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Brief Paper

An Observer-based Compensator for Distributed Delays*

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Key Words—Delayed control systems; integrated control systems; observer design; robustness.

Abstract—This paper presents an algorithm for compensating delays that are distributed between the sensor(s), controller and actuator(s) within a control loop. This observer-based algorithm is specially suited to compensation of network-induced delays in integrated communication and control systems. The robustness of the algorithm relative to plant model uncertainties has been examined.

1. *Introduction*

AN EFFICIENT means for realizing Integrated Control Systems is to interconnect the spatially distributed components by a computer communication network (Ray, 1987). Such a network introduces randomly varying delays in the control loop, which degrade the system dynamic performance and are a source of potential instability.

This paper considers control systems with distributed delays that occur in both measurements and control inputs. The plant and controller dynamics at the sample time k are modeled as:

$$x_{k+1} = Ax_k + Bu_{k-\Delta 1}\text{—plant dynamics with delayed control} \tag{1}$$

$$y_k = Cx_k\text{—sensor measurement} \tag{2}$$

$$w_k = y_{k-\Delta 2}\text{—delayed output} \tag{3}$$

$$u_k = \xi(W_k)\text{—control function} \tag{4}$$

where $x \in R^n$, $u \in R^s$ and $y \in R^m$, and the matrices A, B and C, are of compatible dimensions; the finite non-negative integers $\Delta 1$ and $\Delta 2$ represent the number of delayed samples in control inputs and measurements, respectively. The control law u_k is a linear function of the history $W_k := \{w_k, w_{k-1}, \ldots\}$ of the delayed measurements. The objective is to construct the control function ξ such that the effects of the delays on the control system performance are mitigated.

A major motivation for considering the delayed control system described above is the recent interest in Integrated Communication and Control Systems (ICCS) (Ray, 1987, 1988; Ray and Phoha, 1989; Halevi and Ray, 1988; Ray and Halevi, 1988) for applications to diverse processes. Since the individual system components in ICCS are interconnected via a time-division-multiplexed network, the delays $\Delta 1$ and $\Delta 2$ in (1) and (3) arise because of network-induced delays between the controller and actuator, and the sensor and controller, respectively. Recently Ray and Halevi (1988; also Halevi and Ray, 1988) have reported analysis and design of ICCS (also described in Ray, 1987, 1988; Ray and Phoha, 1989) where the delays $\Delta 1$ and $\Delta 2$ are deterministically or randomly varying. Necessary and sufficient conditions were obtained for the case of periodically varying delays. The problem of delay compensation under the non-periodic and random traffic has been discussed in Halevi and Ray (1988) and Ray and Halevi (1988), but no specific solution has been given.

Several investigators have addressed the problems of delay compensation in closed loop control systems. An intuitive approach (Isermann, 1981) is to augment the system model to include delayed variables as additional states. Unfortunately, this renders some of the states uncontrollable even when the original system is completely controllable (Marianni and Nicoletti, 1973; Drouin *et al.*, 1985). For the case of delayed control inputs, Pyndick (1972) proposed a predictor for the optimal state trajectory based on past control inputs. Zahr and Slivinsky (1974) considered the problem of controlling a computer-controlled system with measurement and computational delays. It was pointed out that the delays in multi-variable systems may result in: (1) an increase in the magnitudes of the transients and poor response during the inter-sampling time, (2) loss of decoupling between individual SISO control loops although decoupling may be restored for a stable process at the steady-state, and (3) a possible decrease in the stability margin. Their algorithm was verified by simulation but the use of an observer to estimate the unavailable states was not discussed.

A significant amount of research work has been reported for observer and controller design (Drouin *et al.*, 1985; Bhat and Kiovo, 1976; Fairman and Kumar, 1986) for processes with inherent constant delays that occur within the process to be controlled. In contrast to the system under consideration in (1)–(4), such processes are described as follows:

$$\mathrm{d}x(t)/\mathrm{d}t = Ax(t) + Dx(t-h) + Gu(t) \tag{5}$$

$$y(t) = Cx(t) \tag{6}$$

where h is a constant. By setting $G = 0$ and $D = BK$, equation (5) reduces to a delayed state-variable-feedback system.

The reported literature in delay compensation does not apparently address the problem of distributed delays in both the input and output variables, which is the case with ICCS. A methodology for compensation of distributed delays, as an essential step to ICCS design, is presented in this paper.

The ICCS network can be designed on the assumption that the induced delays are bounded within a specified confidence interval. It has been established by several investigators that a stable controller designed on the basis of a constant delay which is equal to the supremum of the varying delay may not ensure the system stability (Halevi and Ray, 1988). The proposed delay compensator circumvents the detrimental effects of bounded network-induced delays by using a multi-step predictor. The key idea in the compensator design

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is to monitor the data when it is generated and to keep track of the delay associated with it.

This paper is organized in five sections including the introduction. The delay compensation algorithm in a closed loop representation is derived in Section 2 and its application is illustrated by simple experimentation in Section 3. The analysis of modeling errors for the case of a two-step delay compensator, and their effects on the stability of the closed loop control system, are presented in Section 4. Summary and conclusions are provided in Section 5.

2. *Algorithm of an observer-based delay compensator*

A possible approach (Zahr and Slivinsky, 1974) for compensation of constant delays that affect the input or output variables of a system is to predict the current output. However, if a state-space approach is used for predicting the output, the plant state variables must be obtained first. Moreover, the measurements might be corrupted with noise. In view of the above, we propose to use an observer for estimation of the delayed states and then to predict the current state using the state transition matrix. However, this multi-step observer imposes additional dynamics, and hence phase lag, to the control system, which may not always be desirable; nevertheless, such a sensitive system is likely to be unstable when subject to delays.

The filtering characteristics of the observer would help attenuate the high-frequency noise in the measurements. If the measurement noise is considerable, then the proposed algorithm could be extended to include a stochastic filter (Anderson and Moore, 1979) instead of a deterministic observer. Furthermore, a reduced-order observer or a functional observer (Kailath, 1980) can be used to minimize the observation lag.

Now we proceed to formulate a delay-compensation algorithm using the concept of the multi-step observer. First we present four lemmas that are necessary to derive the algorithm.

Lemma 1

$$z_{k|r} = z_{k|1} - \sum_{i=0}^{r-2} A^i L_{k-i-1} C e_{k-i-1} \quad \text{for } r \geq 2. \qquad \text{(L1.1)}$$

Proof. The proof of Lemma 1 requires Lemmas 2, 3 and 4. We also introduce a definition that will be required to establish recursive relations in the lemmas.

Definition 1

$$f_k := z_{k|1}, \quad g_k := z_{k|2}$$

$$G(k, j) := \begin{cases} \sum_{i=0}^{j-1} A^i g_{k-i+1} & \text{for } k \geq j \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

$$F(k, j) := \begin{cases} \sum_{i=1}^{j} A^i f_{k-i+1} & \text{for } k \geq j \geq 1 \\ 0 & \text{otherwise.} \end{cases} \qquad \blacksquare$$

By use of Definition 1 and Lemma 2, it follows that

$$z_{k|r} = G(k-1, r-1) - F(k-1, r-2). \qquad \text{(L1.2)}$$

Using the expression for $G(\cdot, \cdot)$ in Definition 1 in conjunction with Lemma 3, i.e. the equation (L3.1), we have

$$\begin{aligned} G(k-1, r-1) &= \sum_{i=0}^{r-2} A^i g_{k-i} = \sum_{i=0}^{r-2} A^i (f_{k-i} - L_{k-i-1} C e_{k-i-1}) \\ &= \sum_{i=0}^{r-2} A^i f_{k-i} - \sum_{i=0}^{r-2} A^i L_{k-i-1} C e_{k-i-1}. \qquad \text{(L1.3)} \end{aligned}$$

Using the expression for $F(\cdot, \cdot)$ in Definition 1 in conjunction with (L1.3) yields

$$G(k-1, r-1) = f_k + F(k-1, r-2) - \sum_{i=0}^{r-2} A^i L_{k-i-1} C e_{k-i-1}. \qquad \text{(L1.4)}$$

The proof follows by substituting (L1.4) and the expression for f_k (from Definition 1) in (L1.2). $\blacksquare$

Lemma 2. Following Definition 1,

$$z_{k+1|r} = G(k, r-1) - F(k, r-2). \qquad \text{(L2.1)}$$

Proof. The identities $G(k, 1) = g_{k-1}$ and $F(k, 0) = 0$ are obtained from Definition 1. Using (L1.2) in conjunction with these identities yields

$$z_{k+1|2} = G(k, 1) - F(k, 0). \qquad \text{(L2.2)}$$

Using (P3) with $r = 3$ and $r = 2$ and subtracting,

$$z_{k+1|3} - z_{k+1|2} = A(z_{k|2} - z_{k|1}). \qquad \text{(L2.3)}$$

Using Lemma 3 and (L2.2) in (L2.3)

$$\begin{aligned} z_{k+1|3} &= G(k, 1) - F(k, 0) + A(g_k - f_k) \\ &= (G(k, 1) + A g_k) - (F(k, 0) + A f_k) \\ &= G(k, 2) - F(k, 1). \qquad \text{(L2.4)} \end{aligned}$$

The method of induction is now used to complete the proof of Proposition 1. Using (P3) as in (L2.3) results in

$$z_{k+1|r+1} - z_{k+1|r} = A(z_{k|r} - z_{k|r-1}). \qquad \text{(L2.5)}$$

Substituting (L2.1) in (L2.5)

$$\begin{aligned} z_{k+1|r+1} - z_{k+1|r} &= A[G(k-1, r-1) - F(k-1, r-2) \\ &\quad - G(k-1, r-2) + F(k-1, r-3)] \\ &= A[G(k-1, r-1) - G(k-1, r-2)] \\ &\quad - A[F(k-1, r-2) \\ &\quad - F(k-1, r-3)]. \end{aligned}$$

Setting $j = r - 1$ in (L4.1) of Lemma 4 and similarly $j = r - 2$ in (L4.2), and then substituting these results in the above equation, we obtain

$$\begin{aligned} z_{k+1|r+1} - z_{k+1|r} &= [G(k, r) - G(k, r-1)] \\ &\quad - [F(k, r-1) - F(k, r-2)] \\ &= [G(k, r) - F(k, r-1)] - [G(k, r-1) \\ &\quad - F(k, r-2)]. \qquad \text{(L2.6)} \end{aligned}$$

Using (L2.1) in the right hand side of (L2.6) yields

$$z_{k+1|r+1} = G(k, r) - F(k, r-1). \qquad \blacksquare$$

Lemma 3

$$z_{k|2} = z_{k|1} - L_{k-1} C e_{k-1} \qquad \text{(L3.1)}$$

or equivalently, $g_k = f_k - L_{k-1} C e_{k-1}$.

Proof. From (P1), (P2) and (P6), it follows that

$$z_{k|1} = A z_{k-1|1} + B u_{k-1} + L_{k-1} C e_{k-1}. \qquad \text{(L3.2)}$$

Also, using (P3), we have

$$z_{k|2} = A z_{k-1|1} + B u_{k-1}. \qquad \text{(L3.3)}$$

The proof follows by substituting (L3.2) in (L3.3). $\blacksquare$

Lemma 4

$$G(k, j+1) - G(k, j) = A[G(k-1, j) - G(k-1, j-1)] \qquad \text{(L4.1)}$$

$$F(k, j+1) - F(k, j) = A[F(k-1, j) - F(k-1, j-1)]. \qquad \text{(L4.2)}$$

Proof.

$$\begin{aligned} G(k, j+1) - G(k, j) &= \sum_{i=0}^{j} A^i g_{k-i+1} - \sum_{i=0}^{j-1} A^i g_{k-i+1} \\ &= A^j g_{k-j+1} \qquad \text{(L4.3)} \end{aligned}$$

$$\begin{aligned} G(k-1, j) - G(k-1, j-1) &= \sum_{i=0}^{j-1} A^i g_{k-1} - \sum_{i=0}^{j-2} A^i g_{k-i} \\ &= A^{j-1} g_{k-j+1}. \qquad \text{(L4.4)} \end{aligned}$$

(L4.1) is obtained by substituting (L4.4) into (L4.3). The proof of (L4.2) follows a similar procedure. $\blacksquare$

Now we present a proposition which establishes the closed loop response of a delayed system, consisting of a linear state-variable-feedback control law and a multi-step predictor.

Proposition 1. Given the following predictor controller scheme:

Plant model:

$$x_{k+1} = Ax_k + Bu_k; \quad y_k = Cx_k \tag{P1}$$

Observer model:

$$z_{k+1|1} = Az_{k|1} + Bu_k + L_k(y_k - Cz_{k|1}) \tag{P2}$$

r-step predictor:

$$z_{k+1|r} = Az_{k|r-1} + Bu_k \quad \text{for } r \geq 2 \tag{P3}$$

Predictive control:

$$u_k = \Gamma_k z_{k|p} \text{ for a fixed } p \geq 2 \tag{P4}$$

where

$$z_{k|r} := \hat{x}_{k|k-r} \text{ is the estimation of } x_k \text{ given the measurement history } W_{k-r} \tag{P5}$$

and the estimation error

$$e_k := x_k - z_{k|1}. \tag{P6}$$

Then, the closed loop system equation can be expressed as

$$\begin{bmatrix} x_{k+1} \\ e_{k-p+1} \end{bmatrix} = \begin{bmatrix} (A + B\Gamma_k) & -B\Gamma_k\Lambda_k \\ 0 & (A - L_{k-p}C) \end{bmatrix} \begin{bmatrix} x_k \\ e_{k-p} \end{bmatrix}$$

where

$$\Lambda_k := \begin{cases} \prod_{j=1}^{p} (A - L_{k-p+j-1}C) + \sum_{i=1}^{p-1} \left[A^{i-1}L_{k-i}C \times \prod_{j=1}^{p-i-1} (A - L_{k-p+j-1}C) \right] & \text{if } p \geq 2 \\ I & \text{if } 0 \leq p < 2 \end{cases}$$

and the plant model is assumed to be exact.

Proof. The proof of this proposition is supported by Lemma 1. Subtracting (P2) from (P1) yields

$$x_{k+1} - z_{k+1|1} = A(x_k - z_{k|1}) - L_kC(x_k - z_{k|1})$$
$$\text{or} \quad e_{k+1} = (A - L_kC)e_k. \tag{P7}$$

Substituting (P4) in (P1) and using (L1.1) from Lemma 1 yields

$$x_{k+1} = Ax_k + B\Gamma_k\left[z_{k|1} - \sum_{i=0}^{p-2} A^iL_{k-i-1}Ce_{k-i-1}\right]. \tag{P8}$$

Adding and subtracting $B\Gamma_kx_k$ and using (P6) in (P8)

$$x_{k+1} = (A + B\Gamma_k)x_k + B\Gamma_k\left[e_k - \sum_{i=0}^{p-2} A^iL_{k-i-1}Ce_{k-i-1}\right]. \tag{P9}$$

Using (P7) e_{k+q} for some integer $q \geq 1$ can be expressed as

$$e_{k+q} = \left[\prod_{j=1}^{q} (A - L_{k+j-1}C)\right]e_k. \tag{P10}$$

Replacing e_k and e_{k-i-1} in terms of e_{k-p}, i.e. using (P10) in (P9), yields

$$x_{k+1} = (A + B\Gamma_k)x_k - B\Gamma_k\Lambda_ke_{k-p} \tag{P11}$$

where

$$\Lambda_k = \prod_{j=1}^{p} (A - L_{k-p+j-1}C) + \sum_{i=0}^{p-2}\left[A^iL_{k-i-1}C \prod_{j=1}^{p-i-1} (A - L_{k-p+j-1}C)\right].$$

The proof for $p \geq 2$ follows by combining (P7) and (P11). If $p = 0$ then $B_k = I$ since the delay is zero and the ordinary separation principle is applicable. For $p = 1$ the ordinary separation principle still applies since only a first-order prediction is used, i.e. $u_k = \Gamma_k z_{k|1}$. In other words, an ordinary linear state feedback controller with observer drives the plant using a first-order prediction of the states; thus, a standard observer is naturally suited for compensation of a constant delay of one time step. Thus, if $p = 1$ then $\Lambda_k = I$. ∎

Remark 1. If $L_k = L$ and $\Gamma_k = \Gamma$, i.e. constant observer and controller gains, then Proposition 1 implies separation of the controller and the observer. That is, the eigenvalues of the closed loop delayed system are the same as the combined eigenvalues of the two matrices $(A - LC)$ and $(A + B\Gamma)$. ∎

Remark 2. Let the sensor-to-controller delay be m_kT and the controller-to-actuator delay be n_kT at time k where T is the sampling period. Then Proposition 1 can be applied to the above problem if $m_k + n_k =$ constant although m_k and n_k may individually vary with k. ∎

3. *Testing of the delay-compensation algorithm*

A schematic diagram for implementation of the delay-compensation algorithm is shown in Fig. 1. Following this scheme, real-time velocity control of a DC servomotor is presented below to demonstrate how the delay-compensator can improve the closed loop system performance.

The test facility consists of a DC-motor-tachogenerator assembly which interfaces with a PC-AT compatible microcomputer via a DASH-16 A/D and D/A conversion card. The steady-state characteristics of the plant were experimentally determined as:

$$y_{ss} = \begin{cases} K(u_{ss} - \beta) & \text{for } u_{ss} > \beta \\ 0 & \text{for } -\beta \leq u_{ss} \leq \beta \\ K(u_{ss} + \beta) & \text{for } u_{ss} < -\beta \end{cases}$$

where

y_{ss} = measured steady-state angular velocity of the motor,
u_{ss} = constant input voltage,
K = steady-state plant gain,
β = limit of dead band.

In the linear region, the motor dynamics were represented by a first-order model after an appropriate bias compensation in the control input:

$$x_{k+1} = ax_k + bu_k$$
$$y_k = x_k$$

where $a = \exp(-T/\tau)$, T and τ being the sampling period and the motor time constant, respectively, and

$$b = \int_{jT}^{(j+1)T} \exp(-((j+1)T - t)/\tau)K\,dt/\tau = [1 - \exp(-T/\tau)]K.$$

The plant parameters were identified from experimental data to be: $\tau = 1.7$ s; $K = 3.33$ rad s^{-1} V; and $\beta = 3.4$ V which is equivalent to 11.3 rad s^{-1}.

The state-variable-feedback, in this first-order plant, yields a purely proportional controller where the gain, γ, is obtained by pole placement such that the closed loop pole $\varphi = a - b\gamma$. Similarly the oberver gain l is chosen such that the observer pole is at $f = a - l$. Figure 2 exhibits the test results for comparison of the response of the delay-compensated control system with that of the uncompensated system when a step of 17 rad s^{-1} in the reference input was applied from the initial state of 0 rad s^{-1} at time $t = 0$. The loop delay was chosen to be $p = 2$ sampling periods, and the poles were placed at $\varphi = 0.3$ s^{-1} and $f = 0.1$ s^{-1}. The sampling time T was set to 0.3 s.

The transients in Fig. 2 lie in the linear region of the plant, and the dynamic performance of the compensated system is superior to that of the uncompensated system as expected. However, we have not attempted to experimentally evaluate

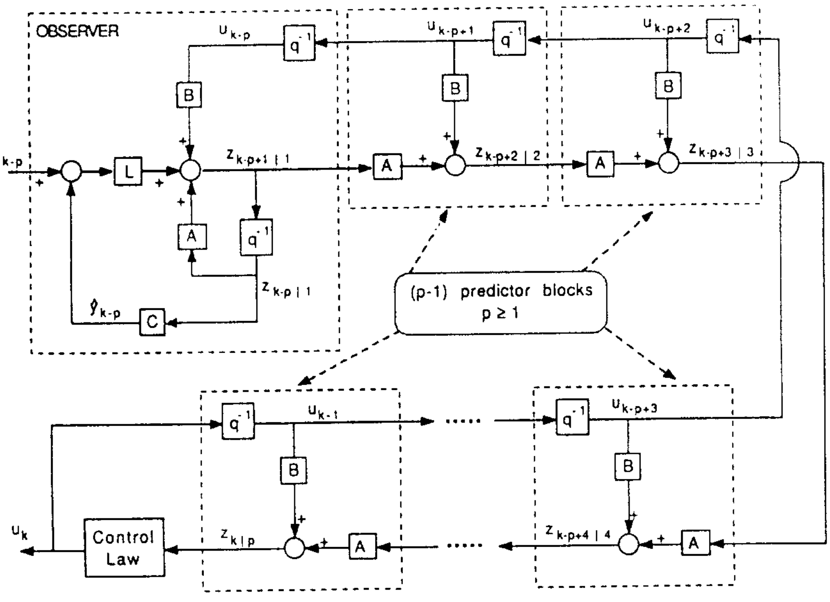

FIG. 1. Schematic diagram of the p-step delay compensation algorithm.

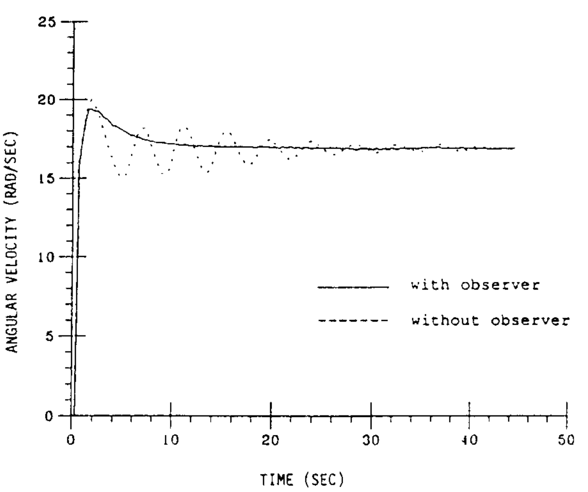

FIG. 2. Dynamic responses of the compensated and uncompensated control system.

robustness properties of the delay-compensator by introducing additional uncertainties in the plant model. This robustness issue needs to be addressed first by rigorous analysis. The next section presents some of the analytical results on robustness of the delay-compensator.

4. *Modeling uncertainty of a two-step predictor/controller*

In this section we investigate the effects of modeling uncertainty upon performance of the predictor/controller algorithm for the case when the delay to be compensated is equal to two sampling periods. Let the plant model described in (P1) have the true dynamical characteristics given as:

$$x_{k+1} = A'_k x_k + B'_k u_k; \quad y_k = C'_k x_k. \tag{E1}$$

In view of the above the modeling uncertainties are defined as:

$$\delta A_k := A'_k - A; \quad \delta B_k := B'_k - B; \quad \delta C_k := C'_k - C. \tag{E2}$$

Remark 3. The postulated model of the plant is time-invariant whereas the uncertainties may be time-varying. ■

The controller structures, given by (P1)–(P6) with $p = 2$ in (P4) and steady-state controller gain Γ and observer gain L, are:

$$u_k = \Gamma z_{k|2} \tag{E3}$$

$$z_{k+1|1} = A z_{k|1} + B u_k + L(y_k - C z_{k|1}) \tag{E4}$$

$$z_{k+1|2} = A z_{k|1} + B u_k \tag{E5}$$

$$e_k = x_k - z_{k|1}. \tag{E6}$$

We intend to obtain a closed loop, augmented state-variable representation in such a way that the modeled system dynamics and the modeling errors are decoupled to the maximum possible extent. From now onwards, for the sake of brevity, the subscript k from the time-varying matrices in (E2) will be omitted.

Substituting (E1) and (E4) into (E6) yields

$$\begin{aligned} e_k &= A' x_{k-1} + B' u_{k-1} - [A z_{k-1|1} + B u_{k-1} \\ &\quad + L(C' x_{k-1} - C z_{k-1|1})] \\ &= \Psi' x_{k-1} - \Psi z_{k-1|1} + \delta B u_{k-1} \end{aligned} \tag{E7}$$

where $\Psi' := A' - LC'$; $\Psi := A - LC$.

Adding and subtracting Ψx_{k-1} in the right hand side of (E7) yields

$$e_k = \Psi e_{k-1} + \Theta x_{k-1} + \delta B u_{k-1} \tag{E8}$$

where $\Theta := \Psi' - \Psi = \delta A - L\,\delta C$.

Substituting (E3) in (E8) yields

$$e_k = \Psi e_{k-1} + \Theta x_{k-1} + \delta B \Gamma z_{k-1|2}. \tag{E9}$$

Substituting (E5) in (E4) yields

$$z_{k+1|1} = z_{k+1|2} + LC' x_k - LC z_{k|1}$$

or

$$\begin{aligned} z_{k|2} &= z_{k|1} - LC' x_{k-1} + LC z_{k-1|1} \\ &= x_k - (x_k - z_{k|1}) - LC(x_{k-1} - z_{k-1}) \\ &\quad - L(C' - C) x_{k-1}. \end{aligned}$$

Substituting (P6) and (E2) in the above equation yields

$$z_{k|2} = x_k - e_k - LC e_{k-1} - L\,\delta C x_{k-1}. \tag{E10}$$

Substituting (E9) in (E10) yields

$$\begin{aligned} z_{k|2} &= x_k - [\Psi e_{k-1} + \Theta x_{k-1} + \delta B \Gamma z_{k-1|2}] \\ &\quad - LC e_{k-1} - L\,\delta C x_{k-1} \\ &= x_k - (\Psi + LC) e_{k-1} - (\Theta + L\,\delta C) x_{k-1} - \delta B \Gamma z_{k-1|2} \\ &= x_k - A e_{k-1} - \delta A x_{k-1} - \delta B \Gamma z_{k-1|2}. \end{aligned}$$

Substituting for $z_{k-1|2}$ in the above equation by using (E10)

$$
\begin{aligned}
z_{k|2} &= x_k - Ae_{k-1} - \delta A x_{k-1} \\
&\quad - \delta B\Gamma[x_{k-1} - e_{k-1} - LCe_{k-2} - L\,\delta C x_{k-2}] \\
&= x_k - (A - \delta B\Gamma)e_{k-1} - \delta G x_{k-1} + \delta B\Gamma L \delta C x_{k-2} \\
&\quad + \delta B\Gamma LCe_{k-2}
\end{aligned} \tag{E11}
$$

where $\delta G := (\delta A + \delta B\Gamma)$.

Substituting for $z_{k-1|2}$ in (E9) by using (E11) yields

$$
\begin{aligned}
e_k &= \Psi e_{k-1} + \Theta x_{k-1} + \delta B\Gamma[x_{k-1} - (A - \delta B\Gamma)e_{k-2} - \delta G x_{k-2} \\
&\quad + \delta B\Gamma L\,\delta C x_{k-3} + \delta B\Gamma LCe_{k-3}] \\
&= \Psi e_{k-1} + \partial_{B2} e_{k-2} + \partial_{B3} e_{k-3} + F_{A\wedge B\wedge C} x_{k-1} \\
&\quad + \partial_{B1} x_{k-2} + \partial_{BC} x_{k-3}
\end{aligned} \tag{E12}
$$

where

$$
\begin{aligned}
F_{A\wedge B\wedge C} &:= (\Theta + \delta B\Gamma) = (\delta A - L\,\delta C + \delta B\Gamma) \\
\partial_{B1} &:= -\delta B\Gamma\delta G = -\delta B\Gamma(\delta A + \delta B\Gamma) \\
\partial_{BC} &:= (\delta B\Gamma)^2 L\,\delta C \\
\partial_{B2} &:= -\delta B\Gamma(A - \delta B\Gamma) \\
\partial_{B3} &:= (\delta B\Gamma)^2 LC.
\end{aligned}
$$

Substituting (E12) into (E11) yields

$$
\begin{aligned}
z_{k|2} &= x_k - (A - \delta B\Gamma)[\Psi e_{k-2} + \partial_{B2} e_{k-3} \\
&\quad + \partial_{B3} e_{k-4} + F_{A\wedge B\wedge C} x_{k-2} \\
&\quad + \partial_{B1} x_{k-3} + \partial_{BC} x_{k-4}] - \delta G x_{k-1} \\
&\quad + \delta B\Gamma L\,\delta C x_{k-2} + \delta B\Gamma LCe_{k-2}.
\end{aligned}
$$

Grouping similar terms

$$
\begin{aligned}
z_{k|2} &= x_k - \delta G x_{k-1} + \partial_{A\wedge B\wedge C} x_{k-2} + \partial_{B4} x_{k-3} + \partial_{B5} x_{k-4} \\
&\quad + (\partial_{B6} - A\Psi)e_{k-2} + \partial_{B7} e_{k-3} + \partial_{B8} e_{k-4}
\end{aligned} \tag{E13}
$$

where

$$
\begin{aligned}
\partial_{A\wedge B\wedge C} &:= -(A - \delta B\Gamma)F_{A\wedge B\wedge C} + \delta B\Gamma L\,\delta C \\
&= -(A - \delta B\Gamma)(\delta A - L\,\delta C + \delta B\Gamma) + (\delta B\Gamma)L\,\delta C \\
\partial_{B4} &:= -(A - \delta B\Gamma)\,\partial_{B1} \\
&= (A - \delta B\Gamma)\,\delta B\Gamma(\delta A + \delta B\Gamma) \\
\partial_{B5} &:= -(A - \delta B\Gamma)\,\partial_{BC} \\
&= -(A - \delta B\Gamma)(\delta B\Gamma)^2 L\,\delta C \\
\partial_{B6} &:= \delta B\Gamma\Psi + \delta B\Gamma LC = \delta B\Gamma(\Psi + LC) = \delta B\Gamma A \\
\partial_{B7} &:= -(A - \delta B\Gamma)\,\partial_{B2} \\
&= (A - \delta B\Gamma)\,\delta B\Gamma(A - \delta B\Gamma) \\
\partial_{B8} &:= -(A - \delta B\Gamma)\,\partial_{B3} \\
&= -(A - \delta B\Gamma)(\delta B\Gamma)^2 LC.
\end{aligned}
$$

Substituting (E13) and (E3) in (E1) yields

$$
\begin{aligned}
x_{k+1} &= A'x_k + B'\Gamma[x_k - \delta G x_{k-1} + \partial_{A\wedge B\wedge C} x_{k-2} + \partial_{B4} x_{k-3} \\
&\quad + \partial_{B5} x_{k-4} + (\partial_{B6} - A\Psi)e_{k-2} + \partial_{B7} e_{k-3} + \partial_{B8} e_{k-4}].
\end{aligned}
$$

Rearranging and grouping similar terms

$$
\begin{aligned}
x_{k+1} &= Yx_k + D_{A\wedge B} x_{k-1} + D_{A\wedge B\wedge C} x_{k-2} \\
&\quad + D_{B1} x_{k-3} + D_{B2} x_{k-4} \\
&\quad + Qe_{k-2} + D_{B3} e_{k-3} + D_{B4} e_{k-4}
\end{aligned} \tag{E14}
$$

where

$$
\begin{aligned}
Y &:= (A' + B'\Gamma) \\
&= (A + B\Gamma) + \delta G \\
Q &:= B'\Gamma(\partial_{B6} - A\Psi) \\
&= B'\Gamma(\delta B\Gamma A - A(A - LC)) \\
D_{A\wedge B} &:= -B'\Gamma\,\delta G \\
&= -B'\Gamma(\delta A + \delta B\Gamma) \\
D_{A\wedge B\wedge C} &:= B'\Gamma\,\partial_{A\wedge B\wedge C} \\
&= B'\Gamma[(\delta B\Gamma)L\,\delta C - (A - \delta B\Gamma)(\delta A - L\,\delta C + \delta B\Gamma)] \\
D_{B1} &:= B'\Gamma\,\partial_{B4} \\
&= B'\Gamma(A - \delta B\Gamma)\,\delta B\Gamma(\delta A + \delta B\Gamma) \\
D_{B2} &:= B'\Gamma\,\partial_{B5} \\
&= -B'\Gamma(A - \delta B\Gamma)(\delta B\Gamma)^2 L\,\delta C \\
D_{B3} &:= B'\Gamma\,\partial_{B7} \\
&= B'\Gamma(A - \delta B\Gamma)\,\delta B\Gamma(A - \delta B\Gamma) \\
D_{B4} &:= B'\Gamma\,\partial_{B8} \\
&= -B'\Gamma(A - \delta B\Gamma)(\delta B\Gamma)^2 LC.
\end{aligned}
$$

The closed loop equation of the two-step delayed system follows from (E12) and (E14)

$$
\begin{bmatrix} x_{k+1} \\ x_k \\ x_{k-1} \\ x_{k-2} \\ x_{k-3} \\ e_{k-1} \\ e_{k-2} \\ e_{k-3} \end{bmatrix}
=
\begin{bmatrix}
Y & D_{A\wedge B} & D_{A\wedge B\wedge C} & D_{B1} & D_{B2} & Q & D_{B3} & D_{B4} \\
I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & I & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & I & 0 & 0 & 0 & 0 \\
0 & 0 & F_{A\wedge B\wedge C} & \partial_{B1} & \partial_{BC} & \Psi & \partial_{B2} & \partial_{B3} \\
0 & 0 & 0 & 0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & I & 0
\end{bmatrix}
\times
\begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \\ x_{k-3} \\ x_{k-4} \\ e_{k-2} \\ e_{k-3} \\ e_{k-4} \end{bmatrix} \tag{E15}
$$

where the subscripts of elements are chosen to describe their relative dependence on the respective errors in formulating the plant model matrices. The above equation can be separated into the nominal and uncertain parts:

$$
X_{k+1} = (V + \delta V)X_k \tag{E16}
$$

where $X_k := [x_k^T x_{k-1}^T x_{k-2}^T x_{k-3}^T x_{k-4}^T e_{k-2}^T e_{k-3}^T e_{k-4}^T]^T$, V is the nominal part which is a constant matrix as it contains the constant matrices A, B and C of the plant model in (1), and δV is the uncertainty part which may be time-varying as it contains the possibly time-varying modeling error matrices δA, δB and δC.

Remark 4. The modeling errors δA, δB and δC constitute a four-fold increase over the order of the augmented system matrix in (E15). However, by setting $\delta B = 0$, the following simplifications result.

$$
\partial_{B1} = \partial_{B2} = \partial_{B3} = \partial_{BC} = D_{B1} = D_{B2} = D_{B3} = D_{B4} = 0
$$

and

$$
\begin{gathered}
Y = (A + \delta A + B\Gamma), \quad D_{A\wedge B} = -B\Gamma\,\delta A, \\
D_{A\wedge B\wedge C} = -B\Gamma A(\delta A - L\,\delta C), \quad Q = -B\Gamma A(A - LC), \\
F_{A\wedge B\wedge C} = (\delta A - L\,\delta C).
\end{gathered}
$$

Accordingly the system is reduced with $\delta B = 0$ as follows

$$\begin{bmatrix} x_{k+1} \\ x_k \\ x_{k-1} \\ e_{k-1} \end{bmatrix} = \begin{bmatrix} A+\delta A+B\Gamma & -B\Gamma\delta A & -B\Gamma A(\delta A - L\,\delta C) & -B\Gamma A(A-LC) \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \delta A - L\,\delta C & A-LC \end{bmatrix} \times \begin{bmatrix} x_k \\ x_{k-1} \\ x_{k-2} \\ e_{k-2} \end{bmatrix}. \quad \text{(E17)}$$

■

Remark 5. By separating the uncertainty part, the above equation can be expressed in the format of (E16) with reduced-order state representation (due to the assumption $\delta B = 0$) as follows:

$$^{r}X_{k+1} = (^{r}V + {^{r}\delta V})^{r}X_k \quad \text{(E18)}$$

where

$$^{r}X_k = [x_k^T x_{k-1}^T x_{k-2}^T e_{k-2}^T]^T,$$

$$^{r}V = \begin{bmatrix} A+B\Gamma & 0 & 0 & -B\Gamma A(A-LC) \\ I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & 0 & A-LC \end{bmatrix},$$

$$^{r}\delta V = \begin{bmatrix} \delta A & -B\Gamma\delta A & -B\Gamma A(\delta A - L\,\delta C) & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \delta A - L\,\delta C & 0 \end{bmatrix}.$$

Remark 6. With $\delta B = 0$ the effects of δA and δC are to increase the order of the closed loop system by two. ■

5. *Conclusions*

The paper addresses the issue of network-induced delays in distributed control systems. The proposed delay-compensation algorithm is based on a deterministic state estimator and a linear state-variable-feedback control law. The deterministic observer can be replaced by a stochastic observer without any structural modifications of the delay-compensation algorithm. However, if a feedforward-feedback control law is chosen instead of the state-variable-feedback control law, then the observer needs to be modified in the same way a conventional non-delayed control system should be. Under these circumstances the delay-compensation algorithm would be accordingly changed.

The separation principle of the classical Luenberger observer (Kailath, 1980) holds true for the proposed delay-compensator. Furthrmore, the structure of the compensator allows for variable delays under the constraints of: (1) the sum of the sensor-to-controller and controller-to-actuator delay is a known constant and (2) the sensor and controller sampling instants are synchronized.

The impact of modeling uncertainties, i.e. errors in the plant model matrices, on the performance of the closed loop compensated system has been investigated. Effects on these modeling errors have been investigated for stability of the compensated system. This approach has a potential for establishing bounds on these modeling errors under specified dynamic performance of the control system.

The proposed delay-compensation algorithm is suitable for Integrated Communication and Control Systems (ICCS) in advanced aircraft, spacecraft, manufacturing automation and chemical process applications. If the individual components of the ICCS communicate with each other via a common communication medium, then the network can be designed such that the induced delays are bounded. In this way the detrimental effects of distributed delays induced by the network can be circumvented by the compensator.

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