An Introduction to Sensor Signal Validation in Redundant Measurement Systems

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This paper presents an introduction to the important topic of sensor signal validation where multiply-redundant measurements of critical variables are available. Pertinent results from a collection of previous publications have been reviewed in this paper, but it does not serve as a survey of the field of signal validation. The intention is to focus on redundancy management in fault detection and isolation with the emphasis on the parity space, which apparently has not been dealt with in details in any other survey or tutorial paper.

Introduction

The successful operation of complex dynamical systems, such as aircraft, spacecraft, chemical plants and nuclear power plants, is largely dependent on the validity of sensor signals providing information for display and control. To enhance safety and improve plant performance, redundant sensors are often installed for measuring critical variables such as neutron flux detectors in nuclear reactors and inertial navigational sensors in aircraft. The task of sensor signal validation in redundant systems can be classified into two broad categories: 1) fault detection and isolation; and 2) measurement calibration and estimation. The objective of this article is to introduce the reader to the first category of sensor signal validation in multiply-redundant measurement systems.

Fault detection and isolation (FDI) algorithms [1]-[8] can be designed to exploit, as much as practicable, all useful redundant information that are available in the plant. Redundancy is broadly classified as direct or analytic. With direct redundancy there are more than the minimum number of sensors (i.e., two or more for scalar variables, and four or more for three-dimensional variables). With analytic redundancy, additional information is obtained from an analytical model formulated on the basis of physical relationships among other direct and analytical measurements. Thus, analytic measurements may supplement sensor redundancy for plant variables of interest. Furthermore, analytic measurements allow detection of (non-sensor) plant component failures and common-mode failures, i.e., simultaneous and identical failures of two similar devices possibly due to a common cause. The failure decisions can be made either solely on the basis of current values of the available measurements or they can be made by sequential tests that use past observations in addition to the current measurements. The single sample approach is applicable only if changes due to failures are large in comparison to measurement noise and statistics. The sequential approach that makes use of the cumulative information provided by the measurement history has the advantage that it can detect moderate degradation of sensors. Methods for on-line calibration and estimation of the measured variable in the framework of this sequential approach are not presented here, but are discussed in [5].

Various methods for fault detection and isolation (FDI) with diverse applications have been extensively reported in literature, notably the excellent survey papers of Willsky [9], Isermann [10], Gertler [11] and Frank [12]. The intent of this review article is to discuss some of the key concepts of sensor redundancy management and illustrate them via simple examples. The next section presents the model of a multiply-redundant measurement system. The subsequent section deals with the generation of residuals and describes the concept of parity space, and the following section focuses on failure detection including sequential testing.

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Measurement Model

The first step toward developing a fault detection algorithm is to formulate a mathematical model of the redundant measurements, which includes the sources of measurement errors. Regardless of whether the measured variables are scalar- or vector-valued, the individual sensor signals are usually scalar quantities. Certain approximations are needed to formulate the measurement model, which we illustrate by the following two examples.

Example of a Scalar Variable

Consider the problem of measuring static pressure of fluid flowing through a pipe, i.e., the pressure of the fluid exerted on the pipe wall. We assume that the pressure transducers follow a linear relationship where $s(t)$ is the transducer signal at time t; h is the scale factor which converts the measured signal from the physical unit, i.e., pressure, into electrical unit; and $p(t)$ is the pressure being measured:

$$s(t) = h \cdot p(t) \tag{1}$$

The signal $s(t)$ includes: i) electrical disturbances in the measurements such as amplifier noise and electromagnetic interference; and ii) time-varying process characteristics due to variations in temperature, mechanical vibrations, and physical deterioration of the transducer; and iii) transducer nonlinearities, i.e., h may not remain constant over the measurement range. Note that a measurement implies that it is either a sensor data or analytically obtained.

Example of a Vector Variable

Consider the problem of measuring the rigid-body acceleration vector $\boldsymbol{a}(t)$ in an aircraft, with scalar components $a_x(t)$ $a_y(t)$ $a_z(t)$ in the x, y, and z directions, respectively. Ideally, the output signal $s(t)$ from a single-axis accelerometer would be proportional to the component of acceleration along the axis of orientation of the accelerometer where the elements, h_{a1}, h_{a2}, and h_{a3}, of the (1×3) scale factor vector $\boldsymbol{h}_T$ is proportional to the direction cosines of respective axes of the accelerometer relative to the reference frame.

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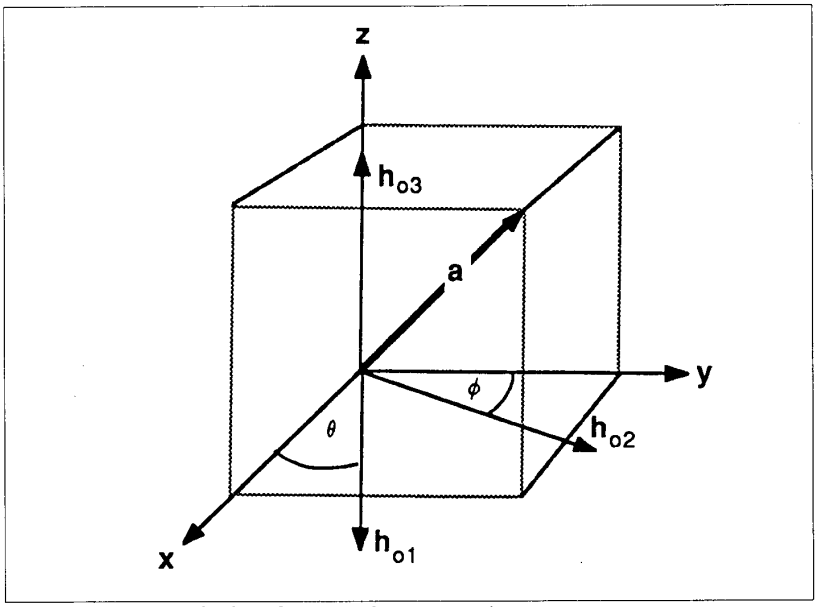

Fig. 1. Measurement of a three dimensional vector quantity.

$$s(t) = \boldsymbol{h}_T \boldsymbol{a}(t). \qquad (2)$$

A specific configuration of three accelerometers in Fig. 1 illustrates how to obtain $\boldsymbol{a}(t)$. These three accelerometers generate the signals $s_1(t), s_2(t), s_3(t)$, respectively. Assuming no measurement errors and a scale factor of one, the following measurement model results where θ and ϕ represent rotation angles for the accelerometer:

$$\begin{bmatrix} s_1(t) \\ s_2(t) \\ s_3(t) \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_x(t) \\ a_y(t) \\ a_z(t) \end{bmatrix}.$$

In summary, the following two assumptions [1] are often imposed to arrive at a measurement model that is convenient for signal validation: 1) Each sensor has a single measurement axis, i.e., a sensor measures the projection of the vector quantity along its own axis. (Complex sensors with more than one axis, e.g., tri-axial accelerometers, can be represented as a combination of highly correlated single measurement-axis sensors); 2) The sensors are assumed to be mutually redundant, i.e., when measuring an n-dimensional vector, every set of n measurements should be linearly independent. In the three dimensional case, no three measurement axes should be coplanar, and in the two dimensional case, no two measurement axes should be collinear. All measurements should be as nearly orthogonal to each other as possible to minimize errors.

Nonetheless, a realistic model must take into account the effects of measurement errors such as: i) time-dependent variations in the orientation due to vibrations of the accelerometer fixture; ii) changes in the proportionality constants due to temperature fluctuations and nonlinearities; and iii) bias and noise in the accelerometer signals due to electronic disturbances. Similar to the scalar case, a measurement can be either a sensor data or analytically obtained.

Considering the sources of error given above, the expressions (1) and (2) for the transducer signal can be rewritten as

$$s(t) = \boldsymbol{h}(t,x)_T x(t) + b(t,x) + n(t) \qquad (3)$$

where $x(t)$ is the $(n\times1)$ measured vector and the terms $b(t,x)$ and $n(t)$ are known as additive errors that represent the bias and (zero-mean) noise, respectively. The scale factor vector $\boldsymbol{h}(t,x) = h_o + \Delta\boldsymbol{h}(t,x)$; the constant vector h_o is a nominal value for the transducer scale factor; and $\Delta\boldsymbol{h}(t,x)$, with the expected value of $\Delta\boldsymbol{h}(t,x)^T x$ equal to zero, is known as the multiplicative error representing nonlinearities and time-varying effects of the measurement process.

For notational simplification, the functional dependence on x and t will not be explicitly displayed except when necessary. If an estimate b^* of the bias b is available such that the expected value of $(b-b^*)$ is zero, then the transducer signal can be compensated as follows:

$$\mu := s - b^* = (\boldsymbol{h}_o+\Delta\boldsymbol{h})^T x + (b-b^*) + n \qquad (4)$$

If the norm of the covariance matrix of $\Delta\boldsymbol{h}$ is small relative to the norm of $\boldsymbol{h}_o$, then the expected value of μ is approximately equal to the scalar product of $\boldsymbol{h}_o$ and the expected value of x. The $\boldsymbol{\ell}$ $(\geq n)$ redundant measurements after bias compensation can be approximately represented in a simple matrix form as shown below:

$$\boldsymbol{m} = \boldsymbol{H}x + \boldsymbol{e} \qquad (5)$$

where $\boldsymbol{H}$ is the $(\boldsymbol{\ell} \times n)$ measurement matrix with its rows representing the scale factor of individual measurements; each of the measurement vector $\boldsymbol{m}$ and the error vector $\boldsymbol{e}$ is of dimension $\boldsymbol{\ell}$, and the expected value of each element of $\boldsymbol{e}$ is equal to zero. Since a minimum $\boldsymbol{n}$, out of the $\boldsymbol{\ell}$ measurements are needed to obtain the measured vector x, each combination of n rows of $\boldsymbol{H}$ must form a non-singular matrix, which suffices that the rank of $\boldsymbol{H}$ is n. As stated earlier, the axes of any three accelerometers must not be co-planar. For a scalar variable, such as pressure, the measurement matrix $\boldsymbol{H}$ is of dimension $(\boldsymbol{\ell}\times1)$ and therefore each element of $\boldsymbol{H}$ must be non-zero. A practical implication is that the scale factors of the individual measurements should not be widely different. In the special case of scalar measured variables, i.e., n=1, the individual measurements can be appropriately scaled to have $\boldsymbol{H} = [1\ 1\ ...\ 1]^T$ without loss of generality.

Generation of Residuals — The Parity Space

Fault detection can be divided in two stages: residual generation and decision-making [13]. In the first stage, outputs from sensors are processed to enhance the effects of possible failures so that they can be detected. The processed measurements are called residuals and the enhanced failure effect on the residuals is called the failure signature. In a Kalman filter-based approach, the innovation sequence can be treated as the residuals. If redundant measurements of a physical variable are available, then residuals are often obtained by taking the difference between functions of the observed sensor outputs and expected values of these variables under the nominal, i.e., no-fail, mode. In the absence of a failure, residuals should be of zero mean showing close agreements between the observed and expected nominal behavior of the

system. A failure signature typically takes the form of residual biases that characterize the specific failure.

Residuals can be obtained directly from the measurement model if multiple redundancy is available. To do this, we first consider the general problem of measuring an n-dimensional quantity using a total of $\ell(n)$ sensors. The measurement model (5) can be adopted for solving this problem. The objective is to obtain a scalar function $f(\bullet)$ of the ℓ-dimensional measurement vector m such that the effects of the errors become prominent:

$$f(\boldsymbol{m}) = f(\boldsymbol{Hx}+\boldsymbol{e}) \approx f(e). \qquad (6)$$

For mathematical tractability, we impose linearity, i.e., $f(\boldsymbol{m}) = f(\boldsymbol{Hx}) + f(\boldsymbol{e})$. Then, $f(\boldsymbol{Hx})$ is forced to be zero. This is satisfied by making $f(\bullet)$ linear such that it can be expressed in a matrix form: $f(\boldsymbol{m}) = \boldsymbol{v}^T\boldsymbol{m}$ where $\boldsymbol{v}$ is an $(\ell\times 1)$ vector determined from the characteristics of $f(\bullet)$ [14]. However, selecting v to satisfy the relation $\boldsymbol{v}^T\boldsymbol{H}=0$ yields $\boldsymbol{v}^T\boldsymbol{m} = \boldsymbol{v}^T\boldsymbol{e}$ independently of x because

$$\boldsymbol{v}^T\boldsymbol{m} = \boldsymbol{v}^T(\boldsymbol{Hx}+\boldsymbol{e}) = \boldsymbol{v}^T\boldsymbol{Hx} + \boldsymbol{v}^T\boldsymbol{e}. \qquad (7)$$

The following example should help clarify this concept.

Example: Consider the problem of comparing the readings from three similar thermocouples to estimate the room temperature, i.e., $\ell=3$. The measurement model is derived as in equation (5), i.e., $\boldsymbol{m} = \boldsymbol{Hx} + \boldsymbol{e}$, where x is the true temperature, and $\boldsymbol{m} = [m_1\ m_2\ m_3]^T$ is the thermocouple measurement vector and $\boldsymbol{e} = [e_1\ e_2\ e_3]^T$ is the measurement error vector, and $\boldsymbol{H} = [1\ \ 1\ \ 1]^T$ is the measurement matrix. The residuals are obtained from vectors $\boldsymbol{v}$ such that $\boldsymbol{v}^T\boldsymbol{H}=0$, i.e., $\boldsymbol{v}$ must be orthogonal to $\boldsymbol{H}=[1\ \ 1\ \ 1]^T$. The equation $v^T\boldsymbol{H} = 0$ have two degrees of freedom meaning that two out of three elements of $\boldsymbol{v}$ can be chosen arbitrarily. The two degrees of freedom can be represented by two linearly independent vectors $\boldsymbol{v}_1 := [v_{11}\ v_{12}\ v_{13}]^T$ and $\boldsymbol{v}_2 := [v_{21}\ v_{22}\ v_{23}]^T$, one for each, such that $\boldsymbol{v}_1^T\boldsymbol{H}=0$ and $\boldsymbol{v}_2^T\boldsymbol{H}=0$.

Thus we have two linearly independent residuals, namely, $\boldsymbol{v}_1^T\boldsymbol{m} = \boldsymbol{v}_1^T\boldsymbol{e}$ and $\boldsymbol{v}_2^T\boldsymbol{m} = \boldsymbol{v}_2^T\boldsymbol{e}$. Any other residuals can be expressed as a linear combination of the above two. This information allows development of alfault detection and isolation (FDI) procedure based on the knowledge of the residuals. To do this, the two residual equations are grouped together in the following manner:

$$\wp := \begin{bmatrix} \boldsymbol{v}_1^T \\ \boldsymbol{v}_2^T \end{bmatrix}\boldsymbol{m} = \begin{bmatrix} \boldsymbol{v}_1^T \\ \boldsymbol{v}_2^T \end{bmatrix}\boldsymbol{e}$$

$$= \begin{bmatrix} v_{11} \\ v_{21} \end{bmatrix} e_1 + \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} e_2 + \begin{bmatrix} v_{13} \\ v_{23} \end{bmatrix} e_3 \qquad (8)$$

The vector $\wp$ on the left side of (8) is called the parity vector according to Potter and Suman [1]. The parity vector can be readily calculated if $\boldsymbol{v}_1, \boldsymbol{v}_2$ and $\boldsymbol{m}$ are known. Notice that, in the right hand side of equation (8), $\wp$ is a linear combination of three (2×1) vectors weighted by the measurement errors e_1, e_2 and e_3, respectively. The column vectors, e_1, e_2 and e_3, are known as failure directions, and the vector space spanned by the failure directions is referred to as the parity space in which the parity vector lies. The physical significance of the failure directions is explained as follows. Let e_2 be much larger than e_1 and e_3 for a particular measurement set, then

$$\wp \approx \begin{bmatrix} v_{12} \\ v_{22} \end{bmatrix} e_2 . \qquad (9)$$

In other words, the parity vector points toward the failure direction $[v_{12}\ v_{22}]^T$. A failure can be detected by computing the normll $\wp$ || of the parity vector and comparing it to a preset threshold. Then, the source of failure can be identified by comparing the orientation of the parity vector to that of the failure directions.

To determine the failure directions, it suffices to specify the six components of the two (3×1) vectors $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$. Two equations are obtained directly from the orthogonality property:

$$\boldsymbol{v}_1^T\boldsymbol{H} = v_{11} + v_{12} + v_{13} = 0$$

and

$$\boldsymbol{v}_2^T\boldsymbol{H} = v_{21} + v_{22} + v_{23} = 0.$$

Since $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ form a basis for the left null space of $\boldsymbol{H}$, it is convenient to make them orthonormal. This generates three more equations, namely,

$$\boldsymbol{v}_1^T\boldsymbol{v}_1 = v_{112} + v_{122} + v_{132} = 1;$$

$$\boldsymbol{v}_2^T\boldsymbol{v}_2 = v_{212} + v_{222} + v_{232} = 1;$$

$$\boldsymbol{v}_1^T\boldsymbol{v}_2 = v_{11}v_{21} + v_{12}v_{22} + v_{13}v_{23} = 0.$$

Now we have five equations and six unknowns and a solution can be obtained by arbitrarily fixing one unknown, say $v_{21}=0$. The solution is: $\boldsymbol{v}_1 = [\sqrt{2}/\sqrt{3}\ \ -1/\sqrt{6}\ \ -1/\sqrt{6}]^T$ and $\boldsymbol{v}_2 = [0\ \ 1/\sqrt{2}\ \ -1/\sqrt{2}]^T$ Then, it follows from (8) that

$$\wp = \begin{bmatrix} \sqrt{2}/\sqrt{3} \\ 0 \end{bmatrix} e_1 + \begin{bmatrix} -1/\sqrt{6} \\ 1/\sqrt{2} \end{bmatrix} e_2 + \begin{bmatrix} -1/\sqrt{6} \\ -1\sqrt{2} \end{bmatrix} e_3 .$$

The failure directions associated with the errors e_1, e_2, and e_3 are symmetrically spaced at 120° as shown in Fig. 2. Assuming that at least two sensors are functioning normally, the parity space, i.e., the space spanned by the failure directions, can be

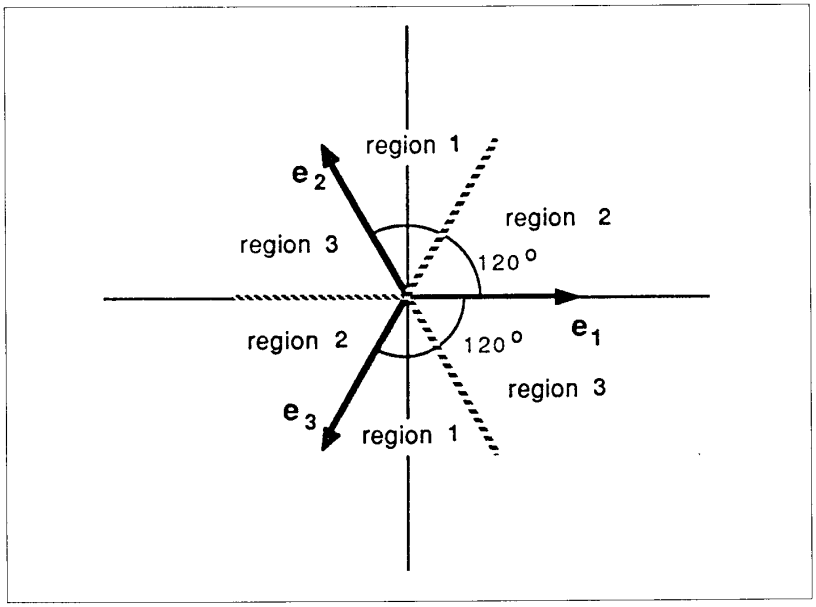

Fig. 2. Failure directions in the parity space.

divided into three failure regions which can be used to identify the remaining failed sensor [4]. If $\wp$ lies in region 2 of Fig. 2, then any possible failure is characterized by the errors e_1 or e_3. Therefore, the measurement, m_2, is least likely to be faulty because the parity vector $\wp$ is most nearly orthogonal to the corresponding failure direction $[v_{21}\ v_{22}]^T$. This is is equivalent to midvalue selection [1],[2] of the set $\{m_1,m_2,m_3\}$.

Now we proceed to extend the above concept to the general case of vector variables where the measurement matrix $\boldsymbol{H}$ is of dimension $(\ell \times n)$ and $\ell > n \geq 1$. Since the rank of $\boldsymbol{H}$ is given to be n, there exists a set, $\{v_1, v_2, ..., v_{\ell n}\}$ of $(\ell - n)$ linearly independent $(\ell \times 1)$vectors such that, for each i, the $(1 \times n)$ vector $v_i^T \boldsymbol{H} = 0$. That is, there are n independent equations involving ℓ unknown elements of the vector v_i. A projection matrix $\boldsymbol{V}$ can be constructed such that v_i^T is the ith row of $\boldsymbol{V}$. The elements of the $(\ell - n)$-dimensional parity vector $\wp$ are then $v_1^T m, v_2^T m, ..., v_{\ell - n}^T m$. That is,

$$\wp = \boldsymbol{V}\boldsymbol{m} \qquad (10)$$

where $\boldsymbol{V} := [v_1\ v_2 \ldots v_{\ell - n}]^T$ (and hence $\boldsymbol{V}\boldsymbol{H} = 0$).

The concept for evaluating the $(\ell - n)$ ℓ unknown elements of the $(\ell - n) \times \ell$ projection matrix $\boldsymbol{V}$ for vector variables is, in principle, similar to that for scalar variables described earlier. The relationship $\boldsymbol{V}\boldsymbol{H} = 0$ provides $(\ell - n)n$ equations which are intrinsic to the development of the parity vector. Therefore, we have $[(\ell - n)\ell - (\ell - n)n] = (\ell - n)^2$ unknowns that can be arbitrarily chosen. A procedure, following Potter and Suman [1], for choosing these variables is presented below.

Additional $(\ell - n)(\ell - n + 1)/2$ equations can be obtained by requiring that the $(\ell - n)$ rows of $\boldsymbol{V}$ are orthonormal, i.e., $\boldsymbol{V}\boldsymbol{V}^T = I_{\ell n}$. The remaining $[(\ell - n)^2 - (\ell - n)(\ell - n + 1)/2] = (\ell - n)(\ell - n - 1)/2$ unknowns are essentially free, and are satisfied by setting each of the $(\ell - n)(\ell - n - 1)/2$ lower diagonal elements of the rectangular matrix $\boldsymbol{V}$ to zero.

The two relationships $\boldsymbol{V}\boldsymbol{H} = 0$ and $\boldsymbol{V}\boldsymbol{V}^T = I_{\ell n}$ are equivalent to having the $(\ell - n)$ rows of $\boldsymbol{V}$ form an orthonormal basis of the left null space of $\boldsymbol{H}$ [14]. This implies that $\boldsymbol{V}^T\boldsymbol{V} = I_\ell - \boldsymbol{H}\boldsymbol{K}$ where $\boldsymbol{K}$ is a left inverse of $\boldsymbol{H}$, i.e., $\boldsymbol{K}\boldsymbol{H} = I_n$. In this process, $\boldsymbol{K}$ can be used to obtain a least squares estimate x^* of the vector variable x where $\boldsymbol{K}\boldsymbol{e}$ is essentially the estimation error:

$$x^* = \boldsymbol{K}\boldsymbol{m} = \boldsymbol{K}\boldsymbol{H}x + \boldsymbol{K}\boldsymbol{e} = x + \boldsymbol{K}\boldsymbol{e} \qquad (11)$$

Moreover, the residue vector

$$\eta := \boldsymbol{V}^T\boldsymbol{V}\boldsymbol{m} = (I_\ell - \boldsymbol{H}\boldsymbol{K})\boldsymbol{m} = \boldsymbol{m} - \boldsymbol{H}x^* = \boldsymbol{H}x + \boldsymbol{e} - \boldsymbol{H}x^* = \boldsymbol{H}(x - x^*) + \boldsymbol{e}. \qquad (12)$$

The above expression also implies that $\boldsymbol{V}^T\boldsymbol{V}\boldsymbol{m} = \boldsymbol{V}^T\wp$ is an estimate of the measurement error $\boldsymbol{e}$, and the estimation error is $\boldsymbol{H}(x - x^*) = \boldsymbol{H}(x - \boldsymbol{K}\boldsymbol{m})$. Furthermore, since the $\ell \times \ell$ matrix $\boldsymbol{V}^T\boldsymbol{V}$ is idempotent (i.e.,the square of $\boldsymbol{V}^T\boldsymbol{V}$ is equal to $\boldsymbol{V}^T\boldsymbol{V}$ itself), the residue vector is its own residue, and the norm $\| \wp \|$ of the parity vector is identically equal to the norm $\| \eta \|$ of the residual vector.

Fault Detection and Isolation

Once the residuals are obtained, a failure decision rule can be put into effect. Such a decision process may consist of a simple threshold test on single samples or moving averages of the residuals, or it can be based on a sequential hypothesis test [15]-[18]. Approaches using both fixed and variable sample size are appropriate for sequential testing [7],[18]. The variable sample size could provide an optimal decision rule for fault detection [15],[16] whereas the fixed sample size is usually simpler in computation although not necessarily optimal.

In general, an optimal decision rule minimizes a composite cost function consisting of the weighted sum of opposing requirements such as the probability of false alarms adjoined with the delay incurred in detecting the fault [15],[16]. Wald's sequential probability ratio test (SPRT) [15] is optimal in the sense that the expected value of the number of samples required for making a decision between two hypotheses, whether the system is in the normal or degraded mode, is minimum for specified probabilities of incorrect decisions. Wald's test is devised on the assumption that either one or other of the two hypotheses holds while the test is running. This restriction is removed in the disruption test of Shiryaeyev [16] where the probability of the change in the hypotheses at any sample is taken into account, and the decision as to whether a transition has occurred from the normal to the degraded mode is made on the basis of the a posteriori probability of failure derived from the past and current observations. In contrast to Wald's test, Shiryaeyev's test provides a smaller expected value of the delay in fault detection at the expense of increased computations. Chien [17] developed a sub-optimal procedure for on-line fault detection in navigational system sensors with Gaussian noise, which is computationally faster and retains the improved features of Shiryaeyev's test.

Sequential Testing for Fault Detection

Let us first assume that only a single sensor is available to measure a scalar variable. Let s_k denote the sensor data at the kth sample. Then, a fault may be detected by analyzing a window of j consecutive measurements, $W_{j,k} := \{s_k, s_{k-1}, ..., s_{k-j+1}\}$. This is done by first identifying the different operational modes where each mode signifies either the normal operation or a degraded operation possibly due to plant component failure(s) or externally induced disturbances, and is usually represented by an explicit model that includes the size (and sometimes the instant of occurrence) of the failure. The objective is to decide which mode is currently active by on-line analysis of the measurement window $W_{j,k}$. Notice that the data window is moving with respect to time, i.e., whenever a new measurement arrives, the time index k is increased by one and the window slides forward to include the recent arrival. An example of nuclear reactor instrumentation is cited below to explain this concept.

Suppose that a radiation flux sensor in a nuclear reactor consistently exhibit abnormally high readings. This leads to two possibilities: i) Hypothesis H_1, i.e., the sensor data is correct implying that the radiation flux is indeed abnormally high; or ii) Hypothesis H_2 — the sensor is erroneous. Selecting H_2 when H_1 is true would delay the detection of the cause of high radiation which may have catastrophic consequences. On the other hand, selecting H_1 when H_2 is true would cause a false alarm hindering the reactor operations. In general, there is a trade-off between detection delays and false alarms [15],[16]. The decisions for selecting H_1 over H_2 and vice versa are made on the basis of a posteriori probabilities, $Pr(H_1|W_{j,k})$ and $Pr(H_2|W_{j,k})$, of H_1 and H_2 given that $W_{j,k}$ is known. For example, it is logical to select H_2 over H_1 if the a posteriori probability of H_2 is relatively large, i.e., highly erratic sensor readings. A candidate procedure for selecting the appropriate hypothesis, H_1 or H_2, is presented below where the scalar constant γ is an a priori determined measure of the importance of H_1 relative to H_2:

If $[Pr(H_2|W_{j,k})/Pr(H_1|Wj_{,k})]$ is greater than γ, then select H_2; else select H_1. (13)

In this example of radiation flux detection, γ should be larger than one indicating that H_1 is more critical than H_2. However, a large γ would increase the probability of false alarms. The above decision rule needs to be modified to obtain the a posteriori probabilities $Pr(H_1|W_{j,k})$ and $Pr(H_2|W_{j,k})$ in a practical implementation. To this effect, if the a posteriori probabilities $Pr(H_i|W_{j,k})$, i=1,2 are expressed in terms of the a priori probabilities $Pr(W_{j,k}|H_i)$, i=1,2, i.e., the probability of $W_{j,k}$ assuming that the mode of operation H_i is active. Applying Baye's rule, the decision (13) is expressed as:

If $[Pr(W_{j,k}|H_2)/Pr(W_{j,k}|H_1)]$ is greater than γ $[Pr(H_1)/Pr(H_2)]$, then select H_2; else select H_1. (14)

The ratio, $Pr(W_{j,k}|H_2)/Pr(W_{j,k}|H_1))$, of a priori conditional probabilities is referred to as the likelihood ratio, and the a priori probabilities $Pr(H_1)$ and $Pr(H_2)$ represent the relative frequencies of H_1 and H_2 to be experimentally obtained or derived via statistical modeling. Since sufficient data for a failure condition may not be available, a priori probability of the failure is often constructed by obtaining a pseudo-estimate of the failure size. Furthermore, if $Pr(H_1)$, $Pr(H_2)$, $Pr(W_{j,k}|H_1)$ and $Pr(W_{j,k}|H_2)$ are Gaussian, the log-likelihood ratio test can be easily implemented by taking the natural logarithm of both sides of the inequalities in (14).

The conditional probabilities $Pr(W_{j,k}|H_i) = Pr(s_k, s_{k-1},...,s_{k-v+1}|H_i)$ are not usually straightforward to obtain in practice. We discuss two alternatives to circumvent this difficulty. In the first approach, a moving average is considered instead of a moving window. Thus, the collection $W_{j,k} = \{s_k, s_{k-1},...,s_{k-j+1}\}$ is substituted by the average $A_{j,k} = (s_k+s_{k-1}+...+s_{k-j+1})/j$ and the joint statistics of $W_{j,k}$ are approximated by the scalar statistics of $A_{j,k}$. A problem with this procedure is that detection of failures can be delayed since the effects of unusually high or low sensor readings are averaged. In particular, detection of soft-failures, e.g., slowly drifting bias errors in the measurements, may suffer from significantly large delays before being detected.

The other approach is to assume that the measurements are conditionally independent. That is,

$$Pr(s_k,s_{k-1},...,s_{k-v+1}|H_i) = Pr(s_k|H_i)Pr(s_{k-1}|H_i..Pr(s_{k-j+1}|H_i). \quad (15)$$

This leads to a recursive relation which has been extensively used for sequential tests [5],[6],[11],[18],[19]. However, if the assumption of conditional independence is invalid, then this technique may also suffer from nullifying the effects of joint statistics. For example, measurements obtained from a gradually drifting bias are conditionally dependent, and an independence assumption may result in a delayed detection.

Applications of sequential testing to fault detection in multiply-redundant measurement systems are reported in the recent publications of Ray [18],[19] where Chien's modified SPRT [17] has been used in the framework of a parity-space-based redundancy management procedure [4]. The reader is referred to an interesting and highly critical article by Kerr [20] on fault detection algorithms based on the sequential probability ratio test.

Remark: Sequential tests may not be appropriate in the absence of relevant statistical information and knowledge of plant dynamics. In that case, if measurement redundancy is unavailable, the only resort is built-in test procedures like limit check and rate check. However, in the presence of measurement redundancy, fault detection and measurement estimation can be carried out either by Potter and Suman's technique [1] of thresholdless redundancy or by approximating the measurement uncertaintiies as being amplitude-limited [3],[4].

Summary and Conclusions

The topic of redundancy management has been addressed in the context of sensor signal validation. The concept of parity space has been reviewed along with sequential tests for fault detection and isolation (FDI). The major requirement for this FDI approach is the availability of multiple measurements, either direct or analytic, of a variable. This is indeed the situation in many complex systems such as nuclear reactors and aircraft guidance, navigation and control.

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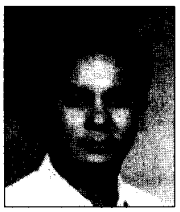

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1991 ICRA

The 1991 IEEE International Conference on Robotics and Automation will be held April 7-12, 1991, at the Hyatt Regency in Sacramento, California. The General Chair is Professor T. C. Hsia of the University of California at Davis, and the Program Chair is Professor T.J. Tarn from Washington University in St. Louis.

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Nominations are open for the new Control Systems Technology Award, which would be awarded for the third time this year. This award is to be given for outstanding contributions to control systems technology, either in design and implementation or in project management. It may be conferred on either an individual or a team. The prize is $1000 and a certificate. Nominations are open to all. The prize is to be awarded annually at the IEEE Conference on Decision and Control. Deadline for nominations is April 30, 1991. Please send you nominations, together with supporting documents, to the Chairman of the Technology Award Committee: Juergen E. Ackermann, DFVLR-Dynamik, D-8031, Oberpfaffenhofen, West Germany; phone: 08153-28423.