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Integrated Communication and Control Systems: Part III—Nonidentical Sensor and Controller Sampling[1]

Luen-Woei Liou

A. Ray
Mem. ASME

Mechanical Engineering Department,
The Pennsylvania State University,
University Park, PA 16802

Networking in Integrated Communication and Control Systems (ICCS) introduces randomly varying delays which degrade the system dynamic performance and are a source of potential instability. In Part I [1] of this sequence of papers we developed a discrete-time, finite-dimensional model of the delayed control system for analysis and design of ICCS where the sensor and controller have identical sampling rates. In Part II [2] we proposed two alternative approaches for ICCS design, namely, identical and nonidentical sampling rates for sensor and controller. This Part III presents extended modeling of ICCS for nonidentical sensor and controller sampling rates. This model is also suitable for analyzing tracking problems, i.e., control systems with time-dependent reference inputs.

1 Introduction

Integrated Communication and Control Systems (ICCS), described in a recent two-part paper [1, 2], are applicable to complex dynamical processes like advanced aircraft, spacecraft, and autonomous manufacturing plants. Time-division-multiplexed networks are employed to interconnect the distributed components and coordinate diverse functions in ICCS. The feedback control loops in ICCS are subject to network-induced delays [1-5] in addition to the delays incurred in digital sampling and data processing. These delays are usually randomly varying and distributed, degrade the system dynamic performance, and are a source of potential instability.

In Part I [1]of the above two-part paper, we have presented the finite-dimensional modeling of ICCS by taking into consideration the effects of network-induced delays. A necessary and sufficient condition for the system stability was established for the special case of periodically varying (nonrandom) delays. The attention was focused on the control loops with identical sampling rates for the sensor and controller. Even though the sensor and controller sampling periods are designed to be identical, a certain difference between them always prevails due to manufacturing tolerances in the clock frequencies. This generates a drift resulting in a time skew Δ_s between the sensor and controller sampling instants. Since the drift is very slow (because of a small difference between the sensor and controller sampling rates), Δ_s can be considered to be a constant over a finite window of time and may significantly contribute to network-induced delays. This aspect has been discussed in view of ICCS design in Part II [2]. One way of circumventing this problem is periodic synchronization of the control system components by which Δ_s is maintained within a desirable range. This could be achieved by transmitting high-priority synchronization signals via the network medium or by additional wiring. Nevertheless this procedure would require additional efforts to meet the system reliability requirements.

An alternative approach to the above synchronization procedure is to deliberately assign nonidentical sampling periods T_s and T_c to the sensor and controller, respectively. If the ratio $\epsilon = T_s/T_c$ is not close to $1/M$, where M is a positive integer, then the skew Δ_s will vary rather rapidly and shall not remain at an undesirable value over a prolonged period. Another benefit of having nonidentical sampling is to reduce the occurrence of vacant sampling slots at the controller, which results from mis-synchronization between the control system components and varying data latency [1, 2]. This can be achieved by selecting an appropriate ϵ as presented in Proposition 2.1 of Part II [2]. (Note: ϵ was defined as $(T_c - T_s)/T_c$ in Part II instead of T_s/T_c).

ICCS with nonidentical sensor and controller sampling, as depicted above, belong to the class of multirate sampling systems [6] and could have significantly different dynamic characteristics from those with identical sampling. The ICCS model developed in Part I [1] for identical sensor and controller sampling cannot be readily adapted to the nonidentical sampling case where the effects of two time frames (due to the absence of synchronization between the sensor and controller sampling instants) must be taken into account and the system is, in general, time-varying even in the absence of network-induced delays. This Part III specifically addresses finite-dimensional modeling of control systems that are subject to randomly varying distributed delays and have nonidentical

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sensor and controller sampling rates. The model is also suitable for analyzing tracking problems, i.e., control systems with time-dependent reference inputs. The reported work is essentially an extension of the two-part paper [1, 2] which provides the necessary background.

A significant amount of research [6-12] has been reported on multirate sampling for nondelayed systems. Some of these techniques are discussed below.

The z-transform method [7] is applicable to multirate linear time-invariant systems where the individual sampling rates are restricted to be integer multiples of the basic sampling rate. This approach is not suitable for ICCS since the network-induced delays make the system time-varying [1].

The state-space approach, originally proposed by Kalman and Betram [6], provides a unified procedure for modeling various types of sampling data systems that consist of both continuous and discrete components. For multirate sampling in the absence of varying delays, if the ratio $\epsilon = L/M$ where L and M are relatively prime integers, then this method produces a time-invariant model at the time epochs that occur with a period of LM samples. Significant modifications of Kalman and Bertram's model are required to develop models of multirate systems with varying delays. Such models may turn out to be very cumbersome because of the additional "bookkeeping" needed to keep track of the varying delays at every sample.

The singular perturbation and time-scale separation [8-10] techniques have been used for analyzing mutlirate systems that have a special structure of separable fast and slow dynamics. Applying the concept of singular perturbation technique for continuous systems where the faster part is assumed to be instantaneous, the time-scale separation technique transforms the system model into two decoupled fast and slow subsystems. This substantially reduces the computational efforts for controller design and yields well-conditioned submatrices. (See Litkouhi and Khalil [8] for example). Although these techniques could be useful for design and performance analysis of various control loops belonging to the integrated control system, the plant and controller dynamics within a given control loop are not separable. Therefore, these methods cannot be applied to modeling of ICCS with nonidentical sensor and controller sampling.

Broussard and Glasson [12] proposed an approach to multirate optimal control synthesis where the system model is restricted to have the ratio of fast and slow sampling rates to be integer multiples and the sampling instants are synchronized. Since the major difficulty in ICCS modeling is the varying network-induced delays, the above approach is not applicable to modeling of multi-rate systems with varying delays.

We are proposing a method for modeling multirate sampled systems that are subjected to randomly or determininstically varying delays. Our approach to analysis of ICCS with nonidentical sampling emphasizes finite-dimensional modeling using the concepts of state transition and keeping track of epochs of random event occurrences. (Keeping track of random events is much less complex for modeling of ICCS with identical sampling [1].) This method is apparently better suited to ICCS modeling than any other reported techniques.

This paper is organized into four sections including the introduction. The finite-dimensional models for control systems with nonidentical sensor and controller sampling periods are developed in Section 2. Simulation results using a simple model are presented in Section 3 to illustrate how the ratio of sensor and controller sampling rates influences the control system dynamics. Summary and conclusions along with recommendations for future work are presented in Section 4.

2 Finite-Dimensional Modeling: Nonidentical Sampling

In the conventional design of digital control systems, the sensor, controller, and actuator are colocated and, therefore, a single sampling rate applies to all components which are essentially synchronized. Often the digital control law is obtained by modifying the continuous-time version. The sampling time T is a critical parameter in this modified control law. Since T_c and T_s, the respective sampling periods of the controller and sensor, are unequal in the present case, the ICCS is modeled on the consideration that the digital control law is based on T_c.

Having the sensor sampling rate larger than the controller's could provide a means for reducing the total delay in the ICCS. Furthermore, probability of vacant sampling at the controller [2] can be reduced to zero by an appropriate choice of the ratio $\epsilon = T_s/T_c$ where arrival of at least one fresh sensor data is assured at the controller during each sampling period. Since the objective is to reduce the detrimental effects of network-induced delays on the system dynamic performance,

Nomenclature

A = plant system matrix $(n\times n)$
A_c = plant state transition matrix $(n\times n)$
A_t = plant state transition matrix $(n\times n)$
B = plant input matrix $(n\times n)$
B_c = plant input matrix $(n\times m)$ in convoluted form
B_t = plant input matrix $(n\times m)$ in convoluted form
C = plant output matrix $(s\times n)$
c_k = instant of control command arrival at the actuator relative to the kth sample of the controller
F = controller system matrix $(q\times q)$
G = controller input matrix $(q\times s)$
H = controller output matrix $(m\times q)$
K = controller direct coupling matrix $(m\times s)$
n_k = number of sensor samples in kth controller period
r_{k_k} = sensor sampling instants in kth controller period
T_c = controller sampling period
T_s = sensor sampling period
t = time
t_k = instant of arrival of the last sensor data at the controller relative to the kth sample
X = augmented state vector $(N\times 1)$
x = plant state vector $(n\times 1)$
u = plant input vector $(m\times 1)$
y = plant output vector, i.e., generated sensor data $(r\times 1)$
z = delayed sensor data $(r\times 1)$
Δ_s = time skew between sensor and controller sampling instants
δ_{ca} = controller-to-actuator data latency
$\delta_{\max}$ = supremum of sensor-to-controller data latency
$\delta_{\min}$ = infinum of sensor-to-controller data latency
δ_p = processing delay at the controller computer
δ_{sc} = sensor-to-controller data latency
ϵ = sampling ratio T_s/T_c note: ϵ was defined as $(T_c - T_s)/T_c$ in Part-II [2]
η = controller state vector $(q\times 1)$
θ_{ca} = sensor-controller delay
θ_{sc} = sensor-controller delay
Υ = input matrix for augmented system $(n\times r)$
Φ = augmented system matrix $(N\times N)$

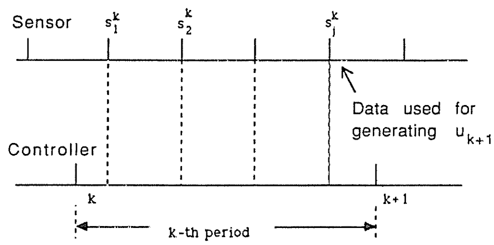

Fig. 1 Multirate sampling without delays

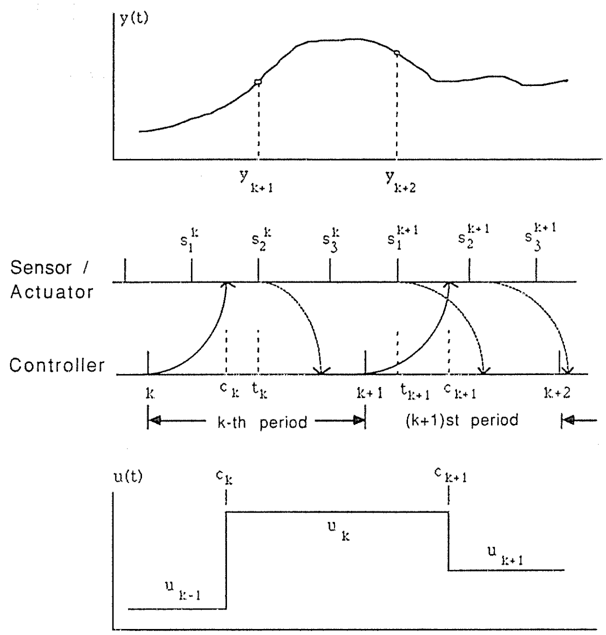

Fig. 2 Multirate sampling with network-induced delays

T_s should not be larger than T_c. Therefore, we only consider the case of $T_s < T_c$, i.e., $\epsilon < 1$, for design of ICCS with nonidentical sampling. However, T_s should not be excessively reduced since this would cause additional traffic which might affect the performance of other control loops that share the network.

The plant and controller models follow the notations in Part I [1]. The linear time-invariant model of the plant is given as:

$$dx/dt = Ax(t) + Bu(t) \tag{2.1}$$

$$y(t) = Cx(t) \tag{2.2}$$

where $x \epsilon R^n$, $u \epsilon R^m$, $y \epsilon R^s$, and the matrices A, B, and C are of compatible dimensions.

The state space model of the linear, time-invariant, discrete-time controller is given as:

$$\eta_{k+1} = F\eta_k - G(z_k - r_k) \tag{2.3}$$

$$u_k = H\eta_k - K(z_k - r_k) \tag{2.4}$$

where $\eta \epsilon R^q$, z_k is the last available measurement at the instant u_k is processed by the controller, r_k is the reference signal, and the matrices F, G, H, and K are of compatible dimensions. In constrast with the sensor data, which may wait at the controller's receiver queue before being processed, the control input u_k acts upon the plant immediately after arriving at the actuator terminal. This happens because the controller is scheduled to periodically generate signals whereas the operations of the zero-order-hold (ZOH) device at the actuator are essentially asynchronous (see Sections 3 and 4 of Part I [1]). The reference signal r_k could either be an external input or may be generated internally by the outer controller loop within the cascaded control system. In the latter case r_k is also subjected to network-induced delays.

The reference time frame of the control system model, i.e., the combined plant and controller, is selected with respect to the controller's sampling instants. Figures 1 and 2 illustrate the concept of relative timing between the sampling instants at the sensor and controller.

Denotation 1: The number of sensor sampling instants that occur within the kth sampling period of the controller (i.e., between the kth and $(k+1)st$ sampling instants) is denoted as n_k.

Denotation 2: The sequence $\{s_j{}^k,\ j=1,\ 2,\ \ldots,\ n_k:\ 0 \le s_j{}^k < T_c\}$ denotes the instants of sensor sampling within the kth sampling period of the controller and measured relative to the beginning of this sampling period.

Remark 1: It follows, from Denotations 1 and 2, that

$$\max_k\ n_k = 1 + [\epsilon^{-1}]$$

where $[\varphi]$ is the largest integer less than φ, e.g., $[4.3]=4$ and $[6]=5$. The observation accrues from the fact that $s_j{}^k < T_c$ and if $s_j{}^k = T_c$, then that occurrence of sensor sampling instant is assigned to the $(k+1)st$ period.

2.1 Ideal Network: Zero Data Latency. To obtain a better understanding of this multirate sampling phenomenom within the control loop, we first consider the ideal case of having infinitesimally small traffic in the network, i.e., we assume the sensor-to-controller and controller-to-actuator data latencies, $\delta_{sc}{}^k$ and $\delta_{ca}{}^k$ (defined in the Nomenclature and explained in [1]), are negligible at every instant k. We also assume that the control signal processing delay $\delta_p \simeq 0$. Later these assumptions will be eliminated to include the network-induced delays in the ICCS model.

Under the assumptions $\delta_{ca} \simeq 0$ and $\delta_p \simeq 0$, it follows from [1] that the controller-to-actuator delay $\theta_{ca} = \delta_{ca} + \delta_p \simeq 0$. Given $s_{n_{k-1}}{}^{k-1}$, the sequence $\{s_j{}^k, j=1, 2, \ldots, n_k\}$ in Denotation 2 can be calculated as:

$$\begin{aligned} &s_1{}^k = s_{nk-1}{}^{k-1} + \epsilon T_c - T_c = s_{nk-1}{}^{k-1} - (1-\epsilon)T_c \\ &s_2{}^k = s_1{}^k + \epsilon T_c \\ &\cdot \\ &\cdot \\ &\cdot \\ &s_m{}^k = s_{m-1}{}^k + \epsilon T_c \qquad \text{if } s_m{}^k < T_c \end{aligned} \tag{2.5}$$

Remark 2: $s_m{}^k = s_1{}^k + \epsilon(m-1)T_c < T_c$ if $m-1 < (T_c - s_1{}^k)/\epsilon T_c)$

Since n_k is the number of sensor sampling instants that occur within the kth period (see Denotation 1), we have

$$n_k = [(T_c - s_1{}^k)/(\epsilon T_c)] + 1 \tag{2.6}$$

$$s_j{}^k = s_{j-1}{}^k + \epsilon T_c, j=2,3,\ \ldots\ ,n_k \tag{2.7}$$

Since the sensor output and controller input sequences, $\{y_k\}$ and $\{z_k\}$, are identical for zero data latency, the sequence $\{z_k\}$ is replaced by $\{y_k\}$. The actuator input $u(t)$ assumes only one value in the kth sampling period. This is illustrated in Fig. 1 which shows the timings of various arrivals.

Denotation 3: The epoch t_k of arrival of sensor data y_{k+1} that is used for generating the control signal at the $(k+1)st$ sampling instant is given as $t_k = s_{nk}{}^k$ and $y_{k+1} = y(kT_c + t_k)$.

Definition 1: Sensor-controller delay $\theta_{sc}{}^k$ is the amount of network-induced delay experienced by the sensor data that is processed at the kth controller sample.

Remark 3: $\theta_{sc}{}^k = T_c - t_{k-1}$. The physical concept of sensor-controller delay for nonidentical sampling is similar to that for nonsynchronous sampling, i.e., $\Delta_s>0$ and $T_s=T_c$, in Part I [1].

Denotation 4: $x_{k+1}=x((k+1)T_c)$, $u_k=u(kT_c)$, $A_c=\exp(AT_c)$, and

$$B_c=\int_0^{T_c}\exp(A(T_c-\tau))d\tau B$$

Integrating (2.1) and using Denotation 4, the plant equation becomes:

$$x_{k+1}=A_cx_k+B_cu_k \tag{2.8}$$

Substituting the expression for u_k from (2.4) in (2.8), we obtain

$$x_{k+1}=A_cx_k+B_c[H\eta_k-K(y_k-r_k)] \tag{2.9}$$

Since y_{k+1} is the most recent data available to the controller at the $(k+1)st$ sampling instant, the continuous-time plant model in (2.2) yields

$$\begin{aligned} y_{k+1} &= Cx(kT_c+t_k) \\ &= C\left[\exp(At_k)x_k+\int_0^{t_k}\exp(A(t_k-s))dsBu_k\right] \\ &= CA_tx_k+CB_tu_k \end{aligned} \tag{2.10}$$

where

$$A_t=\exp(At_k),\ B_t=\int_0^{t_k}\exp(A(t_k-\sigma))d\sigma B.$$

Using (2.4) in (2.10) yields

$$y_{k+1}=CA_tx_k+CB_t[H\eta_k-K(y_k-r_k)] \tag{2.11}$$

Combining equations (2.9) and (2.11) with the controller model (2.3), we have the closed-loop system model as

$$X_{k+1}=\Phi_kX_k+\Upsilon_kr_k \tag{2.12}$$

where the augmented state vector $X_k=[x_k{}^T\ y_k{}^T\ \eta_k{}^T]^T$,

$$\Phi_k=\begin{bmatrix} A_c & -B_cK & B_cH \\ CA_t & -CB_tK & CB_tH \\ 0 & -G & F \end{bmatrix},\ \text{and}\ \Upsilon_k=\begin{bmatrix} B_cK \\ CB_tK \\ G \end{bmatrix}$$

Remark 4: The substate vectors x_k and y_k in the augmented state vector X_k represent plant state and output vectors at different instants since $x_k=x(kT_c)$ and $y_k=((k-1)T_c+t_{k-1})$.

Remark 5: Equation (2.12) is a linear, time-varying state space model. It represents the system having different sampling periods under very light traffic load. If the sampling ratio is a rational number, i.e., $\epsilon=L/M$ where L and M are relative prime integers, then the system in (2.12) becomes periodic. This implies that the varying elements of the matrices are periodic with a period of LM. In this case, $\Phi_{k+LM}=\Phi_k$, and Proposition 4.1 of Part I [1] can be applied to determine the system stability.

2.2 Network With Medium Traffic: Non-Negligible Delays. We now proceed to develop the ICCS model by taking into account the effects of data latencies δ_{sc} and δ_{ca}, induced by the network, and the signal processing delay δ_p at the controller. The network is assumed to operate under the following operating conditions.

1. The ratio ϵ of sampling periods is chosen to be sufficiently small such that the probability of vacant sampling is zero (see Proposition 2.1 in [2]).
2. The offered traffic is sufficiently small relative to its critical value [3, 4] such that the probability of message rejection at the transmitter buffers of the sensor and controller is zero.

Remark 6: The first assumption is necessary to avoid signal distortion [2] unless the appropriate observer is incorporated in the controller [5]. The second assumption is valid from the point of view of network design such that the offered traffic is well below the critical limit (see the conclusions in Section 5 of [3]).

Remark 7: On the basis of the above assumptions it follows that

$0<\delta_{sc}{}^k<T_s$ and $0<\delta_{ca}{}^k<(T_c-\delta_p)$ $\forall k$ with probability 1.

However, this condition can be relaxed to $0<\delta_{ca}{}^k<T_c$ $\forall k$ with a minor modification in the ICCS model.

Figure 2 illustrates the timing relations in the derivations to follow.

Denotation 5: The control command computed at the kth sampling instant reaches the actuator after a delay of c_k.

Remark 8: Since c_k is the controller-actuator delay at the kth sample, $c_k=\delta_{ca}{}^k+\delta_p$. On the basis of Remark 7, $c_k<T_c$.

Now the plant state vector can be expressed as:

$$\begin{aligned} x_{k+1} &= x((k+1)T_c) \\ &= \exp(AT_c)x_k+\int_0^{c_k}\exp(A(T_c-s))dsBu_{k-1} \\ &\quad +\int_{c_k}^{T_c}\exp(A(T_c-s))dsBu_k \\ &= A_cx_k+B_{c1}u_{k-1}+B_{c0}u_k \\ &= A_cx_k+B_{c1}u_{k-1}+B_{c0}[H\eta_k-K(y_k-r_k)] \end{aligned} \tag{2.13}$$

where

$$A_c=\exp(AT_c),\ B_{c1}=\int_0^{c_k}\exp(A(T_c-v))dvB,\ \text{and}$$

$$B_{c0}=\int_{c_k}^{T_c}\exp(A(T_c-v))dvB.$$

For brevity of notations, the superscripts of data latencies are dropped, i.e., we write $s_1{}^k+\delta_{sc}$ and $s_2{}^k+\delta_{sc}$ implies that δ_{sc} in the two expressions represents delays corresponding to the two different messages.

Next we proceed to evaluate the sensor data y_k used in computing the control command u_k. Let r be the smallest positive integer such that $s_{r+1}{}^k+\delta_{sc}\geq T_c$. Then $s_j{}^k+\delta_{sc}<T_c$ $\forall j\leq r$. This implies that the most recent sensor data available to the controller is that sampled at the instant $s_r{}^k$ (relative to the beginning of the kth sample). Therefore, $t_k=s_r{}^k$ in contrast to $t_k=s_{nk}{}^k$ in the absence of any delays. (see denotation 3).

However, the sensor data used at the $(k+1)st$ sampling instant is still $y_{k+1}=Cx(kT_c+t_k)$ albeit the fact that t_k is different in the presence of induced delays.

We consider two cases on the basis of relative magnitudes of c_k and t_k.

Case 1: $t_k\geq c_k$

$$y_{k+1}=C(\exp(At_k)x_k+\int_0^{c_k}\exp(A(t_k-v))dvBu_{k-1}+\int_{c_k}^{t_k}\exp(A(t_k-\nu))d\nu Bu_k) \tag{2.14}$$

Case 2: $t_k<c_k$

$$y_{k+1}=C(\exp(At_k)x_k+\int_0^{t_k}\exp(A(t_k-v))dvBu_{k-1} \tag{2.15}$$

By expressing the control command u_k as $u_k-u_{k-1}+u_{k-1}$ in (2.14), we combine the results of the above two cases as:

$$y_{k+1} = \begin{cases} C(A_t x_k + B_1 u_{k-1} + B_0[u_k - u_{k-1}]) & \text{if } t_k > c_k \\ C(A_t x_k + B_1 u_{k-1}) & \text{if } t_k > c_k \end{cases} \tag{2.16}$$

where $A_t = \exp(At_k)$, $B_1 = \int_0^{t_k} \exp(A(t_k - v))dvB$, and

$$B_0 = \int_{c_k}^{t_k} \exp(A(t_k - v))dvB$$

We define the indicator function p_k as

$$p_k = \begin{cases} I_n & \text{if } t_k \geq c_k \\ 0 & \text{if } t_k < c_k \end{cases}$$

to combine the two expressions in (2.16) as

$$y_{k+1} = C(A_t x_k + (B_1 - p_k B_0)u_{k-1} + p_k B_0 u_k) \tag{2.17}$$

Finally, substituting u_k from (2.4) in (2.17), and combining (2.13), the closed-loop system model is obtained as:

$$X_{k+1} = \Phi_k y_k + \Upsilon_k r_k \tag{2.18}$$

where the augmented state vector $X_k = [x_k{}^T \, y_k{}^T \, \eta_k{}^T \, u_{k-1}]^T$,

$$\Phi_k = \begin{bmatrix} A_c & -B_{c0}K & B_{c0}H & B_{c1} \\ CA_t & -p_k CB_0 K & p_k CB_0 H & C(B_1 - p_k B_0) \\ 0 & -G & F & 0 \\ 0 & -K & H & 0 \end{bmatrix}, \text{ and } \Upsilon_k = \begin{bmatrix} B_{c0}K \\ p_k CB_0 K \\ G \\ K \end{bmatrix}$$

Remark 9: Elements of the matrics Φ and Υ in (2.18) are deterministically or randomly varying depending on the nature of the delays induced by the network traffic. Specifically the time instants t_k and c_k that are imbedded in the system model are determined by the network-induced delays. If the traffic is random, $\{t_k\}$, and $\{c_k\}$ are stochastic sequencies; otherwise they are deterministic functions of time. Even if the network-induced delays are constants, the matrices Φ_k and Υ_k are time-varying except when the ratio $\epsilon = 1/M$, M being a positive integer.

Remark 10: The model for nonidentical sampling case in (2.18) is in a form similar to that for identical sampling in Part I [1], i.e., discrete-time, linear, augmented state space representation with time-varying elements. However, the major difference between the two models is how the frame of reference time is selected. The rationale for using different reference frames for nonidentical sampling is that the sensor arrival instants $\{t_k\}$ relative to the controller sampling instants is a sequence of varying terms even in the absence of network-induced delays whereas the time skew Δ_s between the sensor and controller sampling instants was treated as a constant for identical sampling in [1]. In (2.18) the states x_k are taken at controller sampling instants and y_k is the latest sensor data available at the kth sample of the controller whereas the model in [1] uses both x_k and y_k at the kth sensor sampling instant.

3 Simulation of ICCS With Nonidentical Sampling Periods

For a general, nonperiodic time-varying system, there is no standard technique for stability test. The Lyapunov's method, which is known to have no systematic way of finding suitable V-function(s) can only provide sufficiency conditions. However, in the special case when delays are constants and the ratio $\epsilon = L/M$, (where L, M are positivie integers and $L < M$), the ICCS model becomes periodically varying. Therefore the system stability can be analyzed by the results derived in Proposition 4.1 of Part I [1]. If the delays are nonperiodic or stochastic, sufficiency conditions for stability test could possibly be achieved by the Lyapunov method which is usually very conservative. Until an appropriate analytical technique is available, a viable alternative is simulation of the ICCS using the state-space model in (2.18) which does not require an elaborate structure of combined discrete-event and continuous-time operations. This simulation is very useful for gaining insight into the problem and is often considered as essential for stability analysis.

The effects of nonidentical sensor and controller sampling on the stability of the delayed system were examined by simulation of the finite-dimensional model, developed in Section 2. In order to compare the results of this simulation with those obtained for identical sampling, we selected the first order, proportional control system model which was used in Part II [2].

The plant model is given as

$$dx/dt = -x(t) + u(t), y(t) = x(t) \tag{3.1}$$

where x, u, and y are scalar functions of time. Since the proportional control law (with no dynamics) is given as:

$$u_k = -K(z_k - r_k) \tag{3.2}$$

where r_k is the reference signal, and z_k is the delayed sensor data at the controller whose proportional gain is the scalar K.

In the model developed in Section 2, the corresponding matrices reduce to scalars: $A = -1$, $B = 1$, $C = 1$, $F = G = H = 0$. For no vacant sampling slots (i.e., $\delta_{sc}{}^k + T_s < T_c$), the model in (2.18) becomes:

$$\begin{bmatrix} x_{k+1} \\ y_{k+1} \\ u_k \end{bmatrix} = \begin{bmatrix} A_c & -B_{c0}K & B_{c1} \\ A_t & -p_k B_0 K & (B_1 - p_k B_0) \\ 0 & -K & 0 \end{bmatrix} \begin{bmatrix} x_k \\ y_k \\ u_{k-1} \end{bmatrix} + \begin{bmatrix} B_{c0}K \\ p_k B_0 K \\ K \end{bmatrix} \Upsilon_k \tag{3.3}$$

where

$$\begin{aligned} A_c &= \exp(-T_c) \\ B_{c0} &= 1 - \exp(c_k - T_c) \\ B_{c1} &= \exp(c_k - T_c) - \exp(-T_c) \\ A_t &= \exp(-t_k) \\ B_0 &= 1 - \exp(c_k - t_k) \\ p_k &= \begin{cases} 1 & \text{if } t_k \geq c_k \\ 0 & \text{if } t_k < c_k \end{cases} \\ B_1 &= 1 - \exp(-t_k) \end{aligned}$$

T_c is the sampling period of the controller.

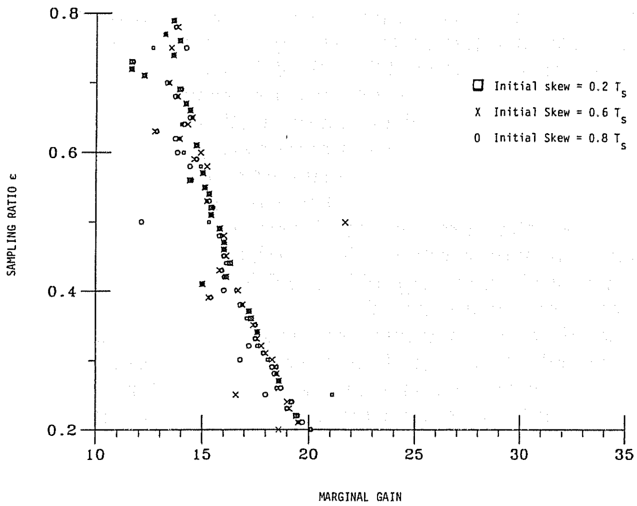

Fig. 3 Stability margin for deterministically varying data latencies

The time-varying nature of this system arises from the dissimilar sampling periods T_c and T_s and the varying delays to which the sensor and control command data are subjected. In terms of the parameters in the finite-dimensional model (3.3), the instants $\{t_k\}$ of sensor data arrival within a controller sampling period are varying even if the network-induced delays are negligible provided that $\epsilon \neq 1/M$, M being a positive integer. If the delays are significant, then t_k depends on both ϵ and network-induced delays, and c_k depends only on these delays.

Numerical data and assumptions used in the simulation of the delayed control system with nonidentical sampling periods are presented below:

The sampling period of the controller was set to $T_c = 0.1$ s. The processing time of the controller was assumed to be $\delta_p = 0.015$ s. The sensor-to-controller and controller-to-actuator data latencies were chosen to follow sinusoidal functions with frequency $\omega = 13$ rad/s [2]. The initial value of the phase difference between the two delay profile was arbitrarily chosen to be 0 rad since it varies with sample instant k.

The discrete sequencies $\{\delta_{sc}\}$ and $\{\delta_{ca}\}$ were generated by sampling the sensor and controller at the instants:

$$\delta_{sc}{}^j = 0.015 + \sin(\omega j T_s)$$

$$j = 1,2,3, \ldots \qquad (3.4)$$

$$\delta_{ca}{}^j = 0.015 + \sin(\omega j T_c)$$

Note that $\delta_{min} = 0.01s = 0.1T_c$ and $\delta_{max} = 0.02s = 0.2T_c$. To satisfy the condition that no vacant slot occurs, the constraint $\delta_{max} + T_s < T_c$ was imposed (see assumption 1 in Section 2.2). Therefore, the upper bound of $\epsilon = T_s/T_c$ was set to 0.8. While there is no restriction on the lower bound of ϵ, excessively high sensor sampling rate results in wastage of the network medium bandwidth. Furthermore if T_s is less than the average cycle time, i.e., the time interval between two consecutive opportunities to transmit a message, some of the sensor data will be rejected before they are transmitted. On this basis the lower bound of ϵ was chosen to be 0.2 in simulation experiments. This implies that the sensor is sampled at most five times faster than the controller.

The data latencies, generated by the functions in (3.4), were incorporated into the delayed control system model in (3.3) following the concept of event scheduling in discrete-event simulation languages [13]. This simulation model can be also be used to represent the operating conditions of random traffic in the ICCS network. In that case the sequence of data latencies can be generated first by a single discrete-event simulation using a standard package like SIMAN [13]. The next step of injecting this delay sequence will follow.

Table 1 Relationshiop between initial skew and marginal gain for $\epsilon = 1/2$

Initial skew	Sensor-controller delay (θ_{sc})	Marginal gain
$0.2T_s$	$0.4T_c$	15.3
$0.6T_s$	$0.2T_c$	21.7
$0.8T_s$	$0.6T_c$	12.1

Simulation experiments were conducted to determine the system stability margin for different sampling ratio $\epsilon = T_s/T_c$ when tracking a sinusoidal reference input $r_k = \sin(k\pi/10)$. In the absence of an exact relationship the system stability criterion was approximately taken to be the limiting condition for the simulation output remaining bounded for a sufficiently long period of time (which was chosen to be 2000 T_c in these simulation runs). Figure 3 shows the stability region of the delayed control systems as a function of the feedback gain K and the sampling ratio ϵ for three values of the initial skew $\Delta_s{}^k$. It follows from Fig. 3 that $\Delta_s{}^k$ has no significant bearing on the stability margin except for certain points with $\epsilon = L/M$, where L and M are small positive integers, e.g., for $\epsilon = 1/2$, 1/4, 2/5, 3/4, etc.

For example, consider the case of $\epsilon = 1/2$ and $\delta_{sc} \epsilon (.01T_c, 0.2T_c)$. Then, if the initial skew is $0.2T_s = 0.1T_c$, the resulting sensor-controller delay (see Definition 1 and Remark 3 in Section 2) is $\theta_{sc} = (0.5T_c - 0.1T_c) = 0.4T_c$, i.e., the sensor data that is used by the controller at time t was generated at $t - 0.4T_c$ since $\delta_{max} < 0.4T_c$. Similarly, if the skew is $0.6T_s = 0.3T_c$, the resulting $\theta_{sc} = 0.2T_c$. However, when the initial skew is $0.8T_s = 0.4T_c$, $\theta_{sc} = (0.5T_c - 0.4T_c) + 0.5T_c = 0.6T_c)$ because the sensor data sampled closest to the controller sampling instant always reaches the controller late as $\delta_{min} > 0.1T_c$ and therefore the previous sensor data is used. This causes a reduction in the marginal gain (which is inversely related to θ_{sc}) as shown in Table 1. Bounds for the marginal gain for other values of initial skew at $\epsilon = 1/2$ can be determined from the information provided in Table 1. For example, if the initial skew is $0.95T_s = 0.475T_c$, then the sensor-controller delay $(0.5T_c - 0.475T_c) + 0.5T_c = 0.525T_c$. Therefore, the marginal gain should lie between 12.1 and 15.3. On the other hand, for those ϵ's at which sampling patterns have long periods, the time time-varying skew $\Delta_s{}^k$ would be scattered in the interval $(0, T_s)$. Under these circumstances the effects of the initial value of $\Delta_s{}^k$ is "averaged out" as expected and the marginal gain becomes independent of the initial skew as seen in Fig. 3.

The general trend is that the expected value of θ_{sc} is an increasing function of ϵ except possibly for the critical points where the effects of the initial skew on θ_{sc} are significant. However, as pointed out in Part II [2], system stability is not solely dependent on the expected value of θ_{sc}. Other factors such as variance of the difference (which is an analog of time-derivative in the mean square sense) of the sensor-controller delays at consecutive samples may be significant for system stability. Analytical and simulation work is being conducted to this effect.

For evaluating the ICCS model performance under randomly varying delays, a large number of simulation runs using different (random number generator) seeds needs to be conducted to obtain useful statistics. Figure 4 shows dependence of stability margin on the sampling ratio ϵ for the model depicted in (3.3) and the conditions similar to those for Fig. 3 except that the data latencies are randomly varying with uniform distribution in $(0.1T_c, 0.2T_c)$. The data latencies δ_{sc}

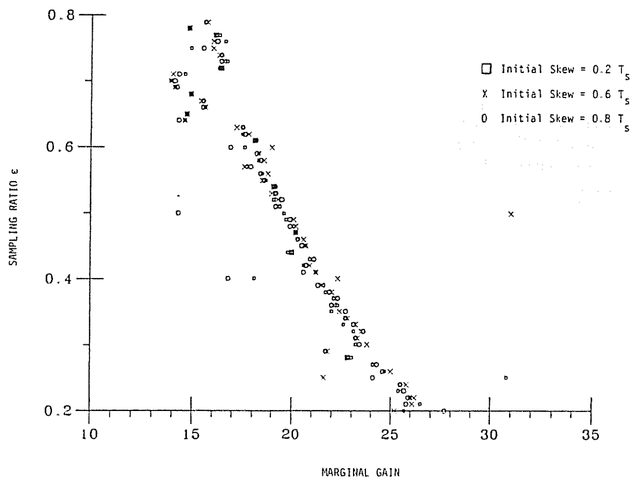

Fig. 4 Stability margin for randomly varying data latencies

and δ_{ca} are assumed to be mutually independent and memoryless, i.e., δ^k is independent of $\delta^{k+i} \forall i \neq 0$. This assumption is an approximate representation of random traffic in a network consisting of a large number of subscribers. The intention is to present a case of completely random data latencies in contrast to that for deterministically varying data latencies in Fig. 3.

A comparison of the results in Figs. 3 and 4 apparently reveals that the system stability is only moderately influenced by the probability distribution of network-induced delays. Further simulation and analytical studies are needed for inferring any such conclusions about systems that are more complex than that considered in this example. While the stability margin under random delays is modestly superior to that under completely deterministic delays with similar upper and lower bounds, critical points, particularly at $\epsilon = 1/M$, $M = 2$, 3, 4, and 5, exist in both cases where the stability margin is strongly dependent on the initial value of the skew $\Delta_s{}^k$. In view of this observation for deterministically and randomly varying delays, the ratio ϵ should not be selected in the vicinity of the critical points where the initial skew may have detrimental effects on the system stability.

The pertinent results derived from simulation experiments (but not yet established by rigorous analysis) are summarized below.

- The impact of the initial value of the skew $\Delta_s{}^k$ on system stability is not significant except at those values of ϵ that yield short periods, namely, $\epsilon = L/M$ where L and M are small positive integers. This indicates that the effects of the varying skew $\Delta_s{}^k$ are averaged out for aperiodic systems as well as for those with large periods.
- The general trend is that the stability margin is improved as ϵ is reduced. This can be explained from the fact that the sensor data generated at fast sampling induces smaller sensor-controller delays on the average.

4 Summary, Conclusions and Recommendations for Future Work

In this Part III of the sequence of papers on Integrated Communication and Control Systems (ICCS) we have presented modeling of a control loop where sensor and controller sampling rates are nonidentical. The control system model is capable of analyzing tracking problems. This is an extension of the work presented in Part I [1] for identical sensor and controller sampling rates. This finite-dimensional model takes into account the effects of network-induced distributed delays that are deterministically or randomly varying. Although the plant and controller models are taken to be linear time-invariant, the phenomenon of multirate sampling renders the system to be time-variant even in the absence of delays (except when the sensor sampling rate is an integer multiple of the controller sampling rate). A model for the nondelayed case has been first developed to establish the model structure which essentially relies on the time frame of reference relative to controller sampling instants. Later network-induced delays have been taken into account. In contrast to the model in Part I [1] under general traffic conditions, the model in this paper has been developed under moderate traffic when the network-induced data latency is small relatively to sensor and controller sampling periods. This assumption has been made to ensure clarity of presentations of the analytical derivations. The methodology applies to the more general case of increased traffic when the data latency could exceed the sampling periods or their multiples.

To better understand the physical phenomena we have presented simulation of a first order system similar to that in Part II [2]. The ratio ϵ of the sensor and controller sampling periods plays a dominant role on the system dynamic performance. The simulation results suggest the possible existence of critical values of ϵ around which the system dynamic performance may sharply deteriorate. Further analytical work is necessary to confirm the conclusions derived from the simulation results and this is a subject of current research.

The integrated control system may have cascaded control loops, each of which is served by the same network and thus subjected to delays induced by the random traffic. In that case the reference input r_k to an inner loop, which is the output of an outer loop, is a stochastic process. Since the input matrix Υ_k is also stochastic, the conventional concepts based on the Wiener integral would no longer be valid. More advanced concepts, such as those based on Itô integral [14, 15], have to be developed for analytically solving the stochastic difference equation.

Another very important problem, albeit much more difficult, is the stochastic stability analysis of the control system under random network traffic. A prerequisite for this research is a thorough understanding of the stochastic phenomena that take place within this multi-rate sampling environment. This can be achieved by simulation of the model derived in Section 2. The input to the model could be stochastic sequences of network-induced data latencies $\{\delta_{sc}{}^k\}$ and $\{\delta_{ca}{}^k\}$ which can be generated by a single discrete-event simulation run of the network under random traffic.

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