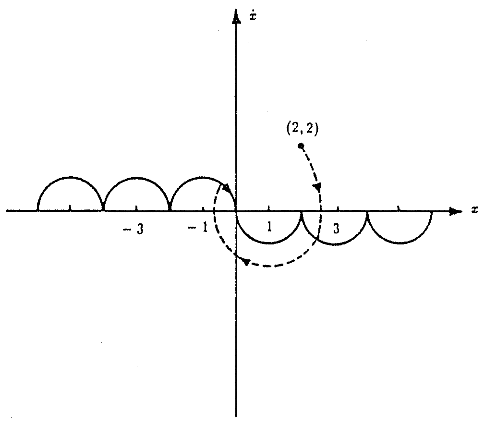

Fig. 5 State trajectories for the illustrative example

The graphical method confirms precisely the above computations for the illustrative example (see Fig. 5).

III Conclusions

A new, explicit solution to the minimum time control of a simple harmonic oscillator facilitates a priori assignment of the parameters needed to construct the bang-bang control. The Fourier series approach can be used for generating feedforward control strategies to achieve rapid maneuvers with minimal residual vibrations. This can be accomplished by eliminating the discontinuities in the bang-bang control by retaining only a finite number of terms in the infinite series. The trade-off is that the time of maneuver increases while minimizing residual vibrations.

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Multi-Level Hypotheses Testing for Fault Detection in Continuous Processes

Asok Ray[1]

This paper presents the analytical development of a multi-level hypotheses testing algorithm and an extension of the sequential bi-level test procedure reported earlier [1]. The multi-level test procedure characterizes M distinct categories of faults in continuous processes like aircraft, spacecraft, and nuclear power plants, and is suitable for generating signals for early warning and failure. The test procedure has been implemented on a commercially available microcomputer for on-line detection of sensor faults in an operating nuclear reactor.

1 Introduction

On-line plant monitoring, fault detection, and decision making are essential features of intelligent instrumentation in continuous processes like advanced aircraft, spacecraft, and hazardous chemical and nuclear power plants. Multi-level hypotheses testing provides a more precise characterization of faults than the bi-level fail/no-fail hypothesis, and is often essential for early warning and timely detection of gradually degrading equipments. In general, if M different types of failures are considered, then $(M+1)$ distinct modes (including the normal mode) could be designated by $(M+1)$ levels of hypotheses.

The major contribution of this paper is the analytical development of a recurrence relation for multi-level hypotheses testing for application to fault detection and condition monitoring of dynamical processes in real time. As an example of implementation, we have considered a fault detection technique [1] where a modified version of Chien's sequential probability ratio test [2,3] has been designed in the framework of the redundancy management procedure of Ray and Desai [4]. First we show how the scope of the above technique is extended from bi-level to multi-level decision-making. Then, the test results for detection of sensor faults in a nuclear reactor are summarized.

2 Algorithm Development for Multi-Level Hypotheses Testing

Let $y(1), y(2), \ldots, y(k), \ldots$ be (conditionally) independent measurements of a plant variable at consecutive sampling instants. M abnormal modes and the normal mode of operation are designated as $(M+1)$ mutually exclusive and exhaustive levels of hypotheses.

${}^0H(k)$: normal operation at the kth sample,
${}^jH(k)$: abnormal operation at the kth sample, $j = 1, 2, \ldots, M$

such that each hypothesis can be treated as a Markov state. The problem is to derive a recurrence relation for the a posteriori probability of faults in individual abnormal modes as the time k progresses.

We define the a posterioiri probability of fault in the j^{th} mode at the instant k as

$$\pi_j(k) := P[{}^jH(k) \mid y(1), y(2), \ldots, y(k)] \qquad (1)$$

based on the measurement history. Let the probability of transition from the state i to the state j at the next sample be denoted as

$$\alpha_{ij}(k) = P[{}^jH(k) \mid {}^iH(k-1)] \qquad (2)$$

such that

$$\sum_{j=0}^{M} \alpha_{ij}(k) = 1 \quad \text{for } \forall i, k.$$

Let the joint probability $\xi_j(k) := P[{}^jH(k), Y(k)]$ where $Y(k)$ denotes the ensemble $\{y(1), y(2), \ldots, y(k)\}$. Then,

$$\xi_j(k) = P[{}^jH(k), y(k), Y(k-1)]$$
$$= P[y(k) \mid {}^jH(k)]\, P[{}^jH(k), Y(k-1)]$$

Since $P[{}^jH(k), Y(k-1)] = \sum_{i=0}^{M} P[{}^jH(k), {}^iH(k-1), Y(k-1)]$

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$$= \sum_{i=0}^{M}(P[Y(k-1)\,|\,{}^iH(k-1)]P[{}^jH(k)\,|\,{}^iH(k-1)]P[{}^iH(k-1)])$$

$$= \sum_{i=0}^{M}(P[{}^jH(k)\,|\,{}^iH(k-1)]P[{}^iH(k-1),Y(k-1)]),$$

it follows that

$$\xi_j(k) = p_j(k)\sum_{i=0}^{M}\alpha_{ij}(k)\,\xi_i(k-1) \tag{3}$$

where the a priori conditional probability is denoted as

$$p_j(k) = P[y(k)\,|\,{}^jH(k)]\ \forall j,k \tag{4}$$

The a posteriori probability of fault in the j^{th} mode, defined in (1), is obtained as

$$\pi_j(k) = P[{}^jH(k)\,|\,Y(k)] = P[{}^jH(k),Y(k)]/\left(\sum_{i=0}^{M}P[{}^iH(k),Y(k)]\right)$$

$$= \xi_j(k)/(\xi_0(k) + \sum_{i=1}^{M}\xi_i(k))$$

$$= \psi_j(k)/\left(1 + \sum_{i=1}^{M}\psi_i(k)\right) \tag{5}$$

where $\psi_j(k) := \xi_j(k)/\xi_0(k)$

$$= \frac{p_j(k)}{p_0(k)}\,\frac{\alpha_{0j}(k) + \sum_{i=1}^{M}\alpha_{ij}(k)\psi_i(k-1)}{\alpha_{00}(k) + \sum_{i=1}^{M}\alpha_{i0}(k)\psi_i(k-1)} \tag{6}$$

The a posteriori probability of fault in one or more of the M abnormal modes can be obtained from (5) as

$$\pi(k) = P\left[\bigcup_{j=1}^{M}{}^jH(k)\,|\,Y(k)\right] = \sum_{j=1}^{M}\pi_j(k) = \frac{\Psi(k)}{1+\Psi(k)} \tag{7}$$

where
$$\Psi(k) := \sum_{i=1}^{M}\psi_i(k) \tag{8}$$

The equations (6) and (8) can be expressed by recurrence relations under the following assumptions.

Assumption 1: A transition from the normal mode to any abnormal mode is equally likely. If ρ is the a priori probability of fault during a sampling period, then $\alpha_{00} = 1-\rho$ and $\alpha_{0i} = \rho/M\ \forall i \neq 0$. ■

Assumption 2: No transition is allowed from an abnormal mode to any other mode, normal or abnormal, i.e., $\alpha_{ij}(k) = \delta_{ij}$ $\forall i \neq 0$ where the Kronecker delta $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ii} = 1$. This implies that once a measurement has failed, it stays in the same condition unless a corrective action is taken. ■

Remark 1: Assumption 2 is a valid representation if a device remains at a certain failed mode for a finite period of interest. The experimental work, reported in Section 4, has been conducted on this basis. An example of the faulty mode under Assumption 2 is saturation of instrument amplifiers. Another example is a slow drift causing gradual degradation of the instrument. The system could eventually fail if this warning is not attended to. ■

Using the above assumptions in (6) yields

$$\psi_j(k) = \frac{\rho/M + \psi_j(k-1)}{(1-\rho)}\,\frac{p_j(k)}{p_0(k)} \tag{9}$$

The recurrence relation for $\Psi(k)$ can be obtained by using (9) in (8).

$$\Psi(k) = \frac{1}{1-\rho}\sum_{j=1}^{M}\left[(\rho/M + \psi_j(k-1))\,\frac{p_j(k)}{p_0(k)}\right] \tag{10}$$

The above recurrence relation can be generated in a different form if Assumption 2 is modified as:

Assumption 2a: No transition is allowed from an abnormal mode to the normal mode but transitions to all abnormal modes (including the present mode) is equally likely, i.e., $\alpha_{i0}(k) = 0$ and $\alpha_{ij}(k) = 1/M\ \forall i,j \neq 0$. This implies that once a measurement has failed, it never comes back to the normal condition unless a corrective action is taken but it may continue to fail in other ways as time progresses. ■

Remark 2: Assumption 2a approximates the abnormal conditions resulting from a low signal-to-noise ratio as well as from erratic behavior of instrumentation components. ■

Using Assumption 2a instead of Assumption 2, equations (9) and (10) are modified as shown below.

$$\psi_j(k) = \frac{\rho + \sum_{j=1}^{M}\psi_j(k-1)}{(1-\rho)M}\,\frac{p_j(k)}{p_0(k)} = \frac{\rho + \Psi(k-1)}{(1-\rho)M}\,\frac{p_j(k)}{p_0(k)} \tag{9a}$$

$$\Psi(k) = \frac{\rho+\Psi(k-1)}{M(1-\rho)}\sum_{j=1}^{M}\left[\frac{p_j(k)}{p_0(k)}\right] \tag{10a}$$

3 Multi-Level Hypotheses for Sequential Testing

We show an application of the above multiple hypotheses algorithm to the sequential fault detection procedure [1] which is based on redundancy management [4]. We consider the abnormal modes to be represented by Assumption 2 and therefore the recurrence relation for $\psi_j(k)$ follows (9).

Given l (calibrated) redundant measurements for an n-dimensional plant variable [5], it follows from [1,4] that the one-dimensional parity vector $p_i(t)$ for the i^{th} $(n+1)$-tuplet S_i at the sampling instant t is:

$$p_i(t) = V_i\mu_i(t) \text{ for } i = 1,2,\ldots,\binom{l}{n+1} \tag{11}$$

where V_i is the $1\text{x}(n+1)$ projection matrix associated with μ_i which is the $(n+1)\text{x}1$ vector representing measurements in S_i. The probability distribution of p_i is assumed to be Gaussian on the justification that p_i is a linear combination of the measurements in S_i, which are usually uncorrelated or weakly correlated. Using the a priori information on the measurement noise covarinace matrix in S_i, $p_i(t)$ is scaled to $q_i(t)$ such that the variance of $q_i(t)$ is unity. For nominally unfailed conditions $E[q_i(t)] = 0\ \forall i,t$.

In the sequential tests, a decision is made between the normal hypothesis, and one or more abnormal hypotheses, on the basis of the information processed at consecutive samples. Instead of postulating only one abnormal hypothesis to represent all abnormal modes [1], M different abnormal modes are considered. Consequently $(M+1)$ distinct modes of operations are designated by $(M+1)$ mutually exclusive and exhaustive hypotheses as the normal hypothesis ${}^0H$ and abnormal hypotheses ${}^jH$, $j = 1,2,\ldots,M$.

${}^0H$: $q_i(t)$ is Gaussian with zero mean and unit variance $\forall t$, $\forall i$.

${}^jH$: $q_i(t)$ is Gaussian with mean ${}^j\theta_i$ and unit variance $\forall t$, $\forall i$.

Remark 3: The mean ${}^j\theta_i$ can be chosen on the basis of the error bounds ${}^j\beta$'s of the measurements in the i^{th} $(n+1)$-tuplet for the j^{th} abnormal mode. No specific functional relationship for obtaining ${}^j\theta_i$ is recommended as they are considered to be flexible parameters for the fault detection system design. The error bounds are usually available from the manufacturer's specifications or from experimental evaluation of the measurement noise statistics. ■

Computation of the a posteriori probability π_j in (5) can be significantly simplified by setting $\rho=0$ in (9) and then compensating for the effects of ρ. This is accomplished by extending the procedure described in [1]. The proposed algorithm is derived below.

The log likelihood ratio at the t-th sample is defined as:

$$ {}^j\phi_i(t) = \ln \frac{P[q_i(t)\,|\,H_j]}{P[q_i(t)\,|\,H_0]}, \quad i=1,2,\ldots,r, \text{ and } j=1,2,\ldots,M \qquad (12) $$

If the measurement noise is stationary, the log likelihood ratio ${}^j\Phi_i(k)$ for k consecutive conditionally independent samples is given by

$$ {}^j\Phi_i(k) = \ln \frac{P[q_i(1),q_i(2),\ldots.,q_i(k)\,|\,H_j]}{P[q_i(1),q_i(2),\ldots.,q_i(k)\,|\,H_0]} = \sum_{t=1}^{k} {}^j\phi_i(t) \qquad (13) $$

A recurrence relation is obtained as

$$ {}^j\Phi_i(k) = {}^j\Phi_i(k-1) - {}^j\theta_i({}^j\theta_i/2 - q_i(k)) \qquad (14) $$

Remark 4: From (9) in Section 2, it follows that ${}^j\Phi_i(k) \approx \ln[\psi_j(k)]$ for the i^{th} $(n+1)$-tuplet S_i if $\rho \approx 0$. ■

The detection algorithm at the k^{th} sampling instant is formulated for every ith $(n+1)$-tuplet, $i=1,2,\ldots.,r$ under the faulty modes $j=1,2,\ldots.,M$ as follows.

Initialization:

$$ {}^j\Phi_i(0) = 0. \qquad \forall i,j \qquad (15) $$

Lower limit setting:

$$ {}^j\Phi_i(k) = \text{Max}[{}^j\Phi_i(k), {}^j\delta_i] \qquad \forall i,j \qquad (16) $$

Consistency of the i^{th} $(n+1)$-tuplet:

$$ {}^j\Phi_i(k) \le {}^j\sigma_i \qquad \forall j \qquad (17) $$

Inconsistency of the i^{th} $(n+1)$-tuplet:

$$ {}^j\Phi_i(k) > {}^j\sigma_i \quad \text{for at least one } j \qquad (18) $$

Upper limit setting:

$$ {}^j\Phi_i(k) = \text{Min}[{}^j\Phi_i(k), {}^j\sigma_i] \qquad \forall i,j \qquad (19) $$

where ${}^j\sigma_i = \ln[({}^j\theta_i)^2 N/2]$ is the detection threshold as derived in [2,3], N being the allowable mean time, i.e., the number of samples, between false alarms $(N \gg 1)$, and the lower limit setting ${}^j\delta_i$ is selected to account for the a priori probability of fault (in the mode j) of any measurement in the i^{th} $(n+1)$-tuplet during one sample interval.

Remark 5: Following Assumption 1, ${}^j\delta_i$ may be set to ρ/M $\forall i,j$, or as an appropriate function of ρ. ■

Remark 6: The upper limits of ${}^j\Phi_i$ are set to ${}^j\sigma_i$ to enhance recovery from a fault after the faulty measurement has been reinstated. Chien [2,3] has presented a detailed discussion on the sensitivity of the parameters N, θ, and σ relative to the system performance (e.g., probability of false alarms and delay in detecting faults). ■

Table 1 Error bounds for individual sensor groups

Sensor group	Multi-level				Bi-level	
	lo I	*hi* I	*lo–lo* I	*hi–hi*	*lo*	*hi*
Power (MWt)	−0.05	+0.05	−0.10	+0.10	−0.075	+0.075
Flow (kg/s)	−2.00	+2.00	−4.00	+4.00	−3.00	+3.00
ΔT (°C)	−0.1	+0.1	−0.50	+0.50	−0.30	+0.30

Table 2 (Normalized) Mean ${}^j\theta$'s for individual sensor groups

Sensor group	Multi-level				Bi-level	
	lo I	*hi* I	*lo–lo* I	*hi–hi*	*lo*	*hi*
Power (MWt)	−2.0	+2.0	−4.0	+4.0	−3.0	+3.0
Flow (kg/s)	−1.5	+1.5	−3.0	+3.0	−2.25	+2.25
ΔT (°C)	−0.05	+0.05	−0.10	+0.10	−0.075	+0.075

4 Synopsis of the Real-Time Experimentation and Results

The multi-level sequential fault detection procedure described in Section 3 was tested for real-time operations at the 5 MWt nuclear reactor MITR-II [6]. The test facility and instrumentation for on-line fault detection are similar to those reported in our earlier publications [1,4,5]. There are four sensors for reactor power ranging up to 5 MWt, four for primary coolant flow ranging up to 140 kg/s, and three for temperature difference (ΔT) between hot and cold legs ranging up to 8°C. The software was updated to modify the bi-level sequential test algorithm [1] to a multi-level algorithm consisting of two warning modes of *lo* and *hi*, and two failure modes of *lo–lo*, and *hi–hi*.

The statistics of measurement noise and error bounds ${}^j\beta$'s for sensors in each group, i.e., power, flow and ΔT, were generated from experimental data. Since the statistics of individual sensors belonging to each group were found to be more or less identical and stationary, a single constant value of the mean ${}^j\theta$, $j=1,2,3,4$, was selected for each j. The selection of θ was based on the respective bounds ${}^j\beta$ of two sensors that form an $(n+1)$-tuplet in this case of scalar measurements. Numerical values of the error bounds and the mean ${}^j\theta$'s are given in Tables 1 and 2, respectively. Having the sampling period equal to 0.5 s, the mean time N between false alarms was set to 10^6 samples. The detection threshold ${}^j\sigma$, $j=1,2,3,4$, for each sensor group was then obtained using the equation next to (19). The a priori probability ρ of a fault during one sample was assumed to be 10^{-8} for all sensors. The lower limit $\delta=\rho/M$ was set accordingly.

The multi-level hypotheses test algorithm requires an execution time (on the LSI-11/23 processor) of about 230 ms per cycle including the time required for data acquisition and signal processing; the increase in processing time over the bi-level algorithm [1] is approximately 30 ms per cycle. Tests were conducted for detecting calibration errors, drift errors and other in-range faults of sensors in the same way as described in our earlier paper on bi-level algorithm [1]. Messages due to detection of *lo–lo* (or *hi–hi*) failures occurred in the event of abrupt failures as well as for gradual drifts whenever early warnings, *lo* (or *hi*), were ignored. These results are as expected in view of the detection threshold settings in Table 2, and are satisfactory on a qualitative basis. A detailed quantitative verification (not presented here) can only be achieved within certain bounds because of: (1) Assumption 2 in Section 2 and other assumptions like Gaussian distribution of q_i's in the algorithm development in Section 3; and (2) uncertainties in the selection of parameters like mean θ for computing the detection threshold σ. However, there is no approximation involved in the analytical derivation in Section 2.

5 Summary and Conclusions

The multiple-level hypotheses test algorithm, presented in this paper, is not restricted to any specific structure of measurement noise statistics. Its computational efficiency is enhanced if the noise distribution in the parity vector, generated from a linear combination of redundant measurements, is assumed to be Gaussian. The algorithm has been implemented on a commercially available microcomputer, and then tested for on-line detection of faulty sensors in a nuclear research reactor. This fault detection procedure is particularly suitable for intelligent instrumentation in continuous processes like spacecraft, aircraft, and nuclear power plants where redundant measurements are usually available for critical plant variables. Further research is recommended to quantify the accuracy and robustness of multi-level fault detection test procedures for different types of processes.

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On Modeling of Integrated Communication and Control Systems[1]

Luen-Woei Liou[2] and Asok Ray[2]

In a two-part paper [1,2], Ray and Halevi reported modeling of Integrated Communication and Control Systems (ICCS). Varying and distributed delays are introduced in the control system due to asynchronous time-division multiplexing in the communication network. This correspondence illustrates the relationship of Ray and Halevi's approach to that of Kalman and Bertram [3] under nonsynchronous sampling.

Nomenclature

A = plant system matrix $(n \times n)$
A_s = plant state transition matrix $(n \times n)$
B = plant input matrix $(n \times m)$
C = plant output matrix $(r \times n)$
F = controller system matrix $(q \times q)$
G = controller input matrix $(q \times r)$
H = controller output matrix $(m \times q)$
J = controller direct coupling matrix $(m \times r)$
l = maximum # of delayed actuator commands in one sampling period
p = maximum delay (# of samples) for sensor data arrival
T = nominal sampling period for the sensor and controller
T_c = controller sampling period
T_s = sensor sampling period
u = plant input vector $(m \times 1)$
X = augmented state vector $((n+q+m) \times 1)$
x = plant state vector $(n \times 1)$
y = plant output vector, i.e., generated sensor data $(r \times 1)$
Δ_s = time skew between sensor and controller sampling instants
δ_{ca} = controller-to-actuator data latency
δ_p = processing delay at the controller computer
δ_{sc} = sensor-to-controller data latency
η = controller state vector $(q \times 1)$
Θ_{sc} = sensor-controller delay
Θ_{ca} = controller-actuator delay
Φ = augmented system matrix

1 Introduction

Varying and distributed delays are introduced in Integrated Communication and Control Systems (ICCS) due to asynchronous time-division multiplexing in the network [1, 2]. The finite-dimensional modeling of systems with varying and distributed delays, reported by Ray and Halevi [1, 2], uses the concept of state transition to transform the continuous-time model into a discrete-time form. In 1959, Kalman and Bertram [3] used this technique to provide a unified approach for modeling different types of sampled data systems that are not subjected to any induced delays. The objective of this correspondence is to illustrate the relationship between these two modeling approaches.

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