasible to control" if the system can be transferred from a given power level and rate of change of power (i.e., period) to a desired steady-state power level without overshoot (or conversely, undershoot) beyond specified tolerance bands, if any. This concept has two important attributes. First, it applies to a reactor and to the specific control mechanism designated for use in accomplishing a given transient. Second, not all states are allowable intermediates through which the system may pass while transiting from some initial to some final power. Excluded are both those states that represent actual overshoots and those from which overshoots could not be averted by manipulation of the specified control mechanism. It should be recognized that the concept of "feasibility of control" is distinct from the more general property of "controllability". That term has a specialized meaning in that a system is said to be controllable if "any initial state can be transferred to any final state in a finite time by some control sequence" [1]. This definition does not place any restrictions on intermediate states.

The necessity for introducing the concept of "feasibility of control" warrants review. Power changes are accomplished by inserting and removing reactivity. Adjustments in reactivity can be loosely thought of as being proportional to variations in the rate of the fractional change in the neutron population per unit time. That population consists of both prompt neutrons which are produced directly from fission and delayed neutrons which are produced following the decay by beta particle emission of certain fission products (i.e., precursors). Delayed neutrons are of extreme importance because, owing to the delay in their production, they lengthen the basic neutron life cycle so that it

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The necessity for introducing the concept of "feasibility of control" warrants review. Power changes are accomplished by inserting and removing reactivity. Adjustments in reactivity can be loosely thought of as being proportional to variations in the rate of the fractional change in the neutron population per unit time. That population consists of both prompt neutrons which are produced directly from fission and delayed neutrons which are produced following the decay by beta particle emission of certain fission products (i.e., precursors). Delayed neutrons are of extreme importance because, owing to the delay in their production, they lengthen the basic neutron life cycle so that it

Absolute Reactivity Constraint

becomes feasible to instrument and monitor changes in reactor power. However, that same delay also complicates reactor control. Specifically, given that precursors have finite lifetimes, the delayed neutron population will lag its equilibrium value during power increases and lead it during decreases. As a result, mismatches exist between the equilibrium and the actual delayed neutron populations during power changes and, owing to these mismatches, the reactor power can only be brought to and kept at steady-state by making time-dependent adjustments in the reactivity. A further complication is that the rate at which reactivity can be removed from a reactor is, under non-scrum conditions, finite. Hence, even with the direction of control mechanism motion reversed, reactor power could still rise if sufficient positive reactivity had been initially present. This explains why it is possible to have a state in which the reactor's power is currently below the allowable but from which an overshoot can not be averted using a given control mechanism. Additional information is given in [2].

Dynamic Period Equation

Reactors which have close-coupled cores may be described by the point kinetics equations. Subsequent to making the prompt-jump approximation, these equations may be combined [2] to obtain:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t) + \frac{\dot{\lambda}_e(t)}{\lambda_e(t)}(\bar{\beta} - \rho(t))} \quad (1)$$

where: $\tau(t)$ is the reactor period,

$\bar{\beta}$ is the effective delayed neutron fraction,

$\rho(t)$ is the reactivity,

$\dot{\rho}(t)$ is the rate of change of reactivity,

$\lambda_e(t)$ is the effective one-group decay constant, and

$\dot{\lambda}_e(t)$ is the rate of change of the effective one-group decay constant.

The effective, one-group decay constant is time dependent because the relative concentrations of the various delayed-neutron precursor groups change depending on whether power is being increased or decreased. Equation (1) is referred to here as the "exact" form of the dynamic period equation. The objective is to use (1) to obtain guidance on when to initiate changes in the signal to the reactor control mechanism. Accordingly, the third term in the denominator can be neglected because, for the transients being studied, that term is small at the time that a signal reversal is required. Also, it changes sign almost immediately following a change in the direction of travel of the control mechanism and therefore need not be considered in a decision on when to change that direction. The result, which is designated here as the "approximate" form of the dynamic period equation, is:

$$\tau(t) = \frac{\bar{\beta} - \rho(t)}{\dot{\rho}(t) + \lambda_e(t)\rho(t)} \quad (2)$$

The quantity $\lambda_e(t)$ is normally approximated by the value that it approaches when the reactor is on an asymptotic period, $\lambda_e^0(t)$. This substitution is desirable because $\lambda_e^0(t)$ can be readily calculated if the net reactivity is known. It is warranted because it results in conservative decisions for power increases. Also, the difference between the two terms is, for the transients of interest, small.

The approximate form of the dynamic period equation can be used to establish quantitative criteria that permit evaluation of the state of a reactor in terms of the definition of "feasibility of control". The first such criterion, the "absolute reactivity constraint", can be derived directly from equation (2). First, note that the reactor period must be made and held infinite if a transient is to be terminated and power kept at steady-state. Second, safety dictates that $(\bar{\beta} - \rho)$ be kept positive (i.e., ρ less than prompt critical). Hence, for the period to be infinite, the expression $(d\rho/dt + \lambda_e\rho)$ must be zero. This can be accomplished if the product of the effective, one-group decay constant and the net reactivity, both that added directly by the control mechanisms and that present indirectly from feedback effects, is maintained less than the maximum available rate of change of reactivity. That is:

$$-\dot{\rho} \leq \lambda_e(t)\rho(t) \leq \dot{\rho} \quad (3)$$

where: $\lambda_e(t)$ is the effective one-group decay constant,

$\rho(t)$ is the net reactivity, both that added deliberately by the control mechanisms and that present indirectly from feedback effects, and

$\dot{\rho}$ is the maximum available rate of change of reactivity that could be obtained at a given rod height were the designated control rod to be moved.

If the reactivity is so constrained, then, by reversal of the direction of motion of the specified control mechanism, it will be possible to negate the effect of the reactivity present and rapidly make the period infinite at any time during the transient.

Sufficient Reactivity Constraint

The absolute reactivity constraint is needlessly conservative since it stipulates that it be possible to level reactor power at any time during a transient when all that is required is that it be possible to level power at the termination point. A less stringent and therefore more efficient condition can be written which permits the presence of additional reactivity beyond the amount specified by the absolute constraint. This less stringent condition specifies that there be sufficient time available to eliminate whatever reactivity is present beyond the amount that can be immediately negated by reversal of direction of the control mechanism before the desired power level is attained. This condition, the "sufficient reactivity constraint", is:

$$-\left(\frac{\dot{\rho}}{\lambda_e} + \dot{\rho} \tau \ln(P_F/P)\right) \leq \rho \leq \left(\frac{\dot{\rho}}{\lambda_e} + \dot{\rho} \tau \ln(P_F/P)\right) \quad (4)$$

The functional dependencies of the variables have been omitted. The symbols are as previously defined except that P_F and P are the desired and current reactor power respectively and τ is either the observed reactor period or the asymptotic period that corresponds to the net reactivity, whichever results in a more conservative decision.

Available and Required Times

Equation (4) may be rewritten for power increases and decreases respectively as:

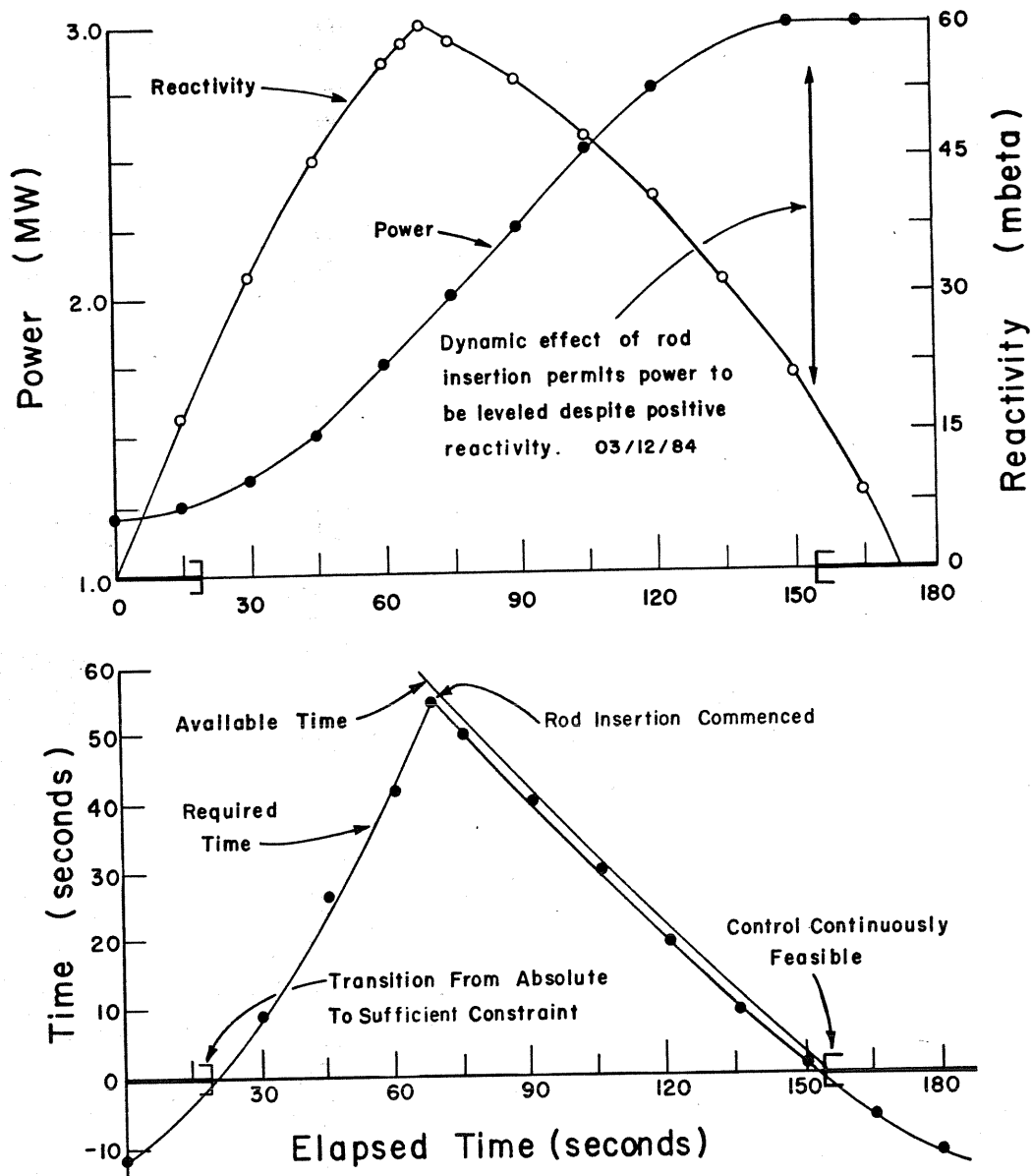


Figure One : Function of Sufficient Reactivity Constraint

$$(\rho - |\dot{\rho}|/\lambda_e)/|\dot{\rho}| \leq \tau \ln(P_F/P) \quad (5(a))$$

$$-(\rho + |\dot{\rho}|/\lambda_e)/|\dot{\rho}| \leq \tau \ln(P_F/P) \quad (5(b))$$

Equation (5) compares two times. The term on the left is the time required to reduce the reactivity present to the amount allowed by the absolute condition. It is the time necessary to establish feasibility of control and is referred to as the "required time" or ΔT_R . The time on the right is that remaining to attain the desired power. It is referred to as the "available time" or ΔT_A . When the actual power equals the desired power, the sufficient constraint reduces to the absolute one. (Note: If the power were to somehow overshoot, the implementing algorithm should set the right side of (5) to zero as a precaution against control instabilities.)

Range of The Reactivity Constraints

The non-linear nature of the differential rod

worth places a restriction on the range of rod heights over which the reactivity constraints can be used. For example, in order to halt a power increase, the specified control rod must be inserted into the core. If the rod is positioned so that as soon as the inward motion starts, it will be traveling from a region of high differential worth to one of low differential worth then use of that rod may not be justified since, while the maximum available rate of change of reactivity will drop rapidly, there may actually be very little corresponding decrease in the reactivity. Noting that the maximum rod speed is fixed and assuming that the effective one-group decay constant does not vary during rod insertion, this restriction becomes:

$$d\rho(t)/dt \geq (1/\lambda_e)[d^2\rho/dt^2] \quad (6(a))$$

The corresponding expression for power decreases is:

$$d\rho(t)/dt \geq (-1/\lambda_e)[d^2\rho/dt^2] \quad (6(b))$$

The terms on the left in (6) are the rates of change of the net reactivity present. Included are feedback ef-

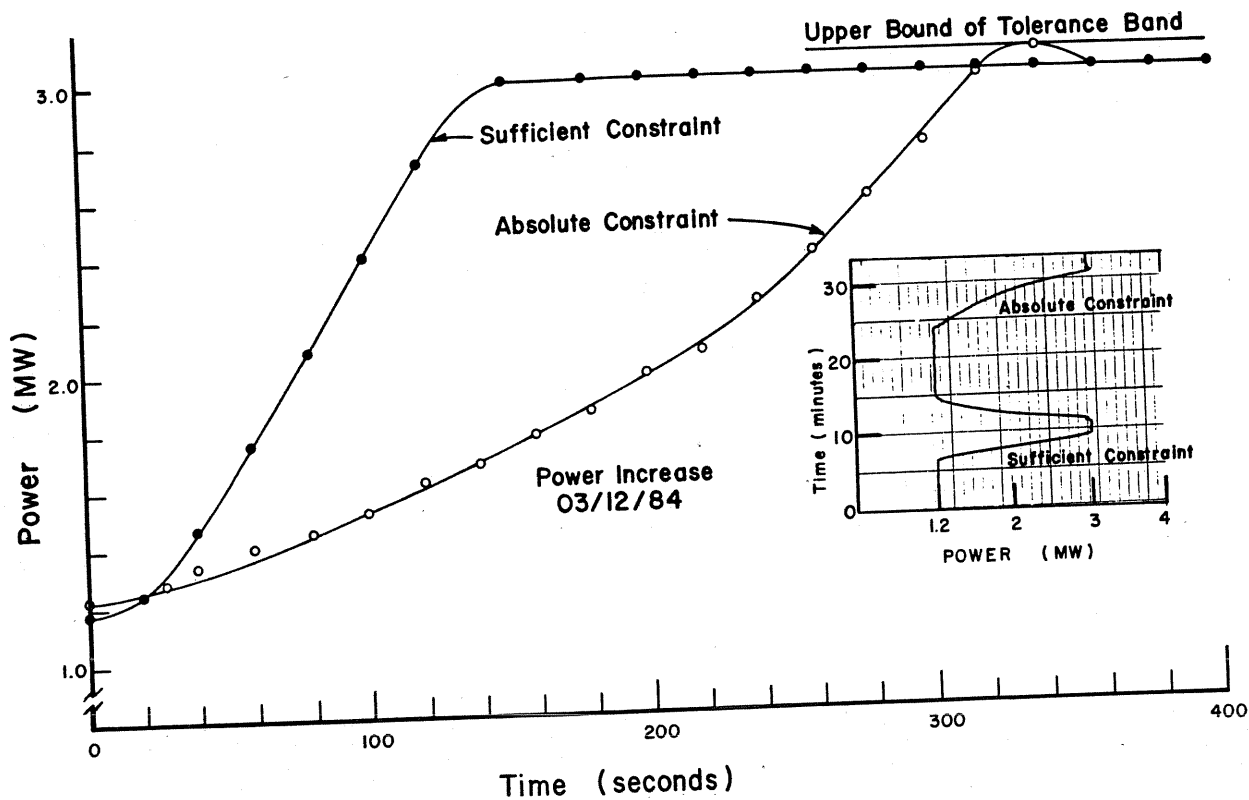


Figure Two: Effect of Absolute and Sufficient Constraints on Power

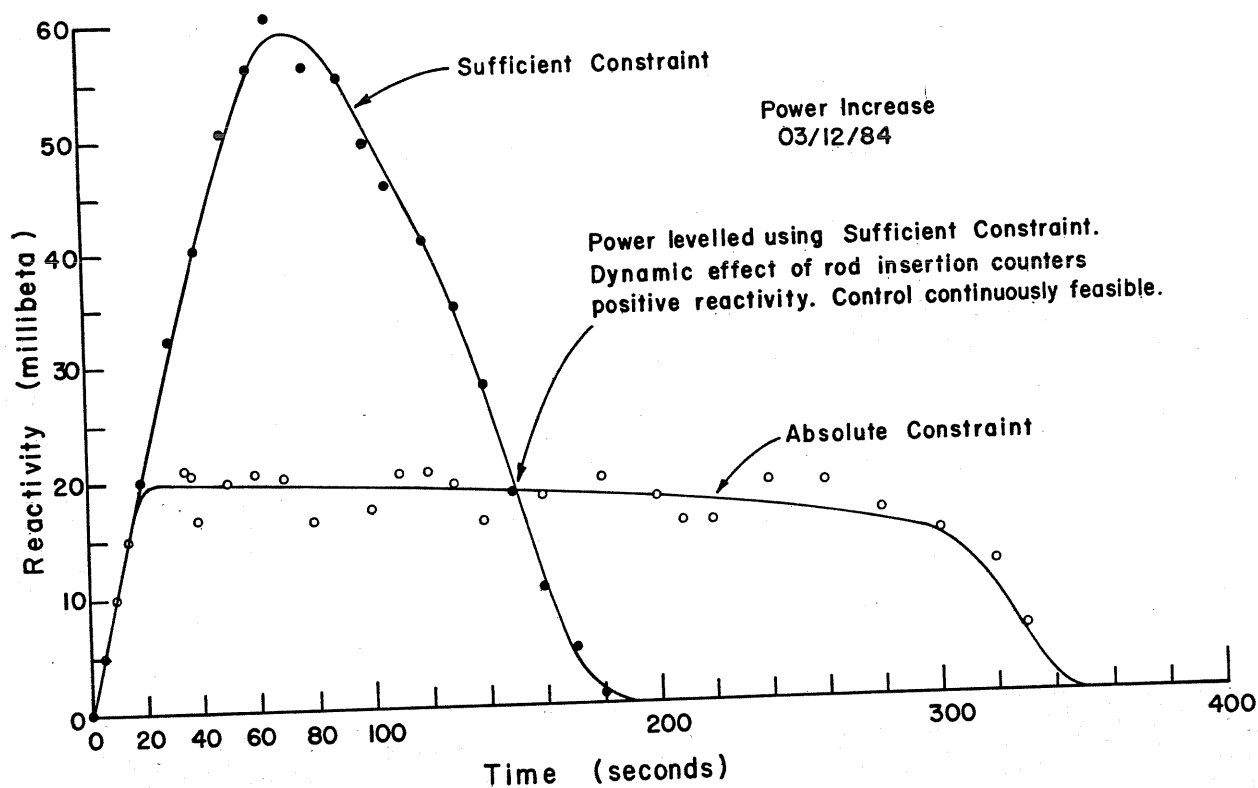


Figure Three: Effect of Absolute and Sufficient Constraints on Reactivity

facts as well as changes in reactivity induced directly by the control rods. This contrasts with the terms on the right which pertain only to the reactivity associated with the designated control mechanism. (Note: Assuming negative coefficients, feedback effects will offset the net reactivity.) Another restriction is that the reactivity associated with moving the control rod from its position when the decision is made to commence halting the transient to the limit of the range of rod heights permitted by (6) must exceed the net reactivity present. It should be recognized that these range restrictions are not unique to closed-loop control. They represent limits on the utility of any control mechanism regardless of whether the control is manual or automatic. However, they may be of special concern to a closed-loop system if either that system lacks access to all available control mechanisms or if it is not programmed to shift from one control mechanism to another when appropriate.

Reactivity Constraints and Reactor Control

A controller based on the sufficient reactivity constraint and on a control law that simulated operator instructions was designed and tested. This algorithm, the MIT-CSDL Non-Linear Digital Controller (NLDC), uses the reactivity constraint to monitor and, if necessary, override the decision of the control law. As far as the experiments described here are concerned, the control law was deliberately chosen so that the constraints would be limiting [3,4]. Reactivity was calculated via a balance. The constraints were evaluated at a frequency of one second.

Control is continuously feasible if the reactivity is within the bounds of the absolute constraint. These bounds are not symmetric about the origin because the magnitude of the effective, one-group decay constant depends on whether power is being raised or lowered. Once the reactivity exceeds these bounds, control is only guaranteed to be feasible at the desired power level. Figure One depicts the available and required times as a function of the elapsed time for a power increase. The corresponding power and reactivity changes are shown for comparison. Initially, the reactivity is zero and, given that there is some rate of change of reactivity available were the designated control rod to be moved, the required time is negative and equal to $(-1/\lambda_e)$ or -12.2 seconds. This indicates that control is already continuously feasible. The available time is initially zero because the desired power equals the actual power. Once the change is made in the demanded power, the available time becomes infinite because the period in the previously steady-state reactor was infinite. Control mechanism motion then commences and the reactivity becomes positive. The required time becomes less negative, passes through zero, and then becomes positive indicating that some finite interval must now elapse before the transient can be halted. The transition from the absolute to the sufficient constraint occurs when the required time is zero. The available time tends towards zero because the period is becoming shorter and the power is rising. Once the time required to restore continuous feasibility of control equals the time remaining to attain full power, continued control mechanism withdrawal is prohibited and insertion is begun. The required time is continuously bounded by the available time indicating that the control mechanism is being more or less constantly inserted. The required and available times eventually both become zero. When this occurs, the reactor power can be leveled since control is again within the range of the absolute constraint. There is still positive reactivity present in the core at this time and the dynamic effect of control mechanism insertion is required to counter this positive reactivity. Hence, the control mechanism must be driven in continuously at this

time. As the reactor power settles out, the available time remains zero. The required time becomes negative, eventually resuming its original value of $(-1/\lambda_e)$.

Figure Two contrasts two power increases of 1.2-3.0 MW. One was accomplished with the sufficient constraint while the other was subject to the absolute constraint. Figure Three contrasts the corresponding reactivity insertions. As expected, the controller that is limited by the absolute constraint requires much more time to accomplish the power change. When using the absolute constraint, the NLDC withdraws the control rod until reactivity equal to $(|\rho|/\lambda_e)$ has been inserted since this is the amount that can be immediately negated by reversal of the direction of rod travel. Note that the reactivity allowed by the absolute constraint decreases slightly during the transient because the value of the effective one-group decay constant is increasing as a result of the short-lived precursors being favored during the power increase.

The results of many other experimental tests of the MIT-CSDL Non-Linear Digital Controller are given in [3] and [4].

Conclusions

The major contribution of this research is the enumeration and experimental demonstration of a set of general principles for the closed-loop control of reactor power. Foremost among these is the idea of restricting the net reactivity so that a power transient can be rapidly halted by merely reversing the direction of travel of the associated control mechanism. Following from this principle are the concepts of feasibility of control, the absolute and sufficient reactivity constraints, and the required and available times. The function of the reactivity constraints is to review the decision of whatever control law is being used to regulate the reactor power and, if necessary, override that decision. The value of this approach is that it provides assurance that no automatic controller will ever challenge the reactor safety system regardless of the control law being employed.

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