A Stochastic Regulator for Integrated Communication and Control Systems: Part II—Numerical Analysis and Simulation[1]

Luen-Woei Liou

Asok Ray

Mechanical Engineering Department,
The Pennsylvania State University,
University Park, PA

A state feedback control law has been derived in Part I [1] of this two-part paper on the basis of an augmented plant model [2, 3, 4] that accounts for the randomly varying delays induced by the network in Integrated Communication and Control Systems (ICCS). The control algorithm was formulated as a linear quadratic regulator problem and then solved using the principle of dynamic programming and optimality. This paper, which is the second of two parts, presents (i) a numerical procedure for synthesizing the control parameters and (ii) results of simulation experiments for verification of the above control law using the flight dynamic model of an advanced aircraft. This two-part paper is concluded with recommendations for future work.

1 Introduction

Integrated Communication and Control Systems (ICCS) utilize time-division-multiplexed computer networks to exchange information between spatially distributed plant components for executing diverse control and decision-making functions. A major problem in ICCS design is to compensate for the randomly varying distributed delays that are induced by the network. To this effect, a state feedback control law has been formulated in Part I [1] of this two-part paper on the basis of an augmented plant model [2, 3, 4] that accounts for the randomly varying induced delays. The stochastic control algorithm is derived in a recursive form using the principle of dynamic programming and optimality. One of the major assumptions in the control law formulation is that the network-induced delays are bounded by the sampling interval which is identical for the sensor and controller in the present context. This assumption is justified in view of the fact that the common practice in ICCS design is to maintain a safe margin between the offered traffic and its critical value [5, 6]. The control law can be simplified if the sensor and controller sampling instants are synchronized such that the respective skew Δ_s is maintained above the specified threshold of δ_{max} (see the nomenclature).

This paper, which is the second of two parts, presents a numerical procedure for synthesizing the above control law (i.e., synthesis of the optimal feedback gain matrix), and the results of simulation experiments for its verification. The stability of the numerical procedure is established via a proposition which shows convergence of the feedback gain matrix to a constant matrix when the time horizon approaches to infinity. The above numerical procedure is based on the following information: (i) a plant dynamic model; (ii) known statistical characteristics of the network traffic; and (iii) specifications of the network access protocol. Performance evaluation of the proposed control algorithm is done by simulation of the longitudinal motion dynamics of an advanced aircraft.

The paper is organized in five sections including the introduction. Section 2 describes the procedures for numerical analyses including solution of the matrix difference equations and computation of the associated expected values of integrals with stochastic limits. Section 3 discusses stability and robustness of the proposed control algorithm. Section 4 presents simulation of the flight control system of an advanced aircraft. This two-part paper is summarized and concluded in Section 5 with recommendations for future research.

2 Numerical Procedure for Control Synthesis

The control algorithm, derived in Section 3 of Part I [1], is expressed in terms of vector-valued difference equations for which no closed-form solutions apparently exist. Therefore, numerical techniques are necessary to solve these equations so that the pertinent parameters can be evaluated for the control system synthesis. This would require computation of the expected values of integrals whose limits are stochastic processes and integrands contain matrix exponentials. Specifically, the tasks of numerical computations are directed toward evalua-

[1]This research was supported in part by: Office of Naval Research under grant No. N0014-90-J-1513; NASA Lewis Research Center under grant No. NAG 3-323; National Science Foundation under grant No. DMC-8707648.

Contributed by the Dynamic Systems and Control Division for publication in the JOURNAL OF DYNAMIC SYSTEMS, MEASUREMENT, AND CONTROL. Manuscript received by the Dynamic Systems and Control Division March 27, 1990; revised manuscript received January 9, 1991. Associate Editor: A. G. Ulsoy.

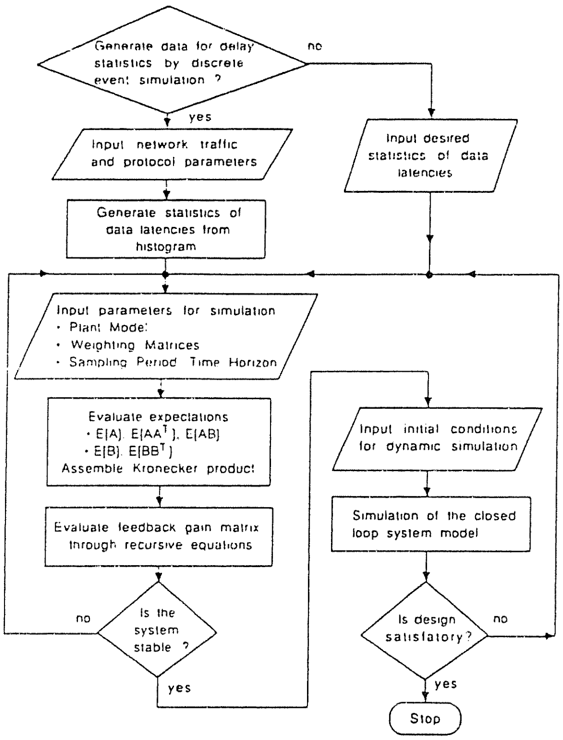

Fig. 1 Flow diagram for the numerical procedure

tion of: (i) probability distribution functions of network-induced delays; and (ii) elements of the optimal feedback gain matrix. A flow diagram of the numerical procedure for off-line design of the proposed stochastic regulator is given in Fig. 1.

2.1 Numerical Evaluation of Probability Distribution Functions. Although analytic evaluation of the first moment of network-induced delays in ICCS has been reported [7] under certain restrictions, their probability distribution functions are difficult, if not impossible, to derive because of the general distribution of data traffic and complexity of the communication protocol. Therefore, numerical techniques are the only recourse to identification of the probability distribution functions of the network-induced delays. To this effect, a database of random delays should be generated via discrete-event simulation [8] of the network traffic and the communication protocol. Finally, the distribution function is numerically generated from the histogram of the delays in the database. This procedure does not require the assumption of ergodicity of the delays because a number of simulation runs can be conducted with several seed numbers in the random number generators, which are equivalent to different sample points.

One or more likely scenarios of network traffic are required in the above off-line design procedure for identification of a probability distribution function. If these scenarios are difficult to generate, then it is logical to synthesize the control system on the basis of postulated probability distributions of the induced delays due to the heterogeneous traffic in the network. For example, in the simulation presented in Section 4, delays are assumed to be uniformly distributed in $(0, T]$ and the step of numerical evaluation of the distribution functions is bypassed as seen in Fig. 1. (Note: The assumption of uniform distribution is somewhat conservative in the sense that the probability density function of network-induced delays is largely a decreasing function of its argument in an ICCS network.)

Remark 2.1: The control parameters, obtained from off-line design, could serve as default values. The network management module may continuously monitor the traffic and then numerically generate the probability distribution functions of induced delays on the basis of collected data under the assumption that these delays are ergodic processes. This step does not require a discrete-event simulation but there is always a waiting time for data collection and identification of the distribution function from the histogram. If the changes in network traffic characteristics are slow compared to the process dynamics, then the above operations can be executed on-line. The control parameters can then be updated using the current statistics of delays, and this procedure is essentially analogous to the well-known practice of gain scheduling [9] for adaptation to relatively slow process dynamics. ■

Nomenclature (For Parts I and II)

a = plant system matrix $(n \times n)$ in continuous time
a_s = plant state transition matrix $(n \times n)$
A_k = augmented plant state transition matrix $(n \times \ell m) \times (n + \ell m)$
b = plant input matrix $(n \times m)$ in continuous time
b_i^k = plant input matrix $(n \times m)$ in discrete-time
B_k = augmented plant input matrix $(n + \ell m) \times m$ in convoluted form
c = plant output matrix $(r \times n)$
F_k = feedback gain matrix $m \times (n + \ell m)$
J', J'' = performance cost
J^* = optimal performance cost w.r.t. $\{u_k\}$
$J^{*\dagger}$ = optimal performance cost w.r.t. $\{u_k\}$ and Δ_s
ℓ = maximum # of delayed actuator command in one sampling time
$\mathcal{L}$ = cost matrix for the case $p(k) = 0\ \forall k$
L = observer gain matrix $(n \times r)$
p = maximum delay (# of samples) of sensor data arrival
N = time horizon (# of sampling periods) over which the performance cost is evaluated
$p(k)$ = sensor delay index (# of samples), i.e., y_k to be used for $u_{k-p(k)}$
P = augmented terminal state weighting matrix
p_k = cost matrix
Q = augmented state weighting matrix
R = weighting matrix for control
S_k = cost matrix
T = sampling period
T_c = controller sampling period
T_s = sensor sampling period
t^k_i = instant of ith control data arrival at the actuator during the kth sample
u = plant input vector $(m \times 1)$
x = augmented plant state vector $((n + \ell m) \times 1)$
y = plant output vector, i.e., generated sensor data $(r \times 1)$
z = delayed output vector $(r \times 1)$
α = probability of sensor data arrived on time, i.e, $p(k) = 0$
η = estimate of plant state vector $(n \times 1)$
ξ = plant state vector $(n \times 1)$
Δ_s = time skew between the sensor and controller sampling instants
δ_{ca} = controller-to-actuator data latency
$\delta_{\max}$ = supremum of sensor-to-controller data latency
$\delta_{\min}$ = infimum of sensor-to-controller data latency
δ_{sc} = sensor-to-controller data latency
Θ_{sc} = sensor-controller delay
Θ_{ca} = controller-actuator delay

2.2 Numerical Evaluation of the Optimal Feedback Gain Matrix. The optimal feedback gain matrix is numerically evaluated in the following three steps:

Step 1: The most critical part in the numerical procedure is accurate and efficient computation of the matrix exponential $\exp(aT)$ and integration of $\exp(a(T-\tau))b$. They are needed for generating the matrices A_k and B_k in the augmented plant model. Especially the matrix integration may have to be carried out at every sample k. Advanced numerical techniques for evaluating the above functions are reported in [10, 11]. However, if the matrix dimension is small as it could often be for individual feedback loops in ICCS, then the integration can be obtained by direct power series expansion followed by appropriate truncation of high order terms:

$$\int_0^t \exp[a\tau]d\tau\, b = (t + at^2/2! + a^2t^3/3! + \cdots)b \qquad (1)$$

In the simulation experiments reported in Section 4, the matrices are of small dimensions and therefore the power series method has been adopted. ■

Step 2: The next step is to numerically evaluate the expectation of the stochastic matrix product $A_k^T \rho_k B_k$ where the individual matrices are defined in the nomenclature as well as in Section 3 of Part I [1]. ρ_k is time-dependent and deterministic, and A_k and B_k are correlated. However, since $\{A_k\}$ and $\{B_k\}$ are identically distributed for every k as a consequence of the assumption #6 in Section 3 of Part I, the numerical integration for evaluating the expectations of A_k, B_k, and $A_k^T \rho_k B_k$ needs to be carried only once at the beginning. This matrix product requires a manipulation by using the concepts of Kronecker product $\otimes$ and stacking operator $\P(\cdot)$ [12] which are defined below.

$$\Phi \otimes \Psi := \begin{pmatrix} \phi_{11}\Psi & \phi_{12}\Psi & \cdots & \phi_{1n}\Psi \\ \phi_{21}\Psi & \phi_{22}\Psi & \cdots & \phi_{2n}\Psi \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \phi_{m1}\Psi & \phi_{m2}\Psi & \cdot & \phi_{mn}\Psi \end{pmatrix},$$

$$\text{and } \P(\Phi) := [\Phi_1^T \Phi_2^T \cdots \Phi_1^T]^T$$

where $\{\phi_{ij}\}$ are elements of Φ and $\{\Phi_i\}$ are columns of Φ. The following relationship based on $\P(\cdot)$ and $\otimes$ holds:

$$\P(\Phi \rho_k \Psi) = (\Psi^T \otimes \Phi)\P(\rho_k) \qquad (2)$$

The quantity $E[B_k^T \otimes A_k]$ needs to be evaluated only once and then stored in memory. Later $E\{A_k^T \rho_k B_k\}$ is calculated $\forall k$ using (2). An appropriate numerical integration routine such as the Gaussian method [13] can be used for evaluation of quantities like $E\{A_k\}$, $E\{B_k\}$, and $E\{A_k^T \rho_k B_k\}$. Since the Gaussian algorithm is basically a weighted sum of the values of the function to be integrated at specific points, it does not pose any difficulty after the matrix exponential term, its integration, and the distribution function of delays are available.

Step 3: The recursive calculations of the cost matrices $\{\rho_k\}$ and $\{S_k\}$ and the feedback gain matrix $\{F_k\}$ are computationally straight-forward after the augmented plant model and the associated expectations are obtained as described above.

Remark 2.2: If the skew Δ_s between the sensor and controller sampling instants is maintained above the threshold of δ_{max}, then $p(k)=0\ \forall k$, i.e., the kth sensor data are always available for computing the control signal u_k. In that case, the results of Corollary 1 to Proposition 3.1 in Part I [1] are directly applicable, and the computational requirement in the above Step 3 is significantly reduced. ■

2.3 Numerical Evaluation of the Optimal Skew Δ_s. Remark 2.2 above justifies the need for maintaining Δ_s (which is slowly varying relative to process dynamics [2, 3]) above the threshold level. However, since Δ_s introduces a transport delay that degrades the system dynamic performance, it is logical to optimize Δ_s by minimizing the performance cost as:

$$J^{*\dagger}_0(z_0) = \min_{\Delta_s} J^*_0(z_0) \qquad (3)$$

where the superscripts * and † denote optimality relative to $\{u_k\}$ and Δ_s, respectively, and $J^*_0(z_0)$ is obtained using the result derived in Section 3.3 of Part I [1]. Because of the complexity of the dependence of $J^*_0(z_0)$ on Δ_s, an analytical relationship for obtaining $J^{*\dagger}_0(z_0)$ in (3) is apparently untractable. Instead, a direct numerical search is a viable approach to achieve this goal. A procedure is outlined in the following.

If the initial value of the sensor delay index $p(0)=0$, then $z_0=x_0$. For a given Δ_s and x_0, the optimal cost at $k=0$ as defined in Eq. (28) of Part I [1] is:

$$J^*_0(x_0) = 1/2\, x_0^T(\rho_0 + S_0)x_0 \qquad (4)$$

We assume that the time horizon N is large enough such that the backward difference equations in the control algorithm converge. (Criteria for convergence are presented later in Section 3.) With $\rho_0=\rho$ and $S_0=S$ and the initial condition being arbitrarily specified, the optimal performance $J^{*\dagger}_0(x_0)$ with respect to Δ_s is obtained by substituting (4) in (3).

The above task of optimization can be recast as a minmax problem where the norm (spectral radius) of the cost matrix $(\rho+S)$ serves as a general measure of the size of the cost. The optimal cost $J^*_0(x_0)$ in (4) is a monotonic function of skew Δ_s due to the fact that inclusion of the model-based prediction

$$E[A_{k-1}]x_{k-1} + E[B_{k-1}]u_{k-1} \text{ if } p(k)=1$$

in Eq. (10) of Part I [1] overcomes the disadvantage of a small skew causing the sensor data to be delayed by one sample; otherwise, the skew should be optimized to reduce the detrimental effects of the loop delay at the minimum. The latter option to avoid the model-based prediction is attractive from the point of view of robust controller design in the presence of disturbances and plant modeling uncertainties as well as for avoiding additional real-time computations. This subject is further discussed in Section 5 as the topic #4 of future research.

Remark 2.3: If all plant states are measurable and the sensor signal-to-noise ratio is acceptable, then the model-based prediction is exercised only when the sensor data are delayed, i.e., $p(k)=1$. Therefore, the proposed algorithm should be less sensitive to modeling uncertainties than the multi-step compensator proposed by Luck and Ray [14, 15]. ■

3 Stability and Convergence of the Numerical Procedure

The proposed stochastic regulator algorithm [1] has been formulated as a finite-time problem. Therefore, its properties as the time horizon $N\to\infty$ need to be evaluated for stability investigation. Since the matrices A_k and B_k in the augmented plant model (see Eq. (5) in Part I) have stochastic elements, conventional definitions and concepts of deterministic stability do not hold. Several concepts of stochastic stability [16] were examined to this effect. For example, the almost sure (a.s.) stability concept would possibly provide sufficiency conditions [17] that are apparently too restrictive to be useful for ICCS design. We adopted the mean square (m.s.) stability concept which is mathematically tractable and yields stable numerical solutions for the controller parameters. The objective is to establish the criteria for convergence of the cost matrices ρ and S which are the key elements for numerical evaluation of the feedback gain matrix F.

Definition 3.1: A homogeneous system $x_{k+1}=A_k x_k$ is mean square stable if $E\{\|x_k\|^2\}\to 0$ as $k\to\infty$ for all x_0 where $\|\bullet\|$ indicates the natural norm. ■

Definition 3.2: A system $x_{k+1}=A_k x_k+B_k u_k$ is mean square stabilizable if ∃ a gain matrix F such that $x_{k+1}=(A_k-B_kF)x_k$ is mean square stable. ■

Now we proceed to establish sufficiency conditions for the cost matrices ρ_0 and S_0 to converge as the time horizon $N\to\infty$. The following proposition is based on De Koning's work [18].

Proposition 3.1: Cost matrices ρ_0 and S_0 converge to the limit matrices ρ and S, respectively, as $N\to\infty$ in the proposed stochastic regulator algorithm [1] if: (i) the augmented plant model $x_{k+1}=A_k x_k+B_k u_k$ is mean square stabilizable; and (ii) $\{A_k\}$ and $\{B_k\}$ are sequences of independent and identically distributed (i.i.d.) stochastic matrices.

Proof of Proposition 3.1: Let Σ^n denote the linear space of $n\times n$ real symmetric positive semi-definite matrices where n is the dimension of the plant model. We define the following transformation in terms of a constant matrix F:

$$\Xi: \Sigma^n-\Sigma^n \text{ such that } \Xi M:=E\{(A_k-B_kF)^T M(A_k-B_kF)\}\ \forall M\in\Sigma^n$$

Since $\{A_k\}$ and $\{B_k\}$ are i.i.d. and F is constant, Ξ is invariant relative to k. It follows from Definition 3.1 that $x_{k+1}=(A_k-B_kF)x_k$ is mean square stable if and only if $\rho(\Xi)<1$ where $\rho(\bullet)$ is the spectral radius. The following lemma is needed to prove the proposition.

Lemma 3.1: The performance cost in Eq. (1) of Part I [1] is bounded, i.e.,

$$J^*_0(z_0)=E\left\{\sum_{i=0}^{N}(x_i^TQx_i+u_i^TRu_i)\,|\,z_k\right\}<\infty$$

Proof of Lemma 3.1: Since the plant is mean square stabilizable, ∃ F such that (A_k-B_kF) is mean square stable, i.e., $\rho(\Xi)<1$. The first part of the above performance cost involving x_k is bounded because

$$E\left\{\sum_{k=0}^{\infty}x_k^TQx_k\right\}=x_0^T[Q+E\{(A_k-B_kF)^TQ(A_k-B_kF)+((A_k-B_kF)^T)^2Q(A_k-B_kF)^2+\cdots\}]x_0<\infty\ \forall x_0$$

Since $u_k:=-F\,x_k$, the infinite summation involving u_k also converges. Therefore, the cost $J^*_0(z_0)$ is bounded. ■

Following Eq. (14) of Part I, the expression of the optimal cost is:

$$J^*_0(z_0)=1/2[E\{x_0^T\rho_0x_0|z_0\}+E\{x_0^T|z_0\}S_0E\{x_0|z_0\}]$$
$$=1/2Tr[\rho_0E\{(x_0-E\{x_0|z_0\})(x_0-E\{x_0|z_0\})^T|z_0\}+(\rho_0+S_0)E\{x_0|z_0\}E\{x_0^T|z_0\}]$$

Since both ρ_0 and (ρ_0+S_0) are positive semi-definite by Corollary 3 to Proposition 3.1 in Part I and J_0^* is bounded by lemma 3.1, ρ_0 and (ρ_0+S_0) form bounded sequences of non-decreasing matrices as N increases. Therefore, ρ_0 and (ρ_0+S_0) converge as $N\to\infty$. Consequently, S_0 must converge. ■

A consequence of Proposition 3.1 is that the cost matrices converge if the augmented system is mean square stabilizable. In this case, the limits ρ and S of the cost matrices can be evaluated by substituting them on both sides of the recursive relations that define these matrices in Part I. Thus ρ and S can be evaluated either by directly solving these simultaneous matrix algebraic equations or via the recursive relations until a desired convergence criterion is satisfied. Once ρ and S are obtained, the steady state feedback gain matrix F can be evaluated accordingly. Then the mean square stability can be verified by checking that $\rho(\Xi)<1$. If this condition is not satisfied, then the procedure is repeated with a new set of weighting matrices as shown schematically in Fig. 1.

4 Simulation of a Flight Control System

The proposed control algorithm has been verified by simulation of the longitudinal motion dynamics of an advanced aircraft. The flight dynamic model is similar to that reported in [5] except that the flight controller has been replaced by the control algorithm developed in Part I [1]. The state-variable model of flight dynamics in continuous time is described below.

Plant Variables and Parameters:

- δ_a = elevator command, i.e., input to the actuator (radian)
- δ_e = elevator deflection, i.e., actuator output (radian)
- W = normal component of linear velocity at the center of mass (ft/s)
- q = pitch rate about the center of mass (radian/s)
- α = angle of attack (radian)
- A_n = normal component of linear acceleration at the sensor location (units of g)
- g = acceleration due to gravity = 32.2 ft/s² (9.81 m/s²)

The dimensional stability derivatives [19] for longitudinal motion dynamics were selected as:

$$\begin{aligned}
Z_{de}&:=(\partial Z/\partial\delta_e)/m = -202.28\ \text{ft/s}^2\ (-61.655\ \text{m/s}^2),\\
Z_q&:=(\partial Z/\partial q)/m = -16.837\ \text{ft/s}\ (-5.132\ \text{m/s}),\\
Z_w&:=(\partial Z/\partial W)/m = -3.1332\ \text{s}^{-1},\\
M_{de}&:=(\partial M/\partial\delta_e)/I_y = -40.465\ \text{s}^{-2},\\
M_q&:=(\partial M/\partial q)/I_y = -2.6864\ \text{s}^{-1},\\
M_w&:=(\partial M/\partial W)/I_y = -0.01429\ (\text{s-ft})^{-1}\ (-0.04688\ (\text{s-m})^{-1}),\\
M_{wd}&:=(\partial M/\partial W)/I_y = -0.00115\ \text{ft}^{-1}\ (-0.00377\ \text{m}^{-1}).
\end{aligned}$$

where

- M is the pitch moment (ft² lbm s⁻²) (m² kgm s⁻²);
- Z is the normal component of the aerodynamic force (ft lbm s⁻²) (m kgm s⁻²);
- m is the lumped mass of the aircraft (lbm) (kgm); and
- I_y is the moment of inertia about the pitching axis (ft² lbm) (m² kgm).

Other constant parameters were:

- ℓ = distance between the center of gravity of the airframe and the accelerometer (12.268 ft) (3.7393 m)
- τ = (actuator time constant (0.05 s))
- U_o = reference flight speed (1005.3 ft/s) (306.42 m/s)

Longitudinal Motion Dynamics in the Continuous Time Domain:

$$d\xi/dt=a\,\xi+b\,u;\ y=c\,\xi \tag{5}$$

where $\xi=[\delta_e\ W\ q]^T$, $u=\delta_a$, $y=[\alpha\ A_n\ q]^T$, and

$$a=\begin{pmatrix}-\tau^{-1}&0&0\\Z_{de}&Z_w&S_0\\S_1&S_2&S_3\end{pmatrix},\ b=\begin{pmatrix}\tau^{-1}\\0\\0\end{pmatrix},$$

$$c=\begin{pmatrix}0&U_o^{-1}&0\\-S_4&-S_5&-S_6\\0&0&1\end{pmatrix},$$

$S_0:=(Z_q+U_o)$, $S_1:=(M_{de}+M_{wd}Z_{de})$, $S_2:=(M_w+M_{wd}Z_w)$, $S_3:=(M_q+M_{wd}S_0)$, $S_4:=(Z_{de}+\ell S_1)/g$, $S_5:=(Z_w+\ell S_2)/g$, $S_6:=(Z_q+\ell S_3)/g$.

The discrete-time realization of the above continuous-time model for a sampling time T and arrival instants $\{t_i^k\}$ of the controller data at the actuator is:

$$\xi_{k+1}=a_s\,\xi_k+\sum_{i=0}^{\ell}b_i^k\,u_{k-i} \tag{6}$$

where

$a_s := \exp[aT]$, $b_i^k := \int_{t_i^k}^{t_{i-1}^k} \exp[-a(T-\tau)]d\tau\, b$, and

$$t_{-1}^k := T \text{ and } t_\ell^k := 0.$$

We assume that the data latencies, $\{\delta_{ca}^k\}$ and $\{\delta_{sc}^k\}$, are white sequences that are mutually independent and uniformly distributed in the interval $(0, T]$. This implies that $\ell = 2$ and $p(k) \in \{0, 1\}\ \forall k$. Next, the model in (6) is augmented to account for the controller-actuator delay as follows:

$$x_{k+1} = A_k x_k + B_k u_k \tag{7}$$

where $x_k^T := [\xi_k^T | u_{k-1} | u_{k-2}]^T$,

$$A_k := \begin{pmatrix} a_s & b_1 & b_2 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \text{ and } B_k := \begin{pmatrix} b_0 \\ 1 \\ 0 \end{pmatrix}$$

The quadratic performance cost is selected to minimize the deviation from the zero state and control effort:

$$J_N'' := 1/2E\left\{\xi_N^T P'' \xi_j + \sum_{j=0}^{N-1} (\xi_j^T Q'' \xi_j + R u_j^2)\right\}$$

$$= 1/2E\left\{x_N^T P x_N + \sum_{j=0}^{N-1} (x_j^T Q x_j + R u_j^2)\right\} \tag{8}$$

where

$$P := \begin{pmatrix} P'' & 0 \\ 0 & 0 \end{pmatrix}, \text{ and } Q = \begin{pmatrix} Q'' & 0 \\ 0 & 0 \end{pmatrix}$$

The optimal control law that minimizes the above performance cost is given by the following set of equations [1]:

$$u^*_k(z_k) = -F_k E\{x_k | z_k\} \text{ for } k < N \tag{9}$$

and the resulting minimum performance cost is

$$J^*_k(z_k) = 1/2[E\{x_k^T \mathrm{p}_k x_k | z_k\} + E\{x_k^T | z_k\} S_k E\{x_k | z_k\}] \tag{10}$$

where

$z_k := x_{k-p(k)}$;
$F_k := [R + E\{B_k^T \mathrm{p}_k B_k\} + E\{B_k^T S_k B_k\}]^{-1} [E\{B_k^T \mathrm{p}_k A_k\} + E\{B_k^T S_k \dot{A}_k\}]$
$\mathrm{p}_k := Q + E\{A_k^T \mathrm{p}_{k+1} A_k\} + E\{\dot{A}_k^T S_{k+1} \dot{A}_k\}$ with $\mathrm{p}_N = P$;
$S_k := [E\{A_k^T \mathrm{p}_{k+1} B_k\} + E\{\dot{A}_k^T S_{k+1} B_k\}] F_k$ with $S_N = 0$;
$\dot{A}_k := \alpha A_k + (1-\alpha) E\{A_k\}$ and $B_k := \alpha B_k + (1-\alpha) E\{B_k\}$;
$\alpha := Pr\{p(k) = 0\}$ and $(1-\alpha) = Pr\{p(k) = 1\}$;
$p(k) = 0$ if kth sensor data arrives at controller on time; $p(k) = 1$ otherwise.

The estimator for generating $E\{x_k | z_k\}$ is formulated through the state transition Eq. (7):

$$E\{x_k | z_k\} = \begin{cases} z_k & \text{if } p(k) = 0 \\ E\{A_{k-1}\} z_k + E\{B_{k-1}\} u_{k-1} & \text{if } p(k) = 1 \end{cases} \tag{11}$$

4.1 Simulation Results and Discussions. In the simulation experiment, the following parameters were used for designing the feedback control system:

$$T = 0.025 \text{ s};\ P'' = 0;\ Q'' = c^T c;\ R = [0.01];\ \text{and } N = 40.$$

The feedback gain matrix sequence $\{F_k: k = N-1, N-2, \cdots\}$ was computed using the above data with Δ_s as a constant parameter for the flight dynamic system subject to the following four scenarios of delays:

Case #1: Distributed random delays derived from $\{\delta_{ca}^k\}$ and $\{\delta_{sc}^k\}$;
Case #2: No delays;
Case #3: Constant loop delay of one sampling period;
Case #4: Constant loop delay of two sampling periods.

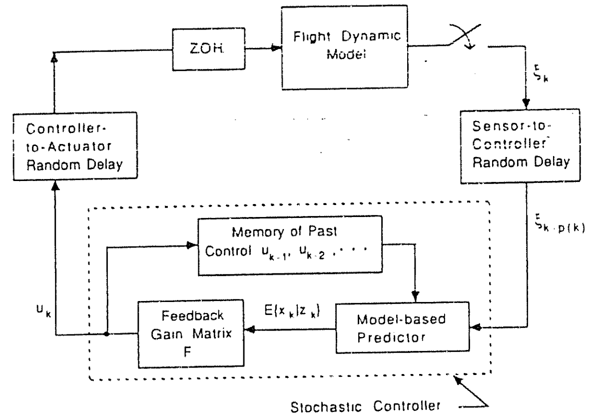

Fig. 2 Schematic diagram of flight dynamic simulation

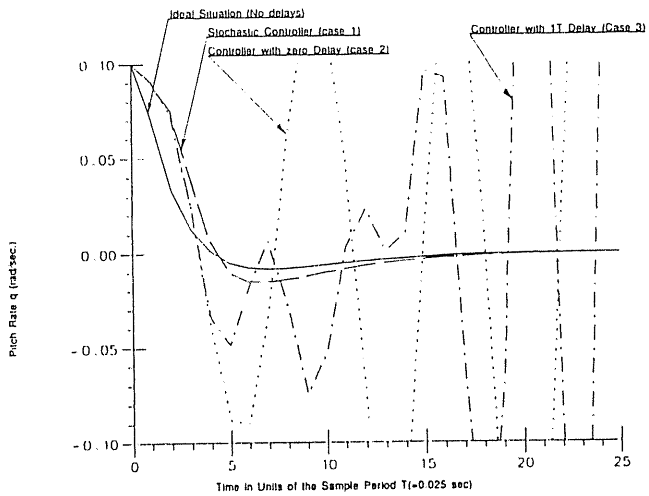

Fig. 3 Transient response of the flight control system for $\Delta_s/T = 0$

The feedback gains for above four cases were employed in the simulation of the flight control system where the feedback system was subjected to random delays. The schematic diagram for the closed loop control system simulation is represented in Fig. 2. The initial condition was set to $x_0 = [0\ 0\ 0.1\ 0\ 0]^T$, i.e., the pitch rate q had an initial value of 0.1 rad/s while the other state variables and past control inputs were set to 0.

With $\Delta_s = 0$, i.e., perfect synchronism of the sensor and controller sampling instants, the transient responses of pitch rate are given in Fig. 3 for the three feedback gain matrices obtained under the cases #1, #2, and #3. The response for the case #4 is omitted because of immediate divergence. For comparison we have also plotted the response of the ideal situation, i.e., the nondelayed system with the gain matrix under the case #2, which is essentially the linear quadratic regulator (LQR) problem in a deterministic setting. Figure 4 presents the transient responses of pitch rate under the same conditions as in Fig. 3 with the exception that $\Delta_s = 0.6T$ was used in the derivation of the gain matrices as well as for simulation. Again, the responses when putting matrices obtained from cases #3 and #4 are omitted because of immediate divergence. The stochastic controller derived under case #1 also yields a less oscillatory response than others at other values of Δ_s.

The simulation results show that, although the LQR controller performs very well in the absence of delays, it yields a poor performance (in the sense of being oscillatory or even unstable) for different values of Δ_s when the plant is subjected to random delays. On the other hand, if a conservative strategy is adopted to obtain the feedback gain matrix assuming a constant delay amounting to the maximum of the stochastic delays, then also the control system performance is unsatis-

Table 1 Feedback gain matrices for the flight dynamic system

Description of Synthesis	Δ_s/T	Steady-state values of the (1×5) feedback gain matrix				
Stochastic controller	0.6	1.8880	−0.00328	−0.92285	0.89091	0.16988
LQR with no delay	0.6	3.8057	−0.00767	−1.8636	1.1409	0.00000
LQR with loop delay T	0.6	3.8129	−0.00661	−1.8619	1.9027	1.1436
LQR with loop delay $2T$	0.6	2.0247	−0.00313	−0.98532	1.0080	1.0124

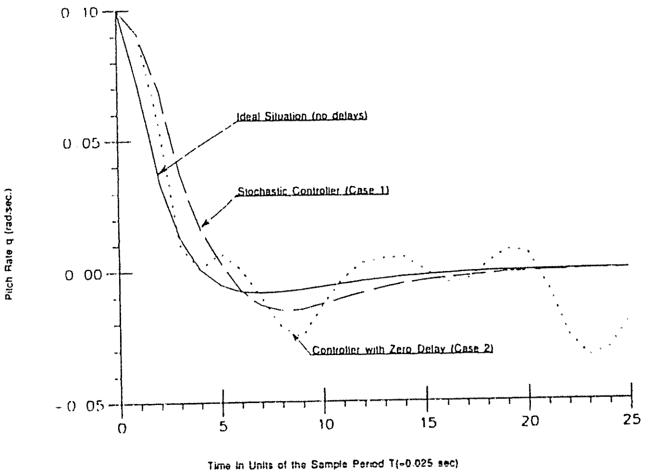

Fig. 4 Transient response of the flight control system for $\Delta_s/T = 0.6$

factory. The situation is further aggravated under the case #4, i.e., if a delay of $2T$ amounting to the sum of the maxima of two delays is considered. In contrast, the performance of the delayed system is vastly improved for all observed values of Δ_s if the feedback gain matrix is obtained via the (proposed) stochastic regulator algorithm in the sense that the resulting responses are very close to the ideal situation of the nondelayed plant controlled by a conventional LQR.

Poor performance of the deterministic controllers accrues from the fact that the LQR algorithm is sensitive to parameter variations. If the model does not closely match the real system, the feedback gain matrix fails to assign correct weights for each state resulting in poor performance. A comparison of the steady-state feedback gain matrices for all four cases for $\Delta_s = 0.6T$ is given in Table 1. (The feedback gain matrices were obtained at $N = 40$ when the steady state was practically reached.)

The stochastic nature of the control system is illustrated by a collection of transient responses of the pitch rate for $\Delta_s = 0.6T$ in Fig. 5 under different network-induced delays that are generated as uniformly distributed random sequences with different seed numbers, i.e., at different sample points. However, any individual response is not guaranteed to be bounded within a fixed envelope since the performance cost being minimized is just the expected value of the quadratic form of the augmented states. Although these responses are dissimilar for different seed numbers, each of them converges to the same steady-state value because of the mean-square convergence.

As seen in Eq. (4) in Section 2.3, the performance cost $J^*(\Delta_s)$ for an arbitrary initial condition x_0 can be measured in terms of the norm (spectral radius) of the matrix $(\mathrm{p} + S)$. Figure 6 shows a plot of this norm, denoted as $\rho(\mathrm{p} + S)$, versus Δ_s. Because of monotonicity of $\rho(\mathrm{p} + S)$, the performance cost J^* is also monotonic relative to Δ_s. The rationale is that the performance cost is constructed to minimize the control effort and the deviation from the target state. Since the loop delay increases with Δ_s, the delayed response to the control command causes the performance cost to increase.

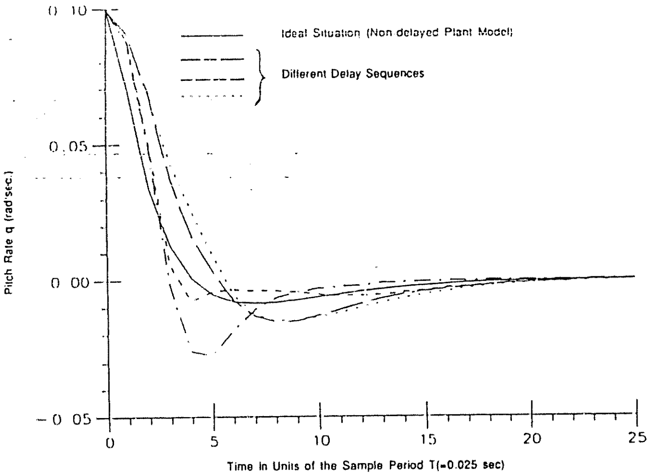

Fig. 5 Transient response of the flight control system under the stochastic regulator for different delay sequences and $\Delta_s/T = 0.6$

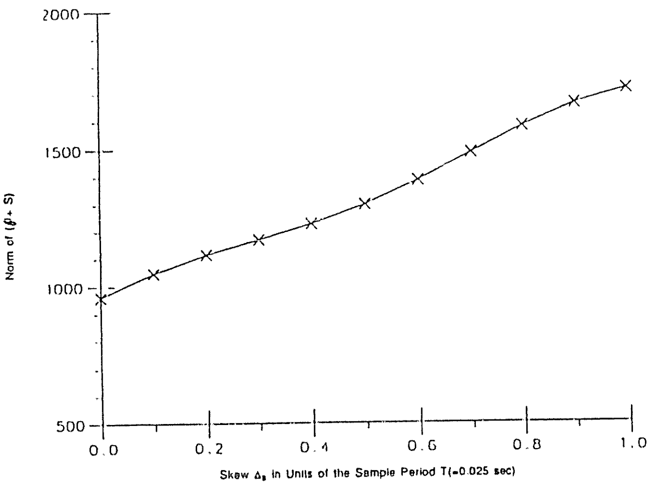

Fig. 6 Performance of the flight control system relative to Δ_s

Remark 4.1: It is well known that, for a nondelayed plant under control of a LQR, the system response can be adjusted to satisfy any desired specification if the weighting matrices are appropriately chosen and if there is no constraint on the control effort. In view of this fact, the first step in off-line design of the proposed controller via simulation experiments is to select a set of suitable weighting matrices for the nondelayed system. The same set of weighting matrices could be used to calculate the feedback gain matrix for the randomly delayed system. Later these gain matrices can be fine-tuned on the basis of simulation results. ■

5 Summary, Conclusions, and Recommendations for Future Research

This two-part paper presents the concept, formulation, and performance evaluation of a delay compensation algorithm under randomly varying, distributed delays that are induced by computer networking in Integrated Communication and Control Systems (ICCS) [2, 3]. The proposed delay-compensation algorithm is suitable for large-scale processes like advanced aircraft, spacecraft, modern automobiles, autonomous manufacturing plants, and chemical plants. The communication network, that interconnects the individual subsystems and components of the ICCS, should be designed such that the offered traffic in the network is bounded within its critical value [5, 6] relative to a specified confidence interval.

The first part [1] deals with the concept and formulation of the stochastic regulator that uses a linear deterministic function of the plant state or its conditional expectation. Specifically, the plant dynamic model is augmented in state space to account for the random delays. Then, the linear state feedback control law is synthesized on the basis of this stochastic plant model by using the principle of dynamic programming and optimality. This second part presents a step-by-step procedure for numerically solving the backward difference equations in the control algorithm to evaluate the feedback gain matrices. Conditions for stability and convergence of the cost matrices have also been established. It might be possible to update the control parameters on-line for adaptation to changes in network traffic statistics which are expected to vary slowly relative to process dynamics. The proposed control algorithm has been evaluated by simulation experiments using the flight dynamic model of an advanced aircraft.

The results of simulation experiments show that if a randomly delayed plant is operated under a controller designed on the basis of the nondelayed plant model, the system dynamic performance would be degraded possibly to the extent of instability. On the other hand, if the random delays are replaced by their maxima and the controller is conservatively designed on the assumption of these constant delays, the system performance is still inferior to that under the proposed controller. Apparently this stochastic regulator performs better than other controllers when the plant is subjected to randomly varying delays. However, this is *not* a final conclusion because robustness of the control system is yet to be investigated under noise, external disturbances, and modeling uncertainties in both plant dynamics and network-induced delays. Further research is needed beyond the analysis presented in Part I [1].

Possible areas for future research in the evolving field of Integrated Communication and Control Systems are innumerable. Some of the topics related to the research reported here are furnished below.

Research Topic #1: Compensation of Data Loss: The proposed stochastic regulator algorithm does not take into account the effects of recurrent loss of sensor and/or control data due to persistent noise corruption in the network or protocol malfunctions. The control system is expected to perform in a gracefully degraded mode if observability and reachability conditions still hold under recurrent loss of data [20]. Given the delay statistics and plant model dynamics, the problem is to find an upper bound of the probability of data loss that will still satisfy the performance and stability specifications. ■

Research Topic #2: State Estimation and Randomly Delayed Measurements: If some of the states are not measurable or if the sensor signal-to-noise ratio is unacceptable, a stochastic filter of special design is needed for state estimation where the sensor delay index $p(k)\in\{0, 1\}$ is random. The problem is formulated as follows:

$$\xi_{k+1}=A_k\xi_k+B_ku_k+n_k$$
$$y_k=C_k\xi_k+v_k$$

where the plant and measurement noises, n_k and v_k, are assumed to be white, mutually independent, and of covariances Q and R, respectively. The delayed sensor data w_k, as input to the filter, is represented as: $w_k=p(k)y_{k-p(k)}+(1-p(k))y_k$. The objective is to obtain an estimate η_k of the state ξ_k, which minimizes the performance cost

$$J=E\{[\xi_k-\eta_k]^T M[\xi_k-\eta_k]\mid W_k\}$$

where W_k is the collection of all past measurements $\{w_0, w_1, \cdots, w_k\}$, $E\{\cdot\mid W_k\}$ is the conditional expectation given W_k, and M is a positive definite weighting matrix.

In the case of missing data, a filter based on the proposed formulation may yield better results than that obtained by Sawagari et al. [21], which may not be effective if ξ_k is under steady state conditions because, assuming that v_k is small, w_k would oscillate between 0 and C_kx_k. In contrast, w_k can vary only between $C_{k-1}x_{k-1}$ and C_kx_k in the proposed formulation. ■

Research Topic #3: Robustness for Compensation of Modeling Uncertainties: An important factor in the controller design is robustness which, in general, means how well the control system would perform in the presence of noise, modeling errors, and parametric and non-parametric uncertainties [22]. Robustness of the control algorithm derived in Part I can be investigated by including additive noise and disturbances in the plant model. In addition to the usual problems, such as loss of gain margin associated with linear quadratic regulators (LQR), robustness with respect to the mismatch in statistics of network-induced delays needs to be investigated. Imperfect knowledge of statistics will cause errors in the expectations and covariances of the plant matrices A_k and B_k, which in turn affect the cost matrices and the feedback gain matrix. To summarize, the robustness problem must address the existence of imprecisely known statistics of network traffic coupled with structured and unstructured uncertainties in plant dynamics. ■

Research Topic #4: Simultaneous Optimization of Controller and Network Parameters: If all plant states are available, model-based prediction to obtain $E\{x_k\mid x_{k-1}\}$ is needed only when the current sensor data are not available at the controller. As pointed out in Section 2.3, robustness of the control system is likely to be improved if this model-based prediction can be avoided. In that case, it is necessary to formulate a control law which will use the most recently available sensor data. Accordingly, the feedback gain matrix should be re-evaluated by minimizing the modified performance cost with respect to both $\{u_k\}$ and Δ_s. The optimal Δ_s^* would minimize the detrimental effects of the increased controller-to-actuator delay by decreasing sensor-to-controller delay. In contrast to the simulation results in Section 4, Δ_s^* in this design should not be zero since there is no estimator to overcome the effects of the delay in sensor data arrival. A brief discussion on how to obtain Δ_s^* is presented in the following.

The sequences $\{A_k\}$ and $\{B_k\}$ of matrices in the augmented plant model are implicitly dependent on Δ_s because the matrices $\{b_i^k\}$ are functions of the arrival instants, $\{t_i^k\}$, of control inputs at the actuator terminal, and each t_i^k is directly affected by Δ_s. Therefore, the optimal cost J^* minimized with respect to $\{u_k\}$ is an implicit function of Δ_s. For a given time horizon of N samples, the optimal cost with respect to Δ_s can be found by a one-dimensional search method over the interval $[\delta_{\min}, \min(\delta_{\max}, T)]$. (Note: we have assumed $\delta_{\max}<T$ in this design procedure.) However, in the above search, each individual guess of Δ_s would require re-evaluation of the cost function, J^*, and the matrices, ρ and S, that are obtained by a set of N recursive computations. This is not considered to be a problem for off-line design unless the dimension of the plant model is large (which is unlikely for individual feedback loops in ICCS). However, on-line updating of Δ_s (as a consequence of changes in the statistics of network-induced delays) might be a concern even though it is not expected to take place frequently. This procedure suggests the need for an analytical method to predict an initial guess of Δ_s which is sufficiently close to its optimal value. The method proposed in Section 4 of [3] where the augmented model includes the controller is apparently a viable approach. ■

Research Topic #5: Regulator Design for Nonidentical Sensor and Controller Sampling: So far we have investigated the case of identical sensor and controller sampling. An alternative procedure for ICCS design is to deliberately make the sensor and controller sampling periods different, i.e., $T_s\neq T_c$. The advantages of this approach have been discussed, in detail, by Ray and Halevi [3] and Liou and Ray [23]. The research problem is to design a stochastic regulator and simultaneously identify an optimal $\epsilon := T_s/T_c$. This would require minimization of a specified performance cost such that stability of the closed loop control system is guaranteed under given network traffic statistics, plant dynamics, and a fixed controller sampling period T_c. ■

References

1 Liou, L-W., and Ray, A., "A Stochastic Regulator for Integrated Communication and Control Systems: Part I—Formulation of Control Law," ASME Journal of Dynamic Systems, Measurement, and Control, published this issue, pp. 604-611.

2 Halevi, Y., and Ray, A., "Integrated Communication and Control Systems: Part I—Analysis," ASME Journal of Dynamic Systems, Measurement, and Control, Dec. 1988, pp. 367-373.

3 Ray, A., and Halevi, Y., "Integrated Communication and Control Systems: Part II—Design Considerations," ASME Journal of Dynamic Systems, Measurement, and Control, Dec. 1988, pp. 374-381.

4 Liou, L-W., and Ray, A., "On Modeling of Integrated Communication and Control Systems," ASME Journal of Dynamic Systems, Measurement, and Control, Dec. 1990, pp. 000-000.

5 Ray, A., "Performance Analysis of Medium Access Control Protocols for Distributed Digital Avionics," ASME Journal of Dynamic Systems, Measurement, and Control, Dec. 1987, pp. 370-377.

6 Ray, A., "Distributed Data Communication Networks for Real-Time Process Control," *Chemical Engineering Communications*, Vol. 65, Mar. 1988, pp. 139-154.

7 Halevi, Y., and Ray, A., "Performance Analysis of Integrated Communication and Control System Networks," ASME Journal of Dynamic Systems, Measurement, and Control, Sept. 1990, pp. 365-371.

8 Banks, J., and Carson, J. S., "Process-Interaction Simulation Languages," *SIMULATION*, May 1985, pp. 225-235.

9 Narendra, K. S., and Annaswamy, A. M., *Stable Adaptive Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

10 Franklin, G. F., Powell, J. D., and Workman, M. L., *Digital Control of Dynamic Systems*, Addison-Wesley, Reading, MA, 1990.

11 Golub, G. H., and Van Loan, C. F., *Matrix Computations*, Johns Hopkins University Press, Baltimore, Maryland, 1983.

12 Bellman, R., *Introduction to Matrix Analysis*, McGraw-Hill, New York, 1970.

13 *User's Manual*, IMSL Problem Solving Software Systems, 1987.

14 Luck, R., and Ray, A., "An Observer-Based Compensator for Distributed Delays," *Automatica*, Vol. 26, No. 5, Sept. 1990, pp. 903-908.

15 Luck, R., and Ray, A., "Delay Compensation in Integrated Communication and Control Systems: Part II—Implementation and Verification," *Proc. American Control Conference*, San Diego, CA, May 1990.

16 Kozin, F., "A Survey of Stability of Stochastic Systems," *Automatica*, Vol. 5, 1969, pp. 95-12.

17 Belle Isle, A. P., "Stability of Systems with Nonlinear Feedback Through Randomly Time-Varying Delays," *IEEE Trans. Aut. Contr.*, Vol. AC-20, No. 1, Feb. 1975, pp. 67-75.

18 De Koning, W. L., "Optimal Control, Estimation, and Compensation of Linear Discrete-Time Systems with Stochastic Parameters," *Control and Dynamic Systems: Advances in Theory and Applications*, Academic Press, San Diego, CA, 1988.

19 Etkin, B., *Dynamics of Flight Stability and Control*, John Wiley, 2nd ed., 1982.

20 Luck, R., Ray, A., and Halevi, Y., "Observability under Recurrent Loss of Data," *AIAA Journal of Guidance, Control, and Dynamics*, in press.

21 Sawagari, Y., Katayama, T., and Fujishige, S., "Sequential State Estimation with Interrupted Observation," *Information and Control*, Vol. 21, pp. 56-71, 1972.

22 Maclejowski, J. M., *Multivariable Feedback Design*, Addison-Wesley, Workingham, England, 1989.

23 Liou, L-W., and Ray, A., "Integrated Communication and Control Systems: Part III—Non-Identical Sensor and Controller Sampling," ASME Journal of Dynamic Systems, Measurement, and Control, Sept. 1990, pp. 357-364.