

DISTRIBUTED DATA COMMUNICATION NETWORKS FOR REAL-TIME PROCESS CONTROL†

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The paper deals with the performance analysis of network access protocols for distributed data communication and control systems (DDCCS) in chemical and processing plants. The selection of an appropriate *data highway protocol* is an important step in the DDCCS network design for large-scale processes because the real-time feedback control loops are subject to time-varying delays resulting from data latencies and possible mis-synchronism between system components.

SAE token ring, SAE linear token bus, and MIL-STD-1553B protocols have been analyzed in view of the DDCCS network design requirements. The results of closed loop control system simulations are presented to demonstrate how data latency generated by the network access protocols could degrade the dynamic performance of the controlled process.

KEYWORDS Data highway protocols Real-time process control Delayed control systems
Discrete-event and continuous-time simulation

1. INTRODUCTION

Large chemical plants require distributed information processing for monitoring and control of the process variables as well as for planning and scheduling of operations. This is achieved by spatially dispersed computers, intelligent terminals, display devices, and sensors and actuators. These interconnected devices must perform, in real time, a set of diverse but mutually related functions ranging from monitoring and closed loop control of essential process variables to routine maintenance support and information display. The requirements for distributed data communication and control systems (DDCCS) may vary for specific applications. For example, in a batch processing environment, the prime objective of a DDCCS network is the integration of computer-controlled complexes of small reactors, automated machines, material handling devices and guided vehicles on the shop floor with inter-person information processing across local and remote stations. In large processing plants, distributed digital control systems could regulate the essential and time-dependent process variables like reactor temperature, feedwater flow and fuel consumption as well as provide the

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necessary support for plant monitoring, energy conservation and management, material scheduling and preventive maintenance. The major challenge in a hazardous chemical plant is to design an integrated information and control system which must ensure safe and reliable operations under all circumstances.

Two basic methods are available for interconnecting the components of a distributed information processing and control system [1,2]: (1) point-to-point dedicated connections, and (2) serial time-division multiplexed data communication network, commonly known as *data highway* in chemical engineering terminology [3,4]. The major advantages of multiplexed networks over conventional point-to-point dedicated connections include the following:

- Reduced wiring and space and power requirements,
- Flexibility of operations,
- Evolutionary design process, and
- Improved maintenance, diagnostics, and monitoring.

However the performance (e.g. data latency) of an asynchronous time-division multiplexed (TDM) data communication network, could be significantly influenced by the intensity and distribution of the network traffic. The traffic in DDCCS is subject to time-varying delays due to data latency of messages in the communication network [5] in addition to the sampling time delay that is inherent in digital control systems. This is evident in very fast processes (e.g., flight dynamics in tactical aircraft [5]). The effects of data latency on the dynamic performance and stability of feedback control systems are often ignored in relatively slow processes which are prevalent in chemical and processing plants. However, as the number of users on the network increases the augmented traffic causes a larger data latency to a point when its impact on the performance of some of the control loops (sharing the network) can no longer be ignored. The detrimental effects of data latency on the dynamic performance of DDCCS are further aggravated by mis-synchronism between the system components as well as by loss of messages resulting from saturation of buffers at the terminals and data corruption by noise in the network medium.

In contrast to aircraft control systems [1,2,5] where both sensor data and actuator commands are communicated via the network medium, the control systems in processing plants are hierarchical where the basic loops are locally controlled and do not normally use the network communication. Sensor data measuring the process variables from these basic loops are transmitted via the network medium to remote controller(s). The resulting feedback control signals serve as set points to basic loops. The schematic diagram in Figure 1 shows the network-induced delays in a distributed control system.

For processes with any response time, the selection of an appropriate network access protocol may be critical for dynamic performance and stability of their real-time distributed data communication and control systems (DDCCS). In general, the requirements for a DDCCS network include low data latency, high throughput, and high reliability and availability. Moreover, the chosen architecture should be flexible and adaptable to future evolution. Selection of an appropriate network access protocol is one of the most important issues in

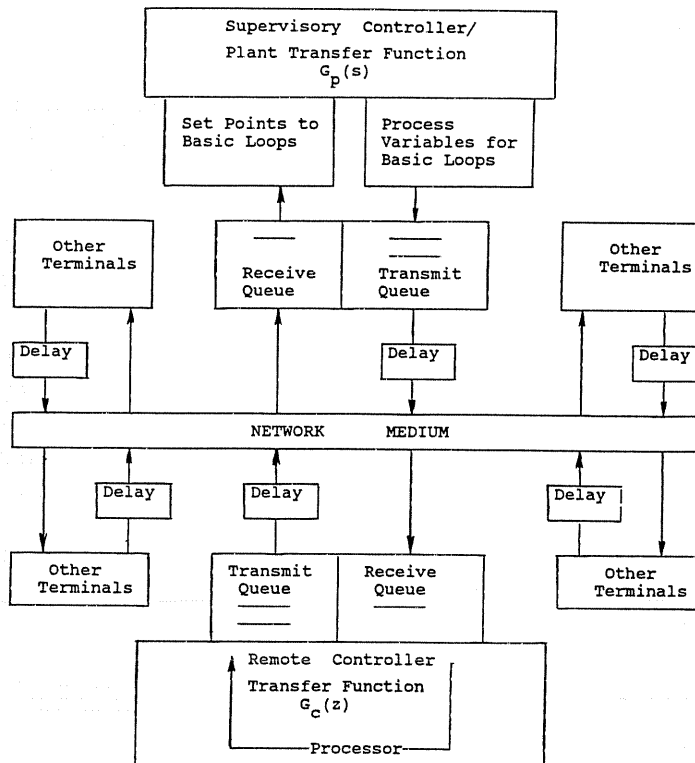


FIGURE 1 Schematic diagram of the DDCCS network.

DDCCS network design. A number of alternate candidate topologies and protocols are potentially viable for this purpose depending on the specific application. The candidate protocols should be identified and systematically evaluated with respect to a set of performance criteria that include data latency, throughput, and reliability [6,7].

The objectives of this paper are (1) to analyze the performance of network access protocols for real-time distributed communication and control system networks, and (2) to demonstrate how transport delays due to time-varying data latency in protocols could degrade the dynamic performance of the controlled process, and be a source of potential instability.

The paper is organized in five sections and one appendix. An overview of candidate protocols for DDCCS networks is presented in Section 2. The structure of the simulation program for network performance evaluation is described in Section 3. Section 4 presents the simulation results to exhibit the time-varying nature of the network-induced delays and their impact on the dynamic performance of the controlled process. Summary and conclusions of this research work are given in Section 5. Appendix A provides definitions of pertinent network variables and supporting propositions that are essential for interpreting the simulation results.

2. NETWORK ACCESS PROTOCOLS

Asynchronous time-division multiplexed protocols are suitable for DDCCS networks that are subject to a combination of periodic and bursty traffic. Commonly used protocols are: Distributed Controlled Access, Centralized Controlled Access, and Random Access [7]. The advantages of distributed controlled access protocols, like token bus and token ring, over random access protocols, like CSMA/CD, for real-time control applications have been discussed in one of our recent publications [5].

Two high-speed distributed controlled access protocols, namely SAE linear token bus [8] and SAE token ring [9], were selected as the candidate protocols for the DDCCS network. Larger communication speed and lesser complexity form the rationale for selecting SAE protocols as opposed to IEEE 802 family protocols [10,11] which have been adopted in other data highway protocols like PROWAY (Process Data Highway) [12] and MAP/EPA [13,14]. Under normal operations, the performance of SAE protocols are largely similar to that of IEEE

Table I. Comparison of Network Access Protocols

Features	SAE Linear Token Bus	IEEE 802.4 Linear Token Bus	SAE-Token Ring	IEEE 802.5 Token Ring	MIL-STD -1553B
Type of medium access	D	D	D	D	C
Transmission rate (Mbit/s)	25/50/100	1/5/10	40/80	1/4	1
Data word length (bits)	16	8	16	8	16
Formatted word length (bits)	16	8	16	8	20
Maximum number of physical address	128	256(suggested)	128	128(suggested)	31
Maximum message size (# of word counts)	4096	8182/8174	4096	no max specified (up to 10 msec)	32
Message overhead (bits)	84	112/176	132	104/168	40/80
Token length (bits)	24	112	36	24	-
Token holding delay (usec)	0.5	-	1	-	-
Response time (usec)	-	-	-	-	12
Intermessage gap (usec)	-	-	-	-	4
Maximum bus length (meter)	1024	-	1000	-	-
Level of priority	4	4	8	8	-

D is Distributed
C is Centralized

802 family protocols but their failure recovery and other station management properties are different. The performance of the SAE linear token bus and token ring protocols were compared with that of MIL-STD-1553B [15] which has been extensively used in digital avionic systems of military aircraft [1,2,5]. Table I summarizes the features of the SAE, IEEE and MIL-STD protocols, that are pertinent to the performance analysis of DDCCS networks.

3. NETWORK PERFORMANCE EVALUATION

Parameters for performance evaluation of communication network protocols in a real-time distributed process control environment are:

- Data latency, i.e., the queueing delay plus the delay due to serial transmission of message bits;
- Data frame errors; and
- Reliability and system availability.

Reliability and availability are largely hardware-dependent and are usually analyzed during the hardware design phase when detailed specifications become available. Data frame error rate depends on the raw bit error rate in the medium and the error detection algorithm provided in the protocol (e.g., 16-bit cyclic redundancy check in SAE token ring and linear token bus). Raw bit error rates in (coaxial cable) network media in a typical industrial environment range for 10^{-12} to 10^{-8} [7]; corresponding values for optical fiber media are significantly lower. Although message retransmissions or rejections (as a result of detected frame errors) apparently have no significant bearing on data latency and throughput under these circumstances, it is the undetected frame errors that degrade control system reliability. Therefore, the issue of undetected frame error rate should be dealt along with reliability.

Data latency is dependent on the traffic pattern which, in turn, is governed by message arrival rate, time sequence of message arrival and transmission on the network medium, message length, buffer size at individual terminal's transmission queue, and number of terminals in the network. In this respect the network performance was evaluated by both simulation and analytical techniques. Appendix A provides definitions of pertinent parameters and two propositions that are useful in interpreting the simulation results.

3.1 *Simulation Models for Network Performance Evaluation*

One of the major objectives of the distributed data communication and control system (DDCCS) simulation is the comparative evaluation of the candidate protocols using one or more models of the process control system. This was achieved by decomposing the DDCCS into subsystems as described below.

The DDCCS model consists of two subsystem models: (1) Discrete-event model of the network, and (2) Continuous-time model of process dynamics and discrete-time model of the process controller. The program structure is modular,

i.e., any one of the protocol models (e.g. SAE linear token bus or SAE token ring or MIL-STD-1553B) can be inserted in the simulation program while operating on the same control system model and vice versa. A schematic diagram for the combined discrete-event and continuous-time model structure of the DDCCS network is shown in Figure 2.

The network subsystem model consists of two independent but interacting submodels: (1) Message generation submodel; and (2) Protocol submodel.

The message generation submodel has an identical structure for all types of protocols and is driven by an external pool of messages that arrive at the network system either periodically or at random intervals of time (Poisson arrival for example). Similarly, the message lengths can be either constants or randomly distributed (exponential for example). When a new message arrives at the system from the external message generator, the message attributes are defined to establish the message identity in the following ways:

- Time of arrival—this is the instant at which the arrival of a message at the transmitter queue is recorded.
- The message information length (overhead not included).
- The source terminal, i.e., the terminal from which the message is generated.
- The destination terminal—this could be any terminal on the network other than the source terminal.
- The message priority if applicable.

The protocol submodel essentially represents the algorithm of the network access protocol under consideration. Different submodels, each of which identically enters the simulation program as a subroutine, have been developed for SAE linear token bus, SAE token ring, and MIL-STD-1553B protocols. Although internal algorithms of the individual protocol submodels are different, their interactions with the message generation submodel and the models of the plant and controller are identical. For example, the attributes of the generated messages are captured by the protocol submodel which, in turn, regulates the delays for exchange of messages between the plant and controller terminals.

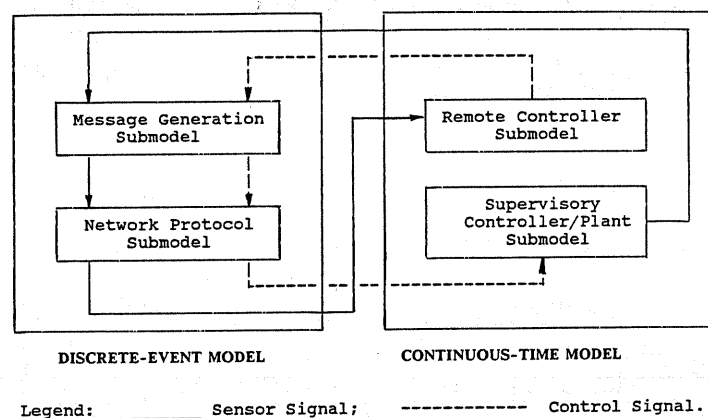


FIGURE 2 Simulation program structure for the DDCCS network.

The plant and controller models were formulated using the standard continuous-time and discrete-time state-variable approaches, respectively. The ordinary differential equations describing the plant model were solved by a standard numerical integration routine provided in the IMSL package. The solution of the difference equation describing the digital controller was straightforward. The interactions between the plant and controller models involve exchange of sensor and control signals which undergo time-varying delays introduced by scheduled events in the network model.

3.2 Simulation Model Coding and Implementation

Key considerations in the choice of a language for the DDCCS simulation were: (1) Combined discrete-event and continuous-time simulation capability; (2) Programming flexibility and software portability; (3) Verification and debugging capability; (4) Built-in statistical testing capability; and (5) Automatic ordering of scheduled events. After considering a number of simulation languages [16], SIMAN which offers all of these key features [17] was chosen as the software for coding and implementation of the DDCCS model.

The network model was operated under steady state while disturbances were applied in the control system model to observe the plant transients. Under each traffic scenario and for each protocol, the simulation model was run for 2 million μsec , i.e., 2 sec. Built-in statistical testing routines provided in the SIMAN [17] were employed to obtain the average values, standard deviations, and confidence intervals at different percentiles of the queueing delay, data latency and throughput. Because of the time-varying nature of the network characteristics, these steady-state statistical parameters do not provide sufficient conditions for the DDCCS design.

4. RESULTS AND DISCUSSION

The effects of data latency on dynamic performance and stability of a closed loop distributed data communication and control system (DDCCS) were demonstrated by simulation. As an example, a single-input single-output control system with unity feedback was considered where the plant transfer function $G_p(s)$ and the analog equivalent $G_c(s)$ of the digital controller transfer function $G_c(z)$ in Figure 1 were selected as

$$G_p(s) = 1/((0.3s + 1)(0.03s + 1))$$

and

$$G_c(s) = 7(s + 5)/s.$$

In a real chemical process, the plant model would be much more complex, have larger time constants, and most likely incorporate a transport delay. However the objective of this simple example is to demonstrate the relative stability of the plant when additional delays due to data latency and loss of messages in the network cannot be ignored. The following network configuration was employed

as the basic model for simulation experiments and comparisons of the simulation results.

- The network consists of 31 terminals or nodes (this is the upper limit for MIL-STD-1553B);
- Terminal #1 operates as the plant supervisory controller with its transmitter queue serving as the sensor for process variables and its receiver queue serving the set point generator for the basic loops as illustrated in Figure 1.
- Terminal #2 operates as the remote controller terminal with its transmitter queue handling control commands and the receiver queue handling sensor data of process variables as illustrated in Figure 1.
- Terminals #1 and #2 have periodic traffic with fixed-length messages with the data part $L = 64$ bits and a sample period of 10 msec. The time skew between the sampling instants of the terminals #1 and #2 is 5 msec.
- Terminals #3 to #31 are modelled as ordinary nodes of the network where the expected value of the message inter-arrival time was set to 10 msec and message lengths were varied to regulate the offered traffic in the network so that its impact on the dynamic performance of the DDCCS can be assessed.

The critical offered traffic G_{cr} (see Definition 10 in Appendix A) for individual protocols was computed on the basis of analytical relationships provided in Appendix A and using the data in Table I. For the traffic scenario described above, G_{cr} for the SAE linear token bus, token ring, and MIL-STD-1553B was computed to be 0.993, 0.986, and 0.523, respectively (see Appendix C of [5]).

The time history of queueing delay at individual terminals was recorded in the form of barcharts to investigate the time-varying nature of data latency. As an example, the queueing delay at the terminal #1 under steady state periodic traffic is shown in Figure 3 for SAE linear token bus at offered traffic $G = 0.5$. The abscissa indicates the instants of time when the sensor data, as a message, arrives at the transmitter queue of terminal #1, and the ordinate indicates the queueing delay for these messages at terminal #1. For example, the message that arrives at the time instant 200 msec in Figure 3 waits in the queue for about 5.25 msec before its transmission is initiated, i.e. its transmission starts at the time instant 205.25 msec.

Figure 3 indicates that the data latency could be time-varying even under steady state periodic traffic. Therefore, the conventional frequency domain analysis which is suitable for linear time-invariant systems may not be valid for analyzing the dynamic performance and stability of the closed loop DDCCS which is subject to time-varying transport delays. Time-domain techniques are needed to solve this problem analytically.

The results of the closed loop simulation of the DDCCS are presented as a series of curves in Figures 4 to 6 illustrating the transient responses of the output for a unit step increase in the reference input from an initial steady state condition under normal operations of the network. Since the data latency characteristics of SAE token ring and linear token bus are similar to a large extent under these circumstances, the transient responses of the DDCCS were

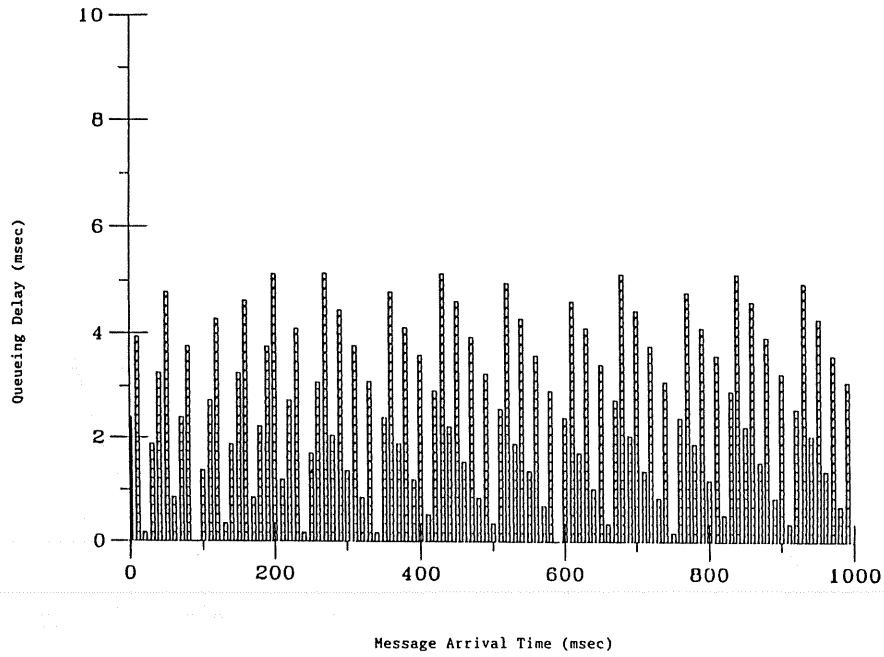


FIGURE 3 Steady state profile of queuing delay for SAE line token bus protocol.

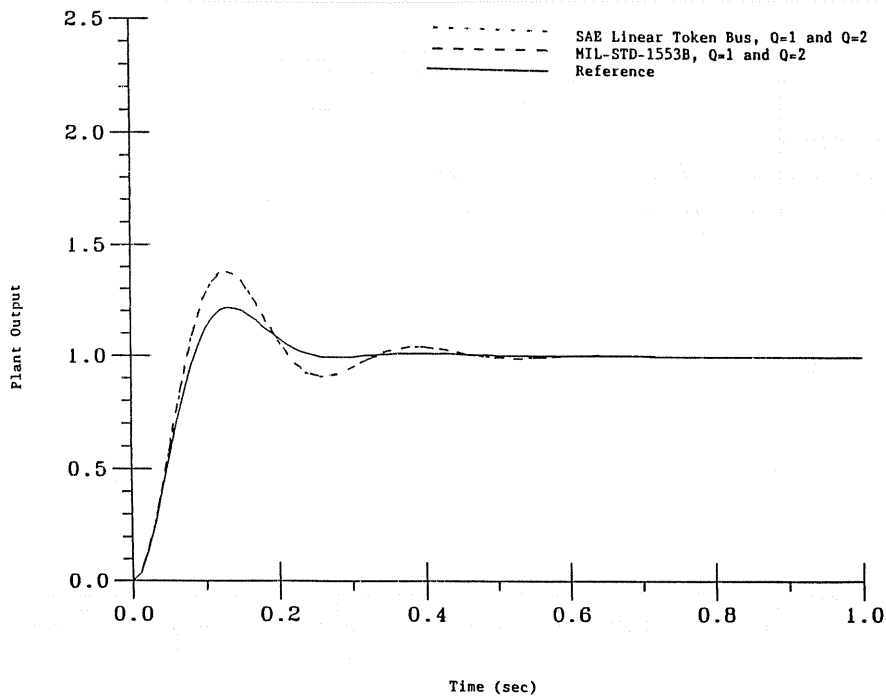


FIGURE 4 Transient response of plant output at $G = 0.2$.

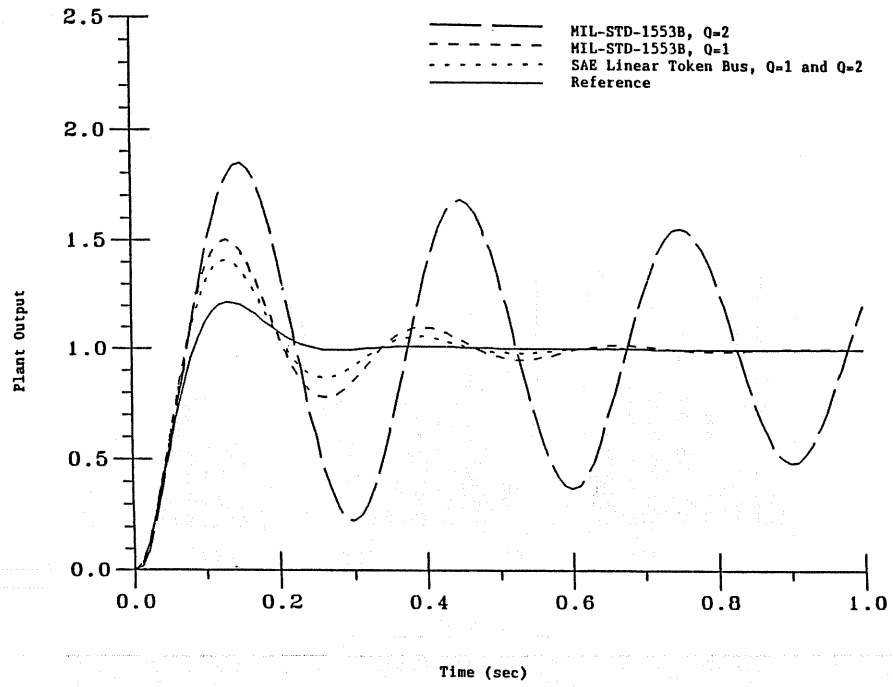


FIGURE 5 Transient response of plant output at $G = 0.7$.

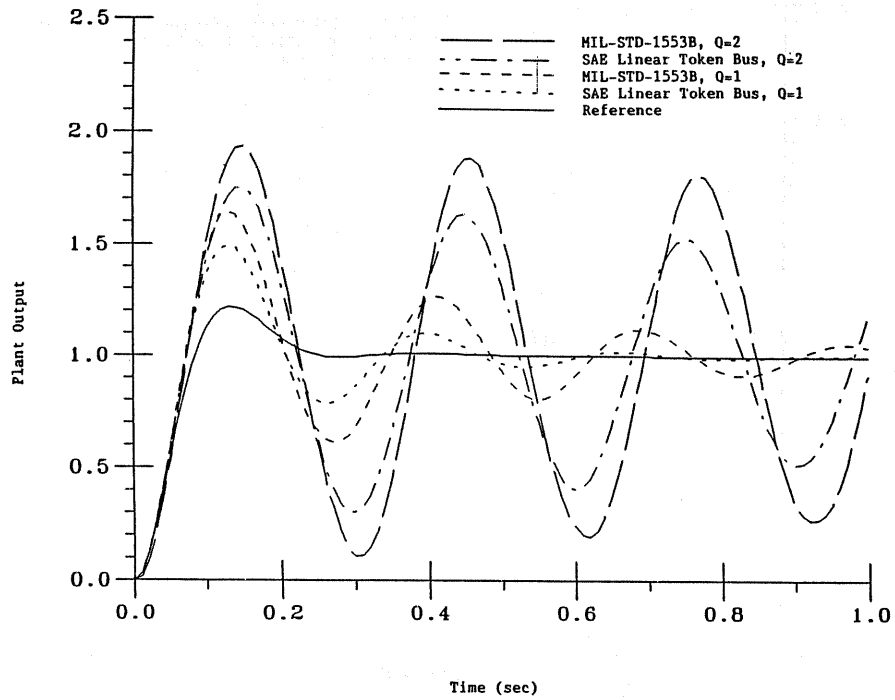


FIGURE 6 Transient response of plant output at $G = 1.2$.

generated with the SAE linear token bus and MIL-STD-1553B as network access protocols. Transient responses under identical conditions were obtained for an equivalent centralized digital controller where the control loop is not subject to any delay except the usual sampling time delay. These responses were used as a reference in Figure 4 to 6 for evaluating the individual protocols in the DDCCS.

Figure 4 shows transient responses for offered traffic $G = 0.2$ (see Definitions 7 and 10 in Appendix A). Since G is less than G_{cr} for both protocols, data latency is independent of the queue limit Q by Proposition 2 in Appendix A, and thus the transient responses for $Q = 1$ and $Q = 2$ are identical for each protocol in Figure 3. The response of the plant output is almost identical for both protocols although there is a noticeable degradation with respect to the reference response obtained from an equivalent centralized control system. This is an evidence of the additional transport delay contributed by the network access protocols.

The transient responses for $G = 0.7$ are given in Figure 5. The performance of MIL-STD-1553B for $Q = 1$ is changed to some extent with respect to that of $G = 0.2$ in Figure 3 since G exceeds G_{cr} only for the MIL-STD-1553B protocol which has a larger overhead and idle time than the linear token bus as shown in Table I. Since $G > G_{cr}$, some messages are lost due to queue saturation for MIL-STD-1553B by Proposition 1. If Q is increased to 2 for MIL-STD-1553B, Proposition 2 states that the steady-state queueing delay at each terminal is increased by an additional amount of one sample time, i.e. 10 ms. In this case the dynamic response of the plant output becomes much worse due to the increased data latency. For the linear token bus, as G is still smaller than G_{cr} , the DDCCS does not suffer from loss of messages due to queue saturation (and additional data latency), and thus exhibits much superior performance.

Figure 6 shows transient responses for $G = 1.2$ where G exceeds G_{cr} for both protocols. For $Q = 1$, MIL-STD-1553B suffers from a long settling time whereas the performance of token bus is not seriously degraded. However, for $Q = 2$ both protocols suffer from increased data latency as a result of the additional queueing delay of 10 ms and exhibit instability.

The overshoot and settling time of transients for SAE linear token bus are distinctly superior to those for MIL-STD-1553B under identical network traffic because MIL-STD-1553B, in general, suffers from larger data latencies due to a larger message overhead and idle time. This also entails a relatively smaller critical offered traffic for MIL-STD-1553B resulting in a larger number of lost messages whenever offered traffic exceeds its critical value.

5. SUMMARY AND CONCLUSIONS

The performance of the SAE linear token bus and MIL-STD-1553B protocols has been analyzed in view of the requirements for real-time distributed data communication and control systems (DDCCS) in chemical and processing plant applications. Performance evaluation was carried out using a combined discrete-event and continuous-time simulation approach. The simulation results were generated to demonstrate how the transport delay due to time-varying data

latency in network access protocols could degrade the dynamic performance of the control system and be a source of potential instability.

The following conclusions regarding DDCCS network design can be derived from the results of this research.

- For all protocols the critical value of offered traffic is important in view of loss of messages due to queue saturation as well as for the data latency especially if the queue limit is larger than 1.
- The network should be designed such that the offered traffic does not exceed the critical value. This implies that any combination of network design parameters (namely, the number of terminals on the network, message arrival rate, message length, medium bandwidth and sample time) should allow for a safe margin between the offered traffic and its critical value.
- Dynamic performance is much better for queue limit of 1 than for larger queue limits for all protocols whenever G exceeds G_{cr} . If a terminal serves M devices that generate M different types of messages then the queue limit has to be set to M to accommodate for M distinct messages in the point-to-point communication mode. Alternatively, the queue limit could be set to 1 by concatenating all M messages into one message and using the broadcast mode of transmission. These options have their own merit and demerit and the choice of a specific option is a design issue.
- The DDCCS, in general, should have better dynamic performance if distributed controlled access protocols, such as SAE linear token bus or token ring) is used instead of a centralized controlled access protocol like MIL-STD-1553B. However, this conclusion is made only on the basis of data latency where failure modes and reliability issues have not been addressed.

The above observations are generic in nature and are applicable to other distributed data communication and control systems (DDCCS) including those for fossil and nuclear power plants, autonomous manufacturing processes, and aircraft. Although the process dynamics in different applications may vary widely, the concept of dimensionless offered traffic and the resulting data latency relative to the sampling period is similar in all cases.

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APPENDIX A

ANALYTICAL MODELS OF NETWORK PERFORMANCE

Definitions of DDCCS network performance parameters and pertinent analytical results under restricted conditions are introduced below.

Definition 1. Network traffic is defined to be deterministic if the message length and the message interarrival time are constants.

Definition 2. Network traffic is defined to be simultaneous if it is deterministic and if the messages arrive at all terminals simultaneously at each sampling period.

Definition 3. For a message, the word count W_c is defined as

$$W_c(L) = \begin{cases} \text{Int}(L/W_d) & \text{if } \text{Rem}(L, W_d) = 0, \\ \text{Int}(L/W_d) + 1 & \text{if } \text{Rem}(L, W_d) > 0, \end{cases}$$

where L = Length (in bits) of the data part of a message excluding the bits due to formatting and overhead, W_d = number of data bits per word, $\text{Int}(\ast)$ indicates the integer part of \ast , and $\text{Rem}(a, b)$ is equal to $a - [\text{Int}(a/b)]b$.

Definition 4. Frame length L' (bits) of a message is defined as

$$L' = W_f W_c(L) + \Omega$$

where W_f = length (bits) of a formatted word, and Ω = overhead (bits) associated with a message.

Definition 5. Queueing delay δ_q of a successfully transmitted message is the difference between the instant of arrival of the message at the transmitter queue of the source terminal and the instant of transmission of its first bit on the medium.

Definition 6. Data latency δ of a successfully transmitted message is defined as the difference between the instant arrival of the message at the transmitter queue of the source terminal and the instant of reception of its last bit at the destination terminal.

Remark 1. Data latency and queueing delay are approximately related as

$$\delta = \delta_q + L'/R$$

where R = data rate in bits/sec.

Definition 7. Offered traffic G is defined as

$$G = (E[L]N)/(R E[T])$$

where N = number of active terminals in the network, T = Message interarrival time at a terminal, and $E[*]$ denotes the expected value of $*$.

Definition 8. Cycle time τ is defined as

$$\tau = \sum_{i=1}^N [L_i'/R] + N\sigma$$

where σ = average bus idle time prior to the beginning of a message transmission, and the subscript i corresponds to the terminal $\#i$.

Remark 2. For a given traffic, cycle time may be interpreted as the total time required to complete the transmission of one message from each of the N terminals.

Definition 9. For a given traffic, normalized cycle time, G' is defined as the ratio of the expected values of the cycle time and the message interarrival time, i.e., $G' = E[\tau]/E[T]$.

Remark 3. For a given G , individual protocols may load the medium to different levels and thus influence the performance of the DDCCS to different degrees. Therefore, G is used as a parameter for selection of network access protocols instead of G' . A limit of G above which a given protocol is expected to overload the medium, resulting in message rejection, needs to be specified.

Definition 10. The critical offered traffic G_{cr} , for a protocol, is defined as the largest offered traffic for which no message frame is rejected due to queue saturation for deterministic traffic under steady state.

Remark 4. $G' \leq 1$ if $G = G_{cr}$.

In a feedback control loop, messages are transmitted via the network medium from the sensor terminal to the controller terminal and from the controller

terminal to the actuator terminal. The the control system is subject to time-varying delays due to data latency, and its performance is dependent on the traffic in the network. Additional definitions of pertinent parameters are given in [5].

Under periodic traffic and constant message lengths, the following propositions which have been derived in our earlier publication [5] are presented below. These propositions are essential for interpretation of the simulation results in Section 4.

Proposition 1. Under steady state the number of messages waiting in the queue of each terminal is either 0 or 1 if $G \leq G_{cr}$, and either $(Q - 1)$ or Q if $G > G_{cr}$, where the positive integer Q is the queue limit, i.e., the maximum number of messages that can wait at the queue of a terminal.

Proof of Proposition 1. For $G \leq G_{cr}$, none of the queues saturate because one message arrives at each terminal over a time interval of T and exactly one message is removed from each queue during the same time interval under steady state. The population of waiting messages at any queue under steady state never exceeds 1.

For $G > G_{cr}$, all queues saturate under steady state since one message always arrives at each terminal during a time interval of T but a terminal may not always have the opportunity to transmit a message during each interval of T . In this process the queue at each terminal will build up to the saturation limit Q and a message will be rejected whenever this limit exceeds. Therefore, starting from a point when the population at a queue is Q , at most one message is removed from the queue due to transmission during an interval of T and exactly one message arrives at the queue during the same time interval. Thus the population of waiting messages at each queue cannot be smaller than $(Q - 1)$.

Corollary to Proposition 1. Under steady state, message rejection rate is independent of the queue limit Q .

Proof of Corollary to Proposition 1. Proof follows directly from the proof of Proposition 1.

Proposition 2. The queueing delay δ_q under steady state is given as

$$\delta_q = \begin{cases} \delta_q^* & \text{for } G \leq G_{cr} \\ \delta_q^* + (Q - 1)T & \text{for } G > G_{cr} \end{cases}$$

where δ_q^* is the queueing delay if $Q = 1$.

Proof of Proposition 2. For $G \leq G_{cr}$, the number of waiting messages at each queue is at most 1. So, δ_q is independent of Q implying that $\delta_q = \delta_q^*$ for all Q .

For $G > G_{cr}$, by Proposition 1, the number of messages at each queue is either $(Q - 1)$ or Q . Since a new message arrives at the queue at an interval of T and occupies a slot at the rear end of the queue. Therefore, every message will have to wait for a period equal to $(Q - 1)T$ before it reaches the tip of the queue. Once it reaches the tip of the queue, the queueing delay is identical to that for $Q = 1$.

Proposition 3. For $G > G_{cr}$, i.e., $\tau > T$, if α_j is defined as $\alpha_j = \text{Rem}(\alpha_0 + j\tau, T)$, then

- (a) $\alpha_0 = 0$ implies that α_j is discretely uniformly distributed in $(0, T]$;
- (b) if α_0 is continuously uniformly distributed in $(0, T]$, then α_j is also continuously uniformly distributed in $(0, T]$.

Proof of Proposition 3. (a) Let γ be the greatest common divisor of τ and T , i.e., $\tau = l\gamma$ and $T = P\gamma$, where l and P are coprime integers and $l > P$. Then $\alpha_j = \text{Rem}(jl, P)\gamma$ and consequently α_j assumes P distinct values $j\gamma$ for $j = 1, 2, \dots, P$. Since $\alpha_{P+i} = \alpha_i$, the sequence $\{\alpha_j\}$ is cyclic with a period of $P\tau$ and the distribution of α_j is discretely uniform.

(b) In this case α_j assumes the P distinct values of $\alpha_0 + k\gamma$ with equal probability, where $-\text{Int}[\alpha_0/\gamma] \leq k \leq P - \text{Int}[\alpha_0/\gamma] - 1$. Since α_0 is continuously uniformly distributed in $(0, T]$, α_j is also continuously uniformly distributed in $(0, T]$.

Corollary to Proposition 3. α_j is identically equal to δ_q^* in Proposition 2.

Proof of Corollary to Proposition 3. Proof follows directly from the proofs of Proposition 3 and Proposition 2.