Table 2 Calculations of the angular distribution function for oxygen atoms passing through an orifice in an orbital plane at various gas temperatures

Temperature, K	Speed ratio,[a] s_r	Angular width FWHM,[b] deg
600	9.87	9.60
700	9.13	10.36
800	8.54	11.06
900	8.06	11.70
1000	7.64	12.32
1100	7.29	12.90
1200	6.98	13.44
1300	6.70	13.97
1400	6.46	14.48
1600	6.04	15.44
1800	5.70	16.32
2000	5.40	17.16

[a]The speed ratio of the satellite velocity (with respect to a stationary atmosphere) taken here as 7.77 km/s to the most probable speed for a Maxwell-Boltzmann distribution for O atoms at the given temperature.
[b]Full width at half maximum.

(full width) in this plane over a significant portion of the orbital exposure.‡ Since the detector system records all angular instabilities on top of one another, time-dependent instabilities cannot be resolved. The result is weighted by oxygen-atom exposure rather than just time averaged. Since the O atom density is exponential with decreasing altitude and the LDEF descended in its orbit at an increasing rate, the bulk of the oxygen exposure was accumulated during the last few months before capture. However, the attitude instabilities themselves are most likely to have been caused by aerodynamic forces. These were also at maximum during the latter portion of the flight as the satellite entered the denser regions of the atmosphere. Thus, the yaw instability of ±0.2 deg may only have occurred late in the flight.

Conclusions

Evidence from the passive attitude detector on experiment A0114 showed that the LDEF spacecraft maintained a highly stable attitude during its 5.75-year flight. There was a small offset yaw of 8.0 ± 0.4 deg clockwise from nominal attitude as viewed from space. There also appeared to be an oscillation of ±0.2 deg about this offset yaw.‡ The satellite was pitched slightly forward by about 1 deg (space end leading). Those experiments on the LDEF that depend on orientation relative to the forward direction, such as atomic oxygen reaction cross-section measurements, may need to be corrected for the angular offset. The gravity-gradient mode of spacecraft stabilization has great cost benefits over those using active systems, particularly for long-lived missions, but uncertainty in yaw stability has been a concern for many applications. With demonstration of the high degree of stability about the yaw axis experienced by the LDEF, the instabilities predicted for passively stabilized spacecraft may be reduced.

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‡It has been observed[7] that the corotation of the atmosphere with the Earth itself would produce a deviation of ±1.5 deg in incidence angle of the atmosphere with the LDEF front surface as it crossed the equator. Such an effect, which we believe to be real, would be indistinguishable from an oscillation in yaw.

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Observability Under Recurrent Loss of Data

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I. Introduction

THE vehicle management system (VMS) in the future generation aircraft would require highly integrated control (e.g., integrated flight-propulsion and flight-fire controls) and decision (e.g., flight trajectory management) functions that will have direct flight-criticality implications. For example, the integrated flight-propulsion controller must take into account the effects of a strong coupling between the propulsion and aerodynamics to take advantage of propulsive moments and forces for flight maneuverability. These functions, combined with new strategies [e.g., self-repairing and reconfigurable flight control systems, management of actuator failures and surface damage, control surface reconfiguration, and applications of artificial intelligence (AI) techniques to distributed decision support systems], would generate significantly large and distributed computational requirements. A communication network [e.g., Society of Automotive Engineers (SAE) token bus] is needed for information processing between the onboard spatially dispersed computers, intelligent terminals, sensors, and actuators to implement the aforementioned functions.

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Recurrent loss of sensor and control signals may randomly occur due to noise corruption in the communication medium or protocol malfunctions if the feedback loop is closed via a computer network.[1] An observable (reachable) system that assumes availability of sensor (control) data at consecutive samples may thus become unobservable (unreachable) due to recurrent loss of data. If the control system is unstable in the open loop as it is with highly maneuverable supersonic aircraft, then recurrent loss of data could render the system undetectable and/or unstabilizable. Therefore, a control system, designed for a conventional (non-networked) application, should be critically examined for the impact of potential loss of data if the system has to operate within a network environment.

The concept of state estimation with recurrent loss of sensor data has been addressed by other investigators in different contexts. For example, Jaffer and Gupta[2] and Sawaragi et al.[3] considered the problem of sequential estimation with interrupted observations within a stochastic setting. The sequential state estimation algorithms in both cases are developed using a Bayesian approach. To the best of the authors' knowledge, the problem of observability under persistent and random loss of data has not been studied before.

This correspondence introduces the concept of extended observability in finite-dimensional linear time-invariant systems under recurrent loss of data where the state vector has to be reconstructed from an ensemble of sensor data at nonconsecutive samples. Given that the computer network is designed to keep the probability of losing more than m data in every set of $(\nu+m)$ successive data, less than an a priori prescribed bound β, the problem is to establish test criteria for observability (reachability) under this condition; ν is the observability (reachability) index, and m is known as a function of β and ν. A fixed-size window of $(\nu+m)$ data from the available collection is considered for each observation, and sample numbers of the missing data are routinely recorded by the computer network protocol. This concept of observability (reachability) based on a set of randomly selected nonconsecutive samples is different from that of extended observability with unknown inputs (Basile and Marro,[4] Emre and Silverman,[5] Kudva et al.,[6] Molinari,[7] and Rappaport and Silverman[8]).

The extended observability can be determined by testing the rank of every possible matrix associated with an augmented set of output data arising from each possible combination of data loss. Although this test is exhaustive, it is time consuming, in general, and could lead to incorrect conclusions due to computational inaccuracy. An alternative approach to determine extended observability in the single-output case is presented. The relevant contributions of this correspondence are 1) a necessary and sufficient condition for extended observability that can be expressed via a recursive relation, 2) necessary conditions (for the aforementioned) that are related to the characteristic polynomial of the state transition matrix in a discrete-time setting or of the system matrix in a continuous-time setting, and 3) a system-theoretic approach for having an insight into the problem of loss of observability.

II. Extended Observability: Concepts and Test Criteria

Let the plant be represented by a discrete-time, linear, time-invariant model in a deterministic setting at the sampling instant k

$$x(k+1) = Ax(k) + Bu(k), \qquad z(k) = Cx(k) \tag{1}$$

where the state vector $x \in R^n$, the input vector $u \in R^r$, the output vector $z \in R^p$, and the constant matrices A, B, and C are of compatible dimensions. Furthermore, rank of C is p and the pair (C,A) is observable with observability index ν. Then,

$$x(k+j) = A^j x(k) + \sum_{i=0}^{j-1} A^i Bu(k+j-1-i) \tag{2}$$

Defining $y(k+j) = CA^j x(k)$, it follows from Eqs. (1) and (2) that

$$y(k+j) = z(k+j) - \sum_{i=0}^{j-1} CA^i Bu(k+j-1-i) \tag{3}$$

The modified output vector sequence $\{y(k)\}$ can be used for state reconstruction in lieu of $\{z(k)\}$ provided that the input sequence $\{u(k)\}$ is available. [The issue of observability with unknown inputs (Basile and Marro,[4] Rappaport and Silverman[8]) is not addressed here.] Under normal circumstances, the state x can be reconstructed from ν consecutive sets of outputs. However, in the event of loss of outputs, $y(k)$ is not available at every k. The problem is to determine whether the state $x(k)$ can be reconstructed by selecting any ν output vectors from the collection $Y^m(k)$ of $(\nu+m)$ output vectors as defined next [Note: $Y^m(k)$ may also be viewed as a $[p(\nu+m)x1]$ vector.]:

$$Y^m(k) = \left[y(k)^T | y(k+1)^T | y(k+2)^T | \cdots | y(k+\nu+m-1)^T\right]^T$$

where m is a fixed finite integer.

Definition 1

The system described in Eq. (1) is said to be m-observable if $x(k)$ can be reconstructed from any ν distinct vectors in $Y^m(k)$.

We now elucidate the concept of m-observability by two second-order, single-output systems that are observable but not m-observable.

Example 1

Let A and c be defined as follows:

Given

$$A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

and $c = [1 \quad 0]$, the observability matrix

$$\Theta = \begin{pmatrix} c \\ cA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

implies that the pair (c,A) is observable. On the other hand, $cA^2 = [-1 \quad 0] = -c$. The system is not 1-observable and hence cannot be m-observable, $m \geq 1$. From a geometric point of view, A is a 90-deg rotation matrix implying that $cA^{\ell+2} = -cA^{\ell}$ for $\ell = 0,1,2,3,\ldots$.

Example 2

Next we consider a 120-deg rotation matrix so that

$$A = \begin{pmatrix} -1/2 & -\sqrt{3}/2 \\ \sqrt{3}/2 & -1/2 \end{pmatrix}, \qquad c = [1 \quad 0]$$

This system is 1-observable but not m-observable, $m \geq 2$.

Next we proceed to determine the conditions for m-observability. The following relationship between $Y^m(k)$ and $x(k)$ is derived from Eqs. (1-3):

$$Y^m(k) = Qx(k) \tag{4}$$

where $Q = [\Theta^T | \Theta_m^T]^T$, which is a $[p(\nu+m)xn]$ matrix,

$$\Theta = \left[(CA^0)^T | (CA^1)^T | \cdots | (CA^{\nu-1})^T\right]^T$$

$$\Theta_m = \left[(CA^{\nu})^T | (CA^{\nu+1})^T | \cdots | (CA^{\nu+m-1})^T\right]^T$$

A simplified necessary and sufficient condition for m-observability is presented next.

Proposition 1

The system described in Eq. (1) is m-observable in the single output case (i.e., C is a $1 \times n$ row vector c and $\nu = n$) if all minors of $O_m O^{-1}$ are nonzero.

Proof: It suffices to show that an arbitrary set of n rows from $Q\Theta^{-1}$ contains linearly independent vectors if all minors of $\Theta_m\Theta^{-1}$ are nonzero. Consider the $(n+m)\times n$ matrix

$$Q\Theta^{-1} = \begin{pmatrix} I_n \\ \hline V \end{pmatrix} \tag{5}$$

where I_n is the $(n\times n)$ identity matrix and $V=\Theta_m\Theta^{-1}$. Let us choose any $(n-k)$ rows from I_n and any k rows from V. If $k=0$, or $k=n$ for $m\geq n$, this proposition automatically holds. Therefore, we consider $1\leq k<n$. Let U represent the $(n-k)$-dimensional subspace spanned by the $(n-k)$ rows selected from I_n and let W represent the subspace spanned by k rows selected from V. First suppose that all minors of V are nonzero. Having all kth order nonzero minors implies that any k rows are linearly independent and each linear combination of these rows must have fewer than k zero elements, i.e., more than $(n-k)$ nonzero elements. Therefore, $\dim W=k$ and $W\cap U=\{0\}$. Consequently, any n rows of $Q\Theta^{-1}$ are linearly independent. This establishes sufficiency. Next suppose that V contains a zero minor of order k. Then it is possible to construct a linear combination of these rows with at least k zero elements. Such a vector can be expressed as a linear combination of $(n-k)$ rows of I_n. Therefore this collection of $k+(n-k)=n$ rows of $Q\Theta^{-1}$ does not form a linearly independent set. This establishes necessity.

Next we proceed to relate certain properties of the matrix A in Eq. (1) to m-observability for the single-output case.

Observation 1

The matrix $\Theta_m\Theta^{-1}$ is completely determined by the coefficients of the characteristic polynomial of A. This can be seen by expressing $\Theta_m\Theta^{-1}$ as follows:

$$\begin{aligned}\Theta_m\Theta^{-1} &= \left[(cA^n)^T(cA^{n+1})^T\cdots(cA^{n+m-1})^T\right]^T\Theta^{-1}\\ &= \left[(cA^n\Theta^{-1})^T(cA^{n+1}\Theta^{-1})^T\cdots(cA^{n+m-1}\Theta^{-1})^T\right]^T\\ &= \left[(c\Theta^{-1}\Theta A^n\Theta^{-1})^T(c\Theta^{-1}\Theta A^{n+1}\Theta^{-1})^T\cdots\right.\\ &\quad\left.\times(c\Theta^{-1}\Theta A^{n+m-1}\Theta^{-1})^T\right]^T\\ &= \left[(cA^n)^T(cA^{n+1})^T\cdots(cA^{n+m-1})^T\right]^T\end{aligned} \tag{6}$$

where $A=\Theta A\Theta^{-1}$ and $c=c\Theta^{-1}$.

This represents a similarity transformation into the standard observability canonical form[9] where

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \vdots & \vdots & & & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_0 & -a_1 & \cdots & & -a_{n-1} \end{pmatrix} \tag{7}$$

and $c=[1\ \ 0\ \ 0\ \ \cdots\ \ 0]$. Therefore, Eq. (6) depends only on the coefficients of the characteristic polynomial of A.

Observation 2

Following Eqs. (6) and (7) in observation 1, the first row of $\Theta_m\Theta^{-1}$ is $[-a_0\ \ -a_1\ \ \cdots\ \ -a_{n-1}]$. By proposition 1, a necessary condition for m-observability, $m\geq 1$, is that all 1×1 minors of $\Theta_m\Theta^{-1}$ be nonzero. This implies that each coefficient of the characteristic polynomial of A must be nonzero for m-observability, $m\geq 1$.

Observation 3

The characteristic polynomial of A in example 1 is A^2+I, i.e., $a_1=0$. Therefore, the system is not m-observable $\forall m\geq 1$. But, the characteristic polynomial in example 2 is A^2+A+I, implying that the necessary condition for m-observability is satisfied.

Observation 4

The degree of m-observability is common to all observable pairs that share the same state transition matrix. That is, if there exists c' such that (c',A) is m-observable then (c,A) is m-observable for each observable pair (c,A). The reason for the above is that the output vector c is only required to establish the invertibility of Θ, i.e., observability. On the other hand, given that Θ^{-1} exists, $\Theta_m\Theta^{-1}$ depends solely on the coefficients of the characteristic polynomial of the A matrix.

The test procedure in proposition 1 requires computation of $V=\Theta_m\Theta^{-1}$. We now show how the coefficients of the $m\times n$ matrix V can be recursively computed. From Eqs. (6) and (7) it follows that the coefficients of V can be expressed as

$$V_{(i,j)} = V(i-1,j-1) - V(i-1,n)a_{j-1}$$

for $1\leq i\leq m$ and $1\leq j\leq n$ by setting the initial conditions $V(\ ,0)=0$ and $V(1,j)=-a_{j-1}$. Then,

$$\begin{aligned}V(i,n) &= V(i-1,n-1) - V(i-1,n)a_{n-1}\\ &= -\sum_{k=0}^{n-1} a_k V(i-n+k,n)\end{aligned}$$

Defining $V_i=V(i,n)$, the above equation can be expressed as

$$V_i = -\sum_{k=0}^{n-1} a_k V_{i-n+k} \qquad \text{for} \qquad i\geq n \tag{8}$$

The range of V_i in Eq. (8) can be extended by using the relationship $V_1=-a_{n-1}$ and defining $V_0=1$ and $V_i=0$ for $i<0$ as follows:

$$V_i = -\sum_{k=0}^{n-1} a_k V_{i-n+k} \qquad \text{for} \qquad i\geq 1 \tag{9}$$

Using Eqs. (6) and (7), the ith row of V can be expressed as

$$\left[V(i,1)\quad V(i,2)\quad \ldots\quad V(i,n)\right] = cA^{n-1+i} \quad \text{for } i\geq 1 \tag{10}$$

Hence, from Eqs. (9) and (10), it follows that cA^{n+i}, for $i\geq 0$, can be expressed as

$$cA^{n+i} = -[a_0\ \ a_1\ \ \cdots\ \ a_{n-1}]\begin{pmatrix} V_i & V_{i-1} & \cdots & V_{i-(n-1)} \\ 0 & V_i & \cdots & V_{i-(n-2)} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & V_i \end{pmatrix} \tag{11}$$

where

$$V_i = \begin{cases} i & \text{for } i<0 \\ 0 & \\ 1 & \text{for } i=0 \\ -\sum_{k=0}^{n-1} a_k V_{i-n+k} & \text{for } i>0 \end{cases}$$

III. Summary, Conclusions, and Recommendations for Future Research

Necessary and sufficient conditions for extended observability of linear, time-invariant systems have been established under recurrent loss of output data. The analysis shows that for an observable system, the extended observability is solely determined by the A matrix for the single-output case, and test

conditions have been formulated to this effect. Although simple in nature, these tests provide an insight on how the characteristic polynomial of the state transition matrix relates to the relative redundancy of the available sensor data.

Extended observability is critical for design of networked control systems such as those for vehicle management systems in advanced aircraft, especially if the plant under control is unstable in the open loop. This concept is also applicable to bad data suppression[10] that may lead to random rejection of signals in feedback control systems.

Some of the areas of future research in control synthesis under recurrent loss of data are briefly discussed below.

1) Extended observability for multiple-output systems: An extension of the work reported in this correspondence is to establish test conditions for extended observability in multiple-output systems. This requires the loss of data to be considered from two different perspectives: all elements of the output vector are unavailable at a random instant, and the more general case where only some of the elements are unavailable and the remaining elements could be used for state reconstruction. In the latter case, the identity of the unavailable outputs may vary with time.

2) Construction of an asymptotic observer: Asymptotic property of the observer is apparently retained for the regulator problem (i.e., without any persistent excitation) in spite of recurrent loss of data because all states can be reconstructed in finite time. However, for command tracking, the dynamic error would naturally be larger under loss of data. Standard design techniques like pole placement need to be examined from this perspective.

3) Performance and stability robustness: Observer-based as well as other types of compensators[11-17] may suffer from performance degradation and instability unless redesigned to compensate for recurrent loss of data in the extendible observable system. The methodology for design of the above robust compensators is yet to be established.

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