A Stochastic Regulator for Integrated Communication and Control Systems: Part I— Formulation of Control Law[1]

Luen-Woei Liou

Asok Ray

Mechanical Engineering Department,
The Pennsylvania State University,
University Park, PA

Integrated Communication and Control Systems (ICCS), recently introduced and analyzed in a series of papers [1–7], are applicable to complex dynamical processes like advanced aircraft, spacecraft, automotive, and manufacturing processes. Time-division-multiplexed computer networks are employed in ICCS for exchange of information between spatially distributed plant components as well as for coordination of the diverse control and decision-making functions. Unfortunately, an ICCS network introduces randomly varying, distributed delays within the feedback loops in addition to the digital sampling and data processing delays. These network-induced delays degrade the system dynamic performance, and are a source of potential instability. This two-part paper presents the synthesis and performance evaluation of a stochastic optimal control law for ICCS. In this paper, which is the first of two parts, a state feedback control law for ICCS has been formulated by using the dynamic programming and optimality principle on a finite-time horizon. The control law is derived on the basis of a stochastic model of the plant which is augmented in state space to take into account the effects of randomly varying delays in the feedback loop. The second part [8] presents numerical analysis of the control law and its performance evaluation by simulation of the flight dynamic model of an advanced aircraft.

1 Introduction

Integrated Communication and Control Systems (ICCS), recently introduced and analyzed in a series of papers [1–7], are applicable to complex dynamical processes like advanced aircraft, spacecraft, automotive, and manufacturing processes. Time-division-multiplexed computer networks are employed in ICCS for exchange of information between spatially distributed plant components as well as for coordination of the diverse control and decision-making functions. Unfortunately, an ICCS network introduces randomly varying, distributed delays within the feedback loops in addition to the digital sampling and data processing delays. The network-induced delays degrade the system dynamic performance, and are a source of potential instability.

A finite-dimensional model of the closed loop control system of ICCS was developed in [1, 2] by taking into consideration the effects of network-induced delays. A necessary and sufficient condition for system stability was established for the special case of periodically varying (non-random) delays. Attention was focused on control systems with identical sampling

rates for the sensor and controller. Although the sensor and controller sampling periods are designed to be identical, a certain difference between them always prevails due to manufacturing tolerances in clock frequencies. This difference in sampling periods causes a slowly varying time-skew, Δ_s, between the sensor and controller sampling instants, which may significantly contribute to the network-induced delays and signal distortion due to vacant sampling slots, i.e., absence of any signal arrival during a sampling interval. This aspect has been discussed in view of ICCS design in [2]. One way of circumventing this problem is periodic synchronization of the control system components by which Δ_s is maintained within a desirable range by transmitting high-priority synchronization signals via the network medium or by additional wiring. However, the control law still needs to be redesigned because of network-induced delays.

An alternative approach to the above synchronization procedure is to deliberately assign nonidentical sampling periods T_s and T_c to the sensor and controller, respectively, such that Δ_s does not remain within an undesirable range for a prolonged period. This is achieved by making the ratio, T_c/T_s, not close to a positive integer. Another benefit of having non-identical sampling is to reduce the occurrence of vacant sampling slots at the controller, which results from mis-synchronization between the system components and varying data latency [1, 2]. The probability of vacant sampling can be arbitrarily reduced

by selecting a sufficiently small value of the ratio, T_s/T_c, as presented in the Proposition 2.1 of [2]. Modeling of ICCS with non-identical sampling has been reported in [3, 4] but design of optimal control systems with non-identical sampling is a subject of future research and is not addressed in this paper.

Luck and Ray [5] have proposed a delay compensator for ICCS in which the sensor and the controller have identical sampling rates with an arbitrary time skew between them, and the sensor and the actuator are synchronized. The detrimental effects of network-induced delays, especially signal distortion due to vacant sampling, are alleviated by using a multi-step predictor where the number of predicted steps in the observer-based control system is determined from the sum of the postulated maxima of the sensor-controller and controller-actuator delays. The key idea is to monitor the data when it is generated and to keep track of the delay associated with it. With this knowledge, the compensation algorithm is formulated to keep the delay constant as seen by the controller. Therefore, the closed-loop control system model is constrained to be finite-dimensional, linear, time-invariant provided that the plant, observer, and controller are linear time-invariant. One major advantage of this delay compensator is that the observer gain and the state-variable-feedback control gain can be designed on the basis of the nondelayed plant model. However, the multi-step prediction makes the control system sensitive to modeling errors, nonlinearities, and uncertainties as revealed in the experimental results on a network testbed. Robustness of the delay compensator relative to structured uncertainties has been analyzed, and the problem of loss of observability under recurrent loss of data is addressed [6].

We propose, in this paper, a stochastic approach for compensation of randomly varying distributed delays as an alternative to the deterministic method of multi-step prediction. The objective is to derive an optimal (or a suboptimal) stochastic control law to compensate for network-induced delays under diverse randomly varying network traffic such that the control parameters can be determined on the basis of statistics of the network-induced delays and the plant model. The key issue in this approach is that a controller designed for a non-networked system may not satisfy the performance and stability requirements in the delayed environment of ICCS networks. We have represented the plant by a linear, finite-dimensional, stochastic model [1, 4] in the discrete-time setting, and the proposed optimal control law is synthesized by using the principle of dynamic programming and optimality.

This paper is the first of two parts, and presents the stochastic regulator problem for ICCS and formulation of an optimal control law using state feedback. The second part concentrates on numerical techniques for solving the difference equations resulting from dynamic programming and presents the simulation results. This Part I is organized in four sections including the introduction. Section 2 presents the status of research in optimal control of delayed systems. The stochastic control law is derived in Section 3. Summary and conclusions are given in Section 4. The nomenclature used in both parts is listed in Part II [8].

2 Research Status of Optimal Control of Delayed Systems

This section focuses on a limited class of optimal control problems for delayed systems, and is not intended to be a survey of this field. Only those publications that have a possible bearing on the synthesis of control laws for randomly varying, distributed delayed systems such as those in ICCS are considered. In particular, we concentrate on the optimal control methodology employing linearity of the plant model and quadratic cost criterion as this approach is suitable for multivariable systems, facilitates formulation of the performance cost and selection of design parameters, and is likely to be mathemat-

ically tractable. A reasonable amount of research effect has been expended [9-22] on extension of the standard linear quadratic regulator (LQR) theory to delayed systems in both continuous-time and discrete-time settings. Some of these results are briefly discussed below.

2.1 Continuous-Time Control of Delayed Systems. A survey paper by Banks and Manitius [9] reviewed time-delayed optimal control problems by employing abstract variational approaches. Manitius and Olbrot [10] proposed a concept for linear systems design with (constant) delays in state and/or control that yields a finite spectrum closed loop system. Since the continuous-time systems under consideration only addressed constant delays, these results are not applicable to the discrete-time control system with randomly varying delays.

Buckalo [11] presented the concept of controllability for systems described by delayed differential equations, and discussed sufficient conditions for controllability in terms of the system and input matrices. Although this approach can be extended to multiple constant delays, it is not apparanetly applicable to varying delays.

Soliman and Ray [12] reviewed their previous work on optimal control of multivariable systems having constant delays. An optimal feedback control policy was derived in a linear-quadratic setting for continuous-time systems. The delays in the problem were presented as transport lags, and modelled by auxiliary partial differential equations. The resulting equations were discretized and approximated by a large number of ordinary differential equations (ODEs). Classical optimal feedback control theory was then applied to the set of ODEs, and the limit was taken as the number of points of discretization approached infinity. This method can be extended to deterministically varying delays. Although the rigor of deriving an optimal feedback control law by discretization of the delay is still open to question, several successful applications were cited.

The problem of optimal control of continuous-time systems with delays in the state and control variables was considered by Koivo and Lee [13]. State and costate equations, obtained by application of the maximum principle for optimal control problems, were transformed into equivalent integral equations. The presence of an integral equation in the optimal feedback gain matrix results in a set of partial differential equations. The feedback control law was numerically obtained by discretization of the integral equation.

The concept of the papers by Soliman and Ray [12] and Koivo and Lee [13] could lead to a design methodology for optimal control of deterministically varying delayed systems by solving additional partial differential equations. However, neither randomly varying delays nor the impact of measurement noise and modeling uncertainties were considered. Furthermore, the control law was obtained in the continuous-time setting which, if used in ICCS, must be discretized for implementation in the controller computer.

2.2 Discrete-Time Control of Delayed Systems. Augmentation of the discrete-time state-space model is a commonly used approach for taking the effects of delays into account. (This approach has also been used in finite-dimensional modeling of ICCS in [1-4].) Diduch and Doraiswami [14] investigated MIMO sampled data systems where delays were modeled as a discrete-time, augmented state representation. Delays were divided into an integer part and a fractional part relative to the sampling time. Whereas the integer part was modeled by augmenting the original state with delayed inputs and outputs, the fractional part was treated via the state transition equation. The conditions for controllability and observability of this augmented discrete-time model were derived by using the Popov-Belevitch-Hautus tests, i.e., the pair (A, B) is completely controllable iff $\text{rank}([zI_n - A \mid B]) = n$. Necessary and sufficient conditions for controllability and observability of the delayed systems were shown to be: (i) a similar

system that is subjected to delays less than the sampling period is controllable and observable, (ii) identical number of inputs and outputs, and (iii) no transmission zero at the origin. Controllability and observability conditions for varying delays were not addressed, and no algorithms for optimal control or estimation were presented.

Chyung [15] has pointed out the problem of potential loss of controllability due to incorporation of additional states. As a recourse to augmentation of the plant model, a discrete-time version of the maximum principle was used for synthesizing the optimal control law for delayed systems. The resulting state and costate equations are analogous to those encountered in the two-point boundary value problem of continuous-time optimal controller design. However, no effective computational technique was prescribed. Furthermore, since the control input $u(k)$ is not specified as a function of state $x(k)$, it cannot be conveniently implemented in the feedback form.

Drouin et al. [16] proposed a decomposition-coordination approach for controller design in linear discrete-time systems with constant delays in both state and input variables. It has been shown that, by an appropriate decomposition of the performance cost, a control law with partial state variable feedback can be formulated. For example, the performance cost can be decomposed as:

$$J = \sum_{k=0}^{N} [x_{k+1}{}^T Q x_{k+1} + u_k{}^T R u_k] (\text{w.r.t.} \ x_{k+1} = A x_k + B u_k)$$

$$= (x_{j+1}{}^T Q x_{j+1} + u_j{}^T R u_j) + \sum_{p=0, \ p \neq j}^{N} (x_{p+1}{}^T Q x_{p+1} + u_p{}^T R u_p)$$

$$= J_j + J''_j$$

Then, for optimality of u_j, it necessitates that, on the optimal trajectory,

$$\partial J / \partial u_j = \partial J_j / \partial u_j + \partial J''_j / \partial u_j = \partial J_j / \partial u_j + \rho_j = 0 \ \forall j$$

Since the coordination vector ρ_j depends on future ρ_p, $p > j$, it is necessary to use an iterative procedure to compute the optimal value of ρ_j. The resulting control law feeds back the current state and the open loop correction term ρ_j. Although this method is good for large systems and can handle constrained control problems, the time needed for convergence of the coordination vector may be too long for on-line applications. Furthermore, varying delays cannot be handled by this approach.

Arthur [17] used an augmented model to deal with delays in state and control variables in a discrete-time setting, and employed the principle of dynamic programming and optimality for synthesizing the control law. The resulting feedback control depends on the solution of matrix Riccati difference equations, which is analogous to that in continuous time. These equations were obtained by partitioning the augmented state matrix according to the original state and control variables. This serves to avoid operations on large matrices of the augmented system and provide a better insight into the structure of the delay problem. Apparently, this approach is restricted to delays that are integer multiples of the sampling period, and its extension to randomly varying delays is not straight-forward.

Now we study discrete-time systems with stochastic parameters with the objective of formulating a control law for randomly varying delayed systems. The majority of publications in stochastic control [18, 19] deal with additive noise where the system, input, and output matrices are deterministic. Since these matrices in the ICCS model [1, 2, 4] contain stochastic elements, the standard techniques for deriving stochastic control laws cannot be readily applied. Bitmead and Anderson [20] investigated sufficiency conditions for exponential stability of linear difference equations with random coefficients

via the Lyapunov technique but no systematic approach to selection of a candidate Lyapunov function was prescribed. De Koning [21] has reported a series of research publications on discrete-time systems with stochastic parameters. The system, input, and output matrices were assumed to be sequences of independent random matrices in addition to the additive white noise in the state and measurement equations. Definitions of stochastic stability, controllability, and observability were given in the sense of mean square convergence. Theorems for solutions of optimal control and optimal estimation problems were given for the infinite-time horizon case. Finally, a set of Riccati-type matrix equations was derived for the optimal compensation problem, i.e., combined control and estimation. However, this algorithm intends to provide a steady-state solution derived from the Hamiltonian based on the expected value of a quadratic cost function. Apparently, the existence of a solution of the resulting set of coupled nonlinear algebraic equations has not been established, and these equations are difficult to solve numerically. Further discussions on control of systems with stochastic parameters can be found in [21, 22].

The control law, proposed in this paper, is derived by modeling the ICCS as a discrete-time system where the sensor and control data are subjected to randomly varying delays. The matrices in the augmented state-space model are stochastic with randomness occurring in the system, input and output matrices instead of being restricted to additive noise. In contrast to the Pontryagin's maximum principle as proposed by De Koning [21], we have adopted the dynamic programming approach to synthesize the stochastic optimal control law via a recursive relation which is not difficult to solve numerically. Conceptually, dynamic programming is more suitable for stochastic problems than deterministic techniques such as calculus of variations and Pontryagin's maximum principle for which a single optimal state and control trajectory ideally exists. The application of dynamic programming in the augmented plant model in our approach is, to some extent, similar to that proposed by Arthur [17] in a deterministic setting.

3 Formulation of the Stochastic Optimal Control Law

Following the previous work on modeling of ICCS [1-4], the network-induced delays are defined below for development of the stochastic control law.

δ_{sc}: Sensor-controller latency, defined as the time interval from the instant of the sensor sampling to the instant that sensor data arrives at the controller receiving queue.

Θ_{sc}: Sensor-controller delay, defined as the interval from instant of the sensor sampling to the instant that data starts to be processed in the controller.

δ_{ca}: Controller-actuator latency, defined as the time interval from the instant that the controller command is generated to the instant that the controller command arrives at the actuator receiving queue.

Θ_{ca}: Controller-actuator delay, defined as the interval from the controller command generation to the instant that data starts to affect the actuator.

For derivation of the control law, the statistical characteristics of δ_{sc}, Θ_{sc}, δ_{ca}, and Θ_{ca}, are assumed to be available via analysis of the network performance. The plant dynamics are represented by a finite-dimensional, linear, time-invariant, continuous-time model:

$$d\xi(t)/dt = a\xi(t) + bu(t) \tag{1}$$

$$y(t) = c\xi(t) \tag{2}$$

where $\xi \in \mathfrak{R}^n$, $u \in \mathfrak{R}^m$ and $u \in \mathfrak{R}^r$. The problem is to formulate a state-feedback control law in the discrete-time setting on the basis of the following assumptions:

1. The sensor and controller have identical sampling periods, T.

2. The discretized version of the plant model in (1) is both reachable and observable.

3. The sampler is ideal and a zero-order-hold (ZOH) is placed between the digital controller output and the plant input.

4. The skew, $\Delta_s \epsilon (0, T]$, between the sensor and controller sampling instants is a slowly varying parameter to be periodically reset [1, 2] and therefore may be treated as a selectable constant parameter.

5. The actuator operates as a continuous-time device, i.e, the control input acts upon the plant immediately after its arrival at the actuator terminal.

6. Network-induced delays, $\{\Theta_{sc}^k\}$ and $\{\Theta_{ca}^k\}$, are bounded, mutually independent, white sequences with identical and a priori known statistics. (The stipulation is that the number of network users is relatively large with diverse requirements for utilization of the communication medium, and that the offered traffic bears a safe margin relative to its critical value [23, 24].)

7. Statistics of plant disturbances and sensor noise are independent of those of Θ_{sc}^k and Θ_{ca}^k.

8. The probability of data loss, due to noise in the communication medium and protocol malfunctions, is zero. ∎

The proposed control synthesis procedure for ICCS is developed according to the steps outlined below.

- Development of an augmented state-space model of the plant to account for the randomly varying delays.
- Formulation of a suitable performance cost that is minimized to obtain the control law.
- Derivation of an optimal control law based on dynamic programming.
- Construction of an estimator for prediction of the delayed states.

3.1 The Augmented Plant Model. Because of the varying (but bounded) controller-actuator delay Θ_{ca}^k, the input $u(t)$ to the plant is piecewise constant during a sampling interval $[kT, (k+1)T)$ where the changes in $u(t)$ occur at the random instants $KT + t_i^k$, $i = 0, 1, \cdots, \ell$, and $t_i^k > t_{i+1}^k$ as illustrated in Fig. 4 of [1]. On this basis, the continuous-time plant model in (1) is discretized to yield:

$$\xi_{k+1} = a_s \xi_k + \sum_{i=0}^{\ell} b_i^k u_{k-i}$$

$$y_k = c \xi_k \tag{3}$$

where

$$a_s := \exp[aT], \quad b_i^k := \int_{t_i^k}^{t_{i-1}^k} \exp[-a(T-\tau)]d\tau \, b, \text{ and}$$

$$t_{-1}^k := T \text{ and } t_\ell^k := 0.$$

We proceed to take into account the effects of the controller-actuator delay Θ_{ca}^k and sensor-controller delay Θ_{sc}^k at the kth sample following the modeling methodology proposed in [1]. Since the delays are assumed to be bounded (see assumption #6 earlier in this section), $\exists \ell \geq 2$ and $_P \geq 0$ such that the following conditions hold:

$$\Theta_{sc}^k <_P T + \Delta_s \text{ and } \Theta_{ca}^k < (\ell - 1)T \; \forall k \text{ with probability 1.} \tag{4}$$

The first condition in (4) suggests that the sensor data y_k may undergo $(_P + 1)$ discretely random delays, i.e., $\Theta_{sc}^k = p(k)T + \Delta_s$, where $p(k) \epsilon \{0, 1, \cdots, _P\}$ and $p(k+1) \geq (p(k) - 1) \; \forall k$. This means that the sensor data, $y_{k-p(k)}$, collected at the $(k-p(k))$th sensor sampling period, is used to generate the control input u_k. The second condition in (4) implies that there are at most ℓ new control input data arrivals at the actuator terminal during any sampling interval $[kT, (k+1)T)$.

Following the modeling methodology given in [1, 4], the discretized model (3) is augmented to take into account the effects of Θ_{ca}^k. The augmented plant model is presented below.

$$x_{k+1} = A_k x_k + B_k u_k \tag{5}$$

where $u_k \epsilon \Re^m$ represents $u(t)$ as defined in (1),

$$x_k := [\xi_k^T | u_{k-1}^T | \cdots | u_{k-\ell}^T]^T \epsilon \Re^L; \quad L := (n + \ell m),$$

$$A_k := \begin{bmatrix} a_s & b_1^k & b_2^k & \cdots & b_{\ell-1}^k & b_\ell^k \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & I_m & 0 & \cdots & 0 & 0 \\ \cdot & & \cdots & & & \cdot \\ \cdot & & \cdots & & & \cdot \\ 0 & 0 & 0 & \cdots & I_m & 0 \end{bmatrix} \text{ and } B_k := \begin{bmatrix} b_0^k \\ I_m \\ 0 \\ \cdot \\ \cdot \\ 0 \end{bmatrix}$$

Remark 3.1: During the kth sampling period, $u_{k-\ell}, \cdots, u_{k-1}$ may affect the plant state ξ_{k+1} in addition to u_k. However, since $u_{k-1}, \cdots, u_{k-\ell}$ are already generated by the controller, they are available at the kth sampling instant. ∎

Remark 3.2: The elements b_i^k of matrices, A_k and B_k, in (5) are stochastic processes because the time epochs, $\{t_i^k\}$, that form the limits of the integration in (3) are random. Therefore, $A_k = A_k(\omega)$, $B_k = B_k(\omega)$, $x_k = x_k(\omega)$, and $u_k = u_k(\omega)$ where ω is a sample point of the random sample space Ω. ∎

3.2 Formulation of the Performance Cost. A standard procedure [18, 19] for obtaining an optimal, linear, state-feedback control law $\{u_k\}$ for the discrete-time plant model (3) would be to minimize the performance cost J'' over a finite time interval from the 0th up to the Nth sampling instant as:

$$J_N'' := 1/2 E \left\{ \left(\xi_N^T P'' \xi_N + \sum_{k=0}^{N-1} [\xi_k^T Q'' \xi_k + u_k^T R'' u_k] \right) \right\} \tag{6}$$

where P'' and Q'' are positive semi-definite symmetric matrices and R'' is a positive definite symmetric matrix, the final time N is selected by the designer, and the expectation is with respect to the statistics of network-induced delays.

The above performance cost J'' needs to be modified to include the augmented plant model (5) by modifying the weighting matrices. The revised performance cost is:

$$J_N'' := 1/2 E \left\{ x_N^T P x_N + \sum_{k=0}^{N-1} [x_k^T Q x_k + u_k^T R u_k] \right\} \tag{7}$$

where

$$P := \begin{pmatrix} P'' & 0 \\ 0 & 0 \end{pmatrix}, \; Q := \begin{pmatrix} Q'' & 0 \\ 0 & 0 \end{pmatrix}, \; R := R''.$$

Remark 3.3: The structure of P and Q matrices in (7) allows the optimal control law u_k to be formulated as a linear, deterministic function of the plant states ξ_k and the delayed control inputs, $u_{k-1}, \cdots, u_{k-\ell}$. ∎

Similar to the standard linear quadratic regulator [18], we propose to formulate an optimal control law for delay compensation on the assumption of availability of the augmented state vector, x_k. The first n elements of x_k are the plant state ξ_k which is readily available if all states are directly measurable (i.e., $r = n$ in (1)) and the sensor signal-to-noise ratio is acceptable; otherwise, a filter [19] is necessary to provide an estimate of ξ_k using the measurement history, and this estimate must be used in lieu of the actual state ξ_k in the optimal control law. This requires the use of the separation property [18] which is valid because of the assumptions #6 and #7 stated at the beginning of this section. (The design of such a state estimator, which must account for the delayed control inputs to the plant, is addressed in Section 3.4.) The remaining elements of x_k, namely, $u_{k-1}, \cdots, u_{k-\ell}$, of x_k are already computed and stored at the controller. At the sampling instant k, a realization of the delayed augmented state $x_{k-p(k)}$ is necessary to generate

u_k. The following notation for the delayed augmented state is used for brevity:

$$z_k := x_{k-p(k)} \tag{8}$$

where $p(k) \in \{0, 1, \cdots, \ell\}$ may be random. The practice of ICCS network design mandates the offered traffic to bear a safe margin relative to its critical value [23, 24]. Therefore, we address the problem of control synthesis under the stipulation that $\delta_{sc}{}^k < T \ \forall k$ with probability 1. This implies that $p(k) \in \{0, 1\} \ \forall k$, and $p(k) = 0$ iff $\delta_{sc}{}^k < \Delta_s$. We consider, in the sequel, the operations of ICCS having $\delta_{sc}{}^k < T \forall k$, i.e, $p(k) \in \{0, 1\}$. Therefore, if $Pr\{p(k) = 0\} = \alpha \ \forall k$, then the expected value of z_k is:

$$E\{z_k\} = \alpha E\{x_k\} + (1-\alpha) E\{x_{k-1}\} \tag{9}$$

and the conditional expectation of x_k given z_k is predicted using the augmented model (5) as:

$$E\{x_k | z_k\} = \begin{cases} x_k & \text{if } p(k) = 0 \\ E\{A_{k-1}\}x_{k-1} + E\{B_{k-1}\}u_{k-1} & \text{if } p(k) = 1 \end{cases} \tag{10}$$

Since the objective is to find an optimal state-feedback law $\{u_k\}$ by minimizing the performance cost in a stochastic setting, dynamic programming and optimality principle [18] are considered to be more appropriate than deterministic methods such as the calculus of variations and Pontryagin's maximum principle [19]. For application of dynamic programming, the unconditional expectation in the performance cost [7] needs to be changed to the conditional expectation based on the measurement history. The rationale is that the control performance at any instant is optimized by utilizing the ensemble of all available measurements up to this instant.

To obtain the optimal control $\{u_k, k = 0, 1, \cdots, N-1\}$ that minimizes the performance cost J via dynamic programming, it is necessary to formulate a backward recursive relation starting from the stage N. Evaluation of u_k with one step at a time is possible because of the Markov property of the state-space model. To this effect, the performance cost in (7) is further modified as follows:

$$J_k(z_k, u_k) = E\left\{ \left(\frac{1}{2} [x_k{}^T Q x_k + u_k{}^T R u_k] \right. \right.$$
$$\left. \left. + J^*_{k+1}(z_{k+1}) \right) | z_k \right\} \text{ if } k < N \tag{11}$$

where $J^*_k(z_k) := J_k(z_k, u^*_k)$ and u^*_k is the optimal state feedback law at the kth sample, i.e., in $[kT, (k+1)T]$. For $k = N$, the terminal state is reached and there is no need for any control. Therefore,

$$J^*_N(z_N) := J_N(z_N, u^*_N) = \frac{1}{2} E\{ (x_N{}^T p_N x_N) | z_N \} \tag{12}$$

where p_N is set to P. The next objective is to determine an optimal control law u^*_k as a function of z_k by minimizing the performance cost (11).

3.3 Derivation of the Optimal State Feedback Control Law.
Now we present the following proposition to arrive at an optimal state feedback control law $\{u^*_k, k = N-1, N-2, \cdots\}$ via a recursive relationship.

Proposition 3.1: Let the stochastic matrices A_k and B_k be independent of $\{A_j, j = k-1, k-2, \cdots\}$ and $\{B_j, j = k-1, k-2, \cdots\}$. Then, given the statistics of the network-induced delays, the optimal control law at the kth stage is:

$$u^*_k(z_k) = -F_k E\{x_k | z_k\} \text{ for } k < N \tag{13}$$

and the resulting minimum performance cost is

$$J^*_k(z_k) = \frac{1}{2} [E\{x_k{}^T p_k x_k | z_k\} + E\{x_k{}^T | z_k\} S_k E\{x_k | z_k\}] \tag{14}$$

where

$$F_k := [R + E\{B_k{}^T p_{k+1} B_k\}$$
$$+ E\{\mathbf{B}_k{}^T S_{k+1} \mathbf{B}_k\}]^{-1} [E\{\mathbf{B}_k{}^T p_{k+1} A_k\} + E\{\mathbf{B}_k{}^T S_{k+1} \mathbf{A}_k\}]$$

$$p_k := Q + E\{A_k{}^T p_{k+1} A_k\} + E\{\mathbf{A}_k{}^T S_{k+1} \mathbf{A}_k\} \text{ with } p_N := P;$$

$$S_k := -[E\{A_k{}^T p_{k+1} B_k\} + E\{\mathbf{A}_k{}^T S_{k+1} \mathbf{B}_k\}]F_k \text{ with } S_N = 0;$$

$$\mathbf{A}_k := \alpha A_k + (1-\alpha) E\{A_k\} \text{ and } \mathbf{B}_k := \alpha \mathbf{B}_k + (1-\alpha) E\{B_k\};$$

$$\alpha := Pr\{p(k) = 0\} \text{ and } (1-\alpha) = Pr\{p(k) = 1\}; \text{ and}$$

each equation is evaluated backward from $K = N-1, N-2, \cdots$.

We need the following lemma to prove the above proposition.

Lemma 3.1: $E\{E\{f(x_k) | z_k\} | z_{k-1}\} = E\{f(x_k) | z_{k-1}\}$ where $f(\cdot)$ is piecewise continuous with at most countable number of discontinuities.

Proof of Lemma 3.1: Since $\{x_k\}$ is a Markov sequence and $z_k = x_{k-p(k)}$ where $p(k)$ is a non-negative integer, it follows that $E\{f(x_k) | z_k\} = E\{f(x_k) | z_k, z_{k-1}\}$ where the expectation is relative to z_k. The proof follows by using the relationship $E\{E\{Z|X, Y\} | Y\} = E\{Z | Y\}$ for conditional expectation. ∎

Proof of Proposition 3.1: Starting at the $(N-1)$th stage, the performance cost in (11) is:

$$J_{N-1}(z_{N-1}, u_{N-1}) = E\{ (\frac{1}{2} (x_{N-1}{}^T Q x_{N-1}$$
$$+ u_{N-1}{}^T R u_{N-1}) + J^*_N(z_N)) | z_{N-1}\} \tag{15}$$

Using lemma 3.1 and the state relationship (5) in (15) yields

$$J_{N-1}(z_{N-1}, u_{N-1}) = \frac{1}{2} E\{ (x_{N-1}{}^T Q x_{N-1} + u_{N-1}{}^T R u_{N-1}$$
$$+ (A_{N-1}x_{N-1} + B_{N-1}u_{N-1})^T p_N (A_{N-1}x_{N-1} + B_{N-1}u_{N-1})) | z_{N-1}\} \tag{16}$$

The optimal control for the $(N-1)$th stage is obtained by minimizing the quadratic equation (16) with respect to u_{N-1}. Setting

$$\partial J_{N-1,N}(z_{N-1}, u_{N-1})/\partial u_{N-1}$$
$$= E\{ (R u_{N-1} + B_{N-1}{}^T p_N (A_{N-1}x_{N-1} + B_{N-1}u_{N-1})) | z_{N-1}\} = 0,$$

the optimal control law is derived as:

$$u^*_{N-1}(z_{N-1}) = -F_{N-1} E\{x_{N-1} | z_{N-1}\} \tag{17}$$

where $F_{N-1} := [R + E\{B_{N-1}{}^T p_N B_{N-1}\}]^{-1} E\{B_{N-1}{}^T p_N A_{N-1}\}$. The following facts have been used in the above derivation:

- A_{N-1} and B_{N-1} are independent of x_{N-1} and z_{N-1} on the basis of the condition laid out in the proposition and the notation in (8).
- u_{N-1} is a deterministically structured function, i.e., if its argument is deterministic, then its value is also deterministic. The optimal performance is then obtained by substituting (17) in (16).

$$J^*_{N-1}(z_{N-1}) := J_{N-1}(z_N, u^*_{N-1})$$
$$= \frac{1}{2} [E\{ (x_{N-1}{}^T p_{N-1} x_{N-1}) | z_{N-1}\}$$
$$+ E\{x_{N-1}{}^T | z_{N-1}\} S_{N-1} E\{x_{N-1} | z_{N-1}\}] \tag{18}$$

where $p_{N-1} := Q + E\{A_{N-1}{}^T p_N A_{N-1}\}$;
$$S_{N-1} := E\{A_{N-1}{}^T p_N B_{N-1}\}F_{N-1}$$

Note: S_{N-1} has been simplified from the expression

$$F_{N-1}{}^T [R + E\{B_{N-1}{}^T p_N B_{N-1}\}]F_{N-1} - F_{N-1}{}^T E\{B_{N-1}{}^T p_N A_{N-1}\}$$
$$- E\{A_{N-1}{}^T p_N B_{N-1}\}F_{N-1}.$$

Now we step back to the $(N-2)$th stage in order to find the required recursive relationship. Steps similar to those in the $(N-1)$th stage were not followed because of the difficulty

in evaluating $E\{J^*_{N-1}(z_{N-1})|z_{N-2}\}$ where the conditional expectation cannot be readily simplified as explained later. Since $p(k)$ is independent of $p(j)$, $j = k - 1$, $k - 2$, $\cdots$, and is also independent of plant dynamics (following the assumption #6 and #7 at the beginning of this section), the conditional expectation in (10) can be expressed as:

$$E\{x_k|z_k\} = \alpha x_k + (1 - \alpha)[E\{A_{k-1}\}x_{k-1} + E\{B_{k-1}\}u_{k-1}]$$

$$= \alpha[A_{k-1}x_{k-1} + B_{k-1}u_{k-1}]$$

$$+ (1 - \alpha)[E\{A_{k-1}\}x_{k-1} + E\{B_{k-1}\}u_{N-2}]$$

or

$$E\{x_k|z_k\} = \mathbf{A}_{k-1}x_{k-1} + \mathbf{B}_{k-1}u_{k-1} \tag{19}$$

where

$$\mathbf{A}_k := \alpha A_k + (1 - \alpha)E\{A_k\}, \quad \mathbf{B}_k := \alpha B_k + (1 - \alpha)E\{B_k\} \tag{20}$$

The performance cost at the $(N - 2)$th stage follows from (11) as:

$$J_{N-2}(z_{N-2}, u_{N-2}) = E\left\{\left[\frac{1}{2}(x_{N-2}^TQx_{N-2} + u_{N-2}^TRu_{N-2})\right.\right.$$

$$\left.\left. + J^*_{N-1}(Z_{N-1})\right]|z_{N-2}\right\}$$

$$= \frac{1}{2}[E\{x_{N-2}^TQX_{N-2} + u_{N-2}^TRu_{N-2}|z_{N-2}\}$$

$$+ E\{E\{x_{N-1}^T p_{N-1} x_{N-1}|z_{N-1}\}|z_{N-2}\}$$

$$+ E\{[E\{x_{N-1}^T|z_{N-1}\}S_{N-1}E\{x_{N-1}|z_{N-1}\}]|z_{N-2}\}] \tag{21}$$

The second term in (21) is expressed by using lemma 3.1 and the state relationship in (5) as:

$$E\{E\{x_{N-1}^T p_{N-1} x_{N-1}|z_{N-1}\}|z_{N-2}\}$$

$$= E\{(A_{N-2}x_{N-2} + B_{N-2}u_{N-2})^T p_{N-1}$$

$$\times (A_{N-2}x_{N-2} + B_{N-2}u_{N-2})|z_{N-2}\} \tag{22}$$

The result of lemma 3.1 cannot be applied to simplify the third term in (21) because of the quadratic expression involving two conditional expectations. This problem is circumvented by using (19) as follows.

$$E\{[E\{x_{N-1}^T|z_{N-1}\}S_{N-1}E\{x_{N-1}|z_{N-1}\}]|z_{N-2}\}$$

$$= E\{(A_{N-2}x_{N-2} + \mathbf{B}_{N-2}u_{N-2})^TS_{N-1}$$

$$\times (\mathbf{A}_{N-2}x_{N-2} + \mathbf{B}_{N-2}u_{N-2})|z_{N-2}\}$$

$$= E\{x_{N-2}^TE\{A_{N-2}^TS_{N-1}A_{N-2}\}x_{N-2}|z_{N-2}\}$$

$$+ E\{x_{N-2}^T|z_{N-2}\}E\{A_{N-2}^TS_{N-1}B_{N-2}\}u_{N-2}$$

$$+ u_{N-2}^TE\{B_{N-2}^TS_{N-1}A_{N-2}\}E\{x_{N-2}|z_{N-2}\}$$

$$+ u_{N-2}^TE\{B_{N-2}^TS_{N-1}B_{N-2}\}u_{N-2} \tag{23}$$

Now J_{N-2} can be obtained by combining (22) and (23) in (21). Following a similar procedure as in the $(N-1)$th stage, the optimal control law at $(N-2)$th stage is obtained by minimizing the quadratic equation of J_{N-2} with respect to u_{N-2}. Setting

$$\partial J_{N-2,N}(z_{N-2}, u_{N-2})/\partial u_{N-2}$$

$$= Ru_{N-2} + E\{B_{N-2}^TP_{N-1}A_{N-2}\}E\{x_{N-2}|z_{N-2}\}$$

$$+ E\{B_{N-2}^T p_{N-1}B_{N-2}\}u_{N-2}$$

$$+ E\{\mathbf{B}_{N-2}^TS_{N-1}\mathbf{A}_{N-2}\}E\{x_{N-2}|z_{N-2}\}$$

$$+ E\{\mathbf{B}_{N-2}^TS_{N-1}\mathbf{B}_{N-2}\}u_{N-2}$$

$$= 0,$$ the optimal control law is derived as:

$$u^*_{N-2}(z_{N-2}) = -F_{N-2}E\{x_{N-2}|z_{N-2}\} \tag{24}$$

where

$$F_{N-2} := [R + E\{B_{N-2}^T p_{N-1}B_{N-2}\} + E\{\mathbf{B}_{N-2}^TS_{N-1}\mathbf{B}_{N-2}\}]^{-1}$$

$$\times [E\{B_{N-2}^T p_{N-1}A_{N-2}\} + E\{\mathbf{B}_{N-2}^TS_{N-1}\mathbf{A}_{N-2}\}]; \text{ and}$$

p_{N-1} and S_{N-1} are as defined in (18).

The minimum cost J^*_{N-2} is obtained by substituting the expression for u^*_{N-2} in J_{N-2} as follows.

$$J^*_{N-2}(z_{N-2}) = \frac{1}{2}[E\{x_{N-2}^TQX_{N-2}$$

$$+ E\{x_{N-2}^T|z_{N-2}\}F_{N-2}^TRF_{N-2}E\{x_{N-2}|Z_{N-2}\}|z_{N-2}\}$$

$$+ E\{(A_{N-2}x_{N-2} - B_{N-2}F_{N-2}E\{x_{N-2}|z_{N-2}\})^T p_{N-1}(A_{N-2}x_{N-2}$$

$$- B_{N-2}F_{N-2}E\{x_{N-2}|z_{N-2}\}) + (\mathbf{A}_{N-2}x_{N-2}$$

$$- \mathbf{B}_{N-2}F_{N-2}E\{x_{N-2}|z_{N-2}\})^TS_{N-1}$$

$$\times (\mathbf{A}_{N-2}x_{N-2} - \mathbf{B}_{N-2}F_{N-2}E\{x_{N-2}|z_{N-2}\})|z_{N-2}\}]$$

Expanding the above equation and collecting coefficient matrices for similar terms yield

$$J^*_{N-2}(z_{N-2}) = \frac{1}{2}[E\{x_{N-2}^T p_{N-2}x_{N-2}|z_{N-2}\}$$

$$+ E\{x_{N-2}^T|_{N-2}\}S_{N-2}E\{x_{N-2}|z_{N-2}\}] \tag{25}$$

where

$$p_{N-2} := Q + E\{A_{N-2}^T p_{N-1}A_{N-2}\} + E\{\mathbf{A}_{N-2}^TS_{N-1}\mathbf{A}_{N-2}\};$$

$$S_{N-2} := -[E\{A_{N-2}^TP_{N-1}B_{N-2}\} + E\{\mathbf{A}_{N-2}^TS_{N-1}\mathbf{B}_{N-2}\}]F_{N-2}$$

The proof is now complete by applying the method of induction using the results in (24) and (25). ∎

Remark 3.4: The condition that A_k and B_k are independent of $\{A_j, j = k - 1, k - 2, \cdots\}$ and $\{B_j, j = k - 1, k - 2, \cdots\}$, as laid out in Proposition 3.1, holds if the time skew $\Delta_s = 0$ or if Δ_s is a random parameter, i.e., an unknown constant. A weak correlation may exist for a known, non-zero, constant value of Δ_s. In that case, the control law derived in Proposition 3.1 should be sub-optimal. ∎

Remark 3.5: Using (20) the second order statistics of $\mathbf{A}_k$ and $\mathbf{B}_k$ in (24) and (25) can be expressed in terms of α, A_k, and B_k as follows:

$$E\{\mathbf{A}_k^TS_{k+1}\mathbf{A}_k\} = \alpha^2 E\{A_k^T S_{k+1}A_k\}$$

$$+ (1 - \alpha^2)E\{A_k^T\}S_{k+1}E\{A_k\};$$

$$E\{\mathbf{A}_k^T S_{k+1}\mathbf{B}_k\} = \alpha^2 E\{A_k^T S_{k+1}B_k\}$$

$$+ (1 - \alpha^2)E\{A_k^T\}S_{k+1}E\{B_k\};$$

$$E\{\mathbf{B}_k^TS_{k+1}\mathbf{B}_k\} = \alpha^2 E\{B_k^T S_{k+1}B_k\}$$

$$+ (1 - \alpha^2)E\{B_k^T\}S_{k+1}E\{B_k\}. ∎ \tag{26}$$

Remark 3.6: The proposed stochastic regulator algorithm does not guarantee an almost sure performance. This implies that, at certain sample points, the control system performance may not satisfy the specified requirements, and the probability of the ensemble of these sample points may not be zero. However, we have not encountered any such situations in extensive simulation experiments. Some of the simulation results are presented in Part II [8]. ∎

Randomness of the sensor-controller delay, Θ_{sc}, can be eliminated by adjustment of the skew Δ_s between sensor and controller sampling instants. If $\Delta_s > \text{Sup } \delta_{sc}$, then $p(k) = 0$ $\forall k$, which implies that the sensor data always arrive at the controller on time. On the other hand, if $\Delta_s \leq \text{Inf } \delta_{sc}$, then $p(k) = 1$ $\forall k$ which implies that the sensor data are always delayed by one sample. The optimal control laws under these two conditions are presented below.

Corollary 1 to Proposition 3.1: If $p(k) = 0$ $\forall k$ with probability 1, then the optimal control law becomes

$$u^*_k(x_k) = -F_kx_k \text{ for } k < N \tag{27}$$

and the resulting minimum performance cost is

$$J^*_k(x_k) = \frac{1}{2}E\{x_k^T\mathcal{L}_kx_k\} \tag{28}$$

where

$$F_k := [R + E\{B_k{}^T\mathcal{L}_{k+1}B_k\}]^{-1}E\{B_k{}^T\mathcal{L}_{k+1}A_k\}$$

$$\mathcal{L}_k := Q + E\{A_k{}^T\mathcal{L}_{k+1}(A_k - B_kF_k)\} \text{ with } \mathcal{L}_N = P; \text{ and}$$

each equation is evaluated backward from

$$k = N-1, N-2, \cdots.$$

Proof of Corollary 1: Since $p(k) = 0$ $\forall k$ with probability 1, we have $\alpha = 1$ and $z_k = x_k$ $\forall k$, which imply that $A_k = A_k$, $B_k = B_k$, and $E\{x_k | z_k\} = x_k$ $\forall k$. The proof is completed by using these results in Proposition 3.1 and setting $\mathcal{L}_k = (\rho_k + S_k)$. ∎

Corollary 2 to Proposition 3.1: If $p(k) = 1$ $\forall k$ with probability 1, then the optimal control law becomes

$$u^*{}_k(x_{k-1}) = -F_k E\{x_k | x_{k-1}\} \text{ for } k < N \qquad (29)$$

and the resulting minimum performance cost is

$$J^*{}_k(x_{k-1}) = \frac{1}{2}[E\{x_k{}^T \rho_k x_k | x_{k-1}\}$$

$$+ E\{x_k{}^T | x_{k-1}\}S_k E\{x_k | x_{k-1}\}] \qquad (30)$$

where

$$F_k := [R + E\{B_k{}^T \rho_{k+1} B_k\}$$

$$+ E\{B_k{}^T\}S_{k+1}E\{B_k\}]^{-1}[E\{B_k{}^T \rho_{k+1}A_k\}$$

$$+ E\{B_k{}^T\}S_{k+1}E\{A_k\}]$$

$$\rho_k := Q + E\{A_k{}^T \rho_{k+1} A_k\} + E\{A_k{}^T\}S_{k+1}E\{A_k\} \text{ with } \rho_N = P;$$

$$S_k := -[E\{A_k{}^T \rho_{k+1} B_k\}$$

$$+ E\{A_k{}^T\}S_{k+1}E\{B_k\}]F_k \text{ with } S_N = 0; \text{ and}$$

each equation is evaluated backward from $k = N-1$, $N-2$, $\cdots$.

Proof of Corollary 2: Since $p(k) = 1$ $\forall k$ with probability 1, we have $\alpha = 0$ and $z_k = x_{k-1}$ $\forall k$, which imply that $A_k = E\{A_k\}$ and $B_k = E\{B_k\}$ $\forall k$. The proof is completed by using these results in Proposition 3.1. ∎

Corollary 3 to Proposition 3.1: Both ρ_k and the sum $(\rho_k + S_k)$ are positive semi-definite $\forall k$.

Proof of Corollary 3: The optimal cost $J^*{}_k$ in (12) can be expressed in terms of traces of matrices as:

$$J^*{}_k(z_k) = \frac{1}{2}[E\{x_k{}^T \rho_k x_k | z_k\} + E\{x_k{}^T | z_k\}S_k E\{x_k | z_k\}]$$

$$= \frac{1}{2} Tr[\rho_k E\{x_k x_k{}^T | z_k\} + S_k E\{x_k | z_k\}E\{x_k{}^T | z_k\}]$$

$$= \frac{1}{2} Tr[\rho_k (E\{(x_k - E\{x_k | z_k\})(x_k - E\{x_k | z_k\})^T | z_k\}$$

$$+ E\{x_k | z_k\}E\{x_k{}^T | z_k\}) + S_k E\{x_k | z_k\}E\{x_k{}^T | z_k\}]$$

$$= \frac{1}{2} Tr[\rho_k E\{(x_k - E\{x_k | z_k\})(x_k - E\{x_k | z_k\})^T | z_k\}$$

$$+ (\rho_k + S_k)E\{x_k | z_k\}E\{x_k{}^T | z_k\}]$$

Since the optimal cost in (11) and (12) are the expected values of quadratic terms, $J^*{}_k(z_k) \geq 0$ for any given initial condition z_k in the above expression. Therefore, both ρ_k and $(\rho_k + S_k)$ must be positive semi-definite. ∎

Remark 3.7: The results (27) and (28) show that if the time skew Δ_s is adjusted [2] such that $\Delta_s > \text{Sup } \delta_{sc}$, then the optimal control law can be directly obtained from x_k at the expense of the transport delay introduced by Δ_s in the feedback loop. On the other hand, if the above transport delay is avoided by setting Δ_s to zero, the results (29) and (30) are applicable where the control law is derived from $E\{x_k | x_{k-1}\}$. ∎

3.4 Construction of an Observer for State Estimation.

In the previous sections, the optimal control algorithm is derived in a linear state feedback form. A state estimator is needed unless all plant states are measurable and the sensor signal-to-noise ratio is acceptable. The state estimator can be designed by utilizing the knowledge of the system input and output provided that the system is observable. Although the state-of-the-art in this field and the attendant task of robustness analysis are relatively well known, the presence of (network-induced) randomly varying delays in both input and output complicates the problem of state estimation. We discuss in this section how the information available in the communication network can be utilized to solve the problem of state estimation via existing analytical techniques.

To illustrate the concept of observer design, we consider a discretized version of the linear, time-invariant model of the plant as set forth in (3).

$$\xi_{k+1} = a_s \xi_k + \sum_{i=0}^{\ell} b_i{}^k u_{k-i}$$

$$y_{k+1} = c \xi_k \qquad (31)$$

If the sensor and plant are subjected to additive noise and disturbance, then the state and measurement equations should be appropriately modified by including the noise terms. The basic structure of a deterministic (or the steady-state version of a stochastic) observer for the above can be expressed as:

$$\eta_{k+1} = a_s \eta_k + L[y_k - c\eta_k] + \sum_{i=0}^{\ell} b_i{}^k u_{k-i} \qquad (32)$$

where η_k is an estimate of the plant state ξ_k, and L is the observer gain matrix.

Since the pair $\{a_s, c\}$ is observable by the assumption #2 at the beginning of Section 3, the estimated state η_k should asymptotically approach the actual state ξ_k (or its expected value) in the absence of any modeling errors provided that the input matrices are exactly known. To implement this observer, the control input u_k and measurement y_k need to be known up to the kth instant. Since the control and sensor data, $\{u_k\}$ and $\{y_k\}$, are subjected to random delays in ICCS, the observer needs to be implemented as described below.

The observer can be constructed if (i) the sensor and control data, albeit delayed, are not lost during transmission, (ii) the exact value of the random delay $p(k)$, along with the sensor data $y_{k-p(k)}$, is available at the controller at every sampling instant k, and (iii) the arrival instants $\{t_j{}^k\}$ of the control data at the actuator during the kth sampling interval are known to the observer. The first condition is equivalent to the assumption #8 given at the beginning of Section 3. The second condition is easily accomplished by appending the sensor data message with the counter reading of its sequence number modulo ρ, where ρ is an upper bound of $p(k)$. The third condition can be achieved if the instant of completion of transmission of each control data u_k is accurately monitored and the constant delay due to signal propagation from the controller to the actuator and software execution at the actuator is known. Even though the queueing delay is random, the instants of arrival of u_k at the actuator are exactly known at the controller. Since the observer is located at the controller, the input matrices $\{b_i{}^k\}$ and $\{u_k\}$ in (32) are known.

Having known $\{b_i{}^k\}$ and $\{u_k\}$, the estimate η_{k+1} is correct if the sensor data is available up to the kth instant in the correct sequence regardless of any delays. Seen at the controller terminal at the sampling instant k, if the sensor data is delayed by p samples, then η_{k-p} is guaranteed to be available at the controller, and then the control command u_k can be generated.

Remark 3.8: A major factor in the implementation of the observer is the computation load because the observer in (32) requires each of the input matrices $\{b_i{}^k\}$ to be computed on-

line by integrating the matrix exponential in (3) whose limits are t_i^k and t_{i-1}^k. ∎

4 Summary and Conclusions

A state feedback control law has been synthesized by using the dynamic programming and optimality principle on a finite-time horizon. The stochastic control algorithm has been specifically developed for Integrated Communication and Control Systems (ICCS) [1–7] where random delays are induced by the network for exchange of information between the system components. The control law is derived on the basis of the plant dynamics and the statistics of randomly varying network traffic. Specifically, the plant model is augmented in state space to take into account the effects of the delays in the feedback loop.

In general, the proposed method can be used for synthesis of sampled data control of dynamics systems with random parameters in their governing equations. The resulting control algorithm apparently satisfies the property of separation (i.e., the state feedback control law can be separately formulated from state estimation) under the assumptions #6 and #7 at the beginning of Section 3. However, this concept may not comply with the principle of certainty equivalence [18] which allows optimal design by separately considering the controller and observer based on the deterministic part of the plant model. That is, the optimal control law could be different if an equivalent deterministic model, $E[x_{k+1}] = \Gamma[A_k] E[x_k] + E[B_k] u_k$ is used instead of the stochastic model in Eq. (5).

If some of the plant states are not measurable or if the sensor signal-to-noise ratio is unacceptable, the proposed controller shall require a state estimator like any other state feedback controller. However, the task of state estimation in this case is more complex than that under the non-delayed environment of conventional state feedback control systems because of random arrival of control commands at the actuator. Although an observer that is located at the controller can be constructed for state estimation, its implementation may impose a significant amount of real-time computation at the controller computer.

References

1 Halevi, Y., and Ray, A., "Integrated Communication and Control Systems: Part I—Analysis," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Dec. 1988, pp. 367–373.

2 Ray, A., and Halevi, Y., "Integrated Communication and Control Systems: Part II—Design Considerations," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Dec. 1988, pp. 374–381.

3 Liou, L-W., and Ray, A., "Integrated Communication and Control Systems: Part III—Non-Identical Sensor and Controller Sampling," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Sept. 1990, pp. 364–371.

4 Liou, L-W., and Ray, A., "On Modeling of Integrated Communication and Control Systems," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Dec. 1990, pp. 790–794.

5 Luck, R., and Ray, A., "An Observer-Based Compensator for Distributed Delays," Automatica, Vol. 26, No. 5, Sept. 1990, pp. 903–908.

6 Luck, R., Ray, A., and Halevi, Y., "Observability under Recurrent Loss of Data," AIAA Journal of Guidance, Control, and Dynamics, in press.

7 Halevi, Y., and Ray, A., "Performance Analysis of Integrated Communication and Control System Networks," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Sept. 1990, pp. 365–371.

8 Liou, L-W., and Ray, A., "A Stochastic Regulator for Integrated Communication and Control Systems: Part II—Numerical Analysis and Simulation," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, published in this issue, pp. 612–619.

9 Banks, H. T., and Manitius, A., "Application of Abstract Variational Theory to Hereditary Systems—A Survey," IEEE Trans. Automatic Control, AC-19, Oct. 1974, pp. 524–533.

10 Manitius, A. Z., and Olbrot, A. W., "Finite Spectrum Assignment Problem for Systems with Delays," IEEE Trans. Automatic Control, AC-24, No. 4, Aug. 1979, pp. 541–553.

11 Buckalo, A., "Explicit Conditions for Controllability of Linear Systems with Time Lag," IEEE Trans. Automatic Control, AC-13, Apr. 1968, pp. 193–195.

12 Soliman, M. A., and Ray, W. H., "Optimal Control of Multivariable Systems with Pure Time Delays," Automatica, Vol. 7, 1971, pp. 681–689.

13 Koivo, H. N., and Lee, E. B., "Controller Synthesis for Linear Systems with Retarded State and Control Variables and Quadratic Cost," Automatica, Vol. 8, 1972, pp. 203–208.

14 Diduch, C. P., and Doraiswami, R., "Controllability and Observability of Multivariable Sampled Data Systems with Input and Output Delays," Proc. American Control Conference, Pittsburgh, PA, June 1989.

15 Chyung, D. H., "Discrete Optimal Systems with Time Delay," IEEE Trans. Automatic Control, AC-13, 1968, p. 117.

16 Drouin, M., Abou-Kandil, H., and Bertrand, P., "Feedback Control for Linear Discrete-time Systems with Time Delays," Automatica, Vol. 21, No. 3, 1985, pp. 323–327.

17 Arthur, W. B., "Control of Linear Processes with Distributed Lags Using Dynamic Programming from First Principles," Journal of Optimization Theory and Applications, Vol. 23, No. 3, 1977, pp. 429–443.

18 Maybeck, P. S., Stochastic Models, Estimation and Control, Vol. 3, Academic Press, New York, 1982.

19 Kwakernaak, H., and Sivan, R., Linear Optimal Control Systems, Wiley Interscience, New York, 1972.

20 Bitmead, R. R., and Anderson, B. D. O., "Lyapunov Techniques for the Exponential Stability of Linear Difference Equations with Random Coefficients," IEEE Transactions on Automatic Control, Vol. AC-25, No. 4, 1980, pp. 782–787.

21 De Koning, W. L., "Optimal Control, Estimation, and Compensation of Linear Discrete-Time Systems with Stochastic Parameters," Control and Dynamic Systems: Advances in Theory and Applications, Academic Press, San Diego, CA, 1988.

22 Akoi, M., Optimization of Stochastic Systems, Academic Press, New York, 1967.

23 Ray, A., "Performance Analysis of Medium Access Control Protocols for Distributed Digital Avionics," ASME JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL, Dec. 1987, pp. 370–377.

24 Ray, A., "Distributed Data Communication Networks for Real-Time Process Control," Chemical Engineering Communication, Vol. 65, Mar. 1988, pp. 139–154.