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Robust Wide-Range Control of Nuclear Reactors by Using the Feedforward-Feedback Concept

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Abstract — *A robust feedforward-feedback controller is proposed for wide-range operations of nuclear reactors. This control structure provides (a) optimized performance over a wide operating range resulting from the feedforward element and (b) guaranteed robust stability and performance resulting from the feedback element. The feedforward control law is synthesized via nonlinear programming, which generates an optimal control sequence over a finite-time horizon under specified constraints. The feedback control is synthesized via the structured singular value μ approach to guarantee robustness in the presence of disturbances and modeling uncertainties. The results of simulation experiments are presented to demonstrate efficacy of the proposed control structure for a large rapid power reduction to avoid unnecessary plant trips.*

I. INTRODUCTION

Operational safety and high performance are the driving forces for automation and control of nuclear power plants.[1,2] In the absence of an appropriate control system, the current practice of wide-range control of nuclear reactors such as scheduled shutdown favors manual control, which is dependent on the standard prescribed procedure as well as on the expertise of the human operator(s). An optimal feedforward control (FFC) coupled with a robust feedback control (FBC) to compensate for uncertainties and disturbances would significantly reduce the risk of the detrimental effects of human errors while the operator plays the role of a top-level supervisor. Since FFC is based on the nominal plant model, its ability to overcome any perturbations is rather limited. Therefore, FBC is necessary to compensate for external disturbances and also modeling uncertainties. On the other hand, without FFC, FBC for wide-range operations suffers from loss of performance as the control system becomes excessively conservative to guarantee stability over the specified operating range. The proposed control structure unifies FFC and FBC to overcome these disadvantages. The feedforward-feedback control (FF/FBC) structure is shown in Fig. 1. In this control structure, the main function of FFC is to provide good nominal performance, and the objective of FBC is to achieve the robustness. Although similar concepts have been used for control of chemical processes,[3] fossil-fueled power

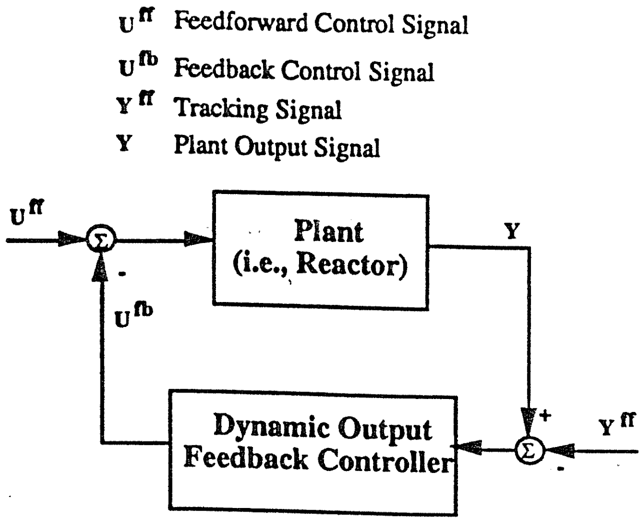

Fig. 1. The proposed FF/FBC system.

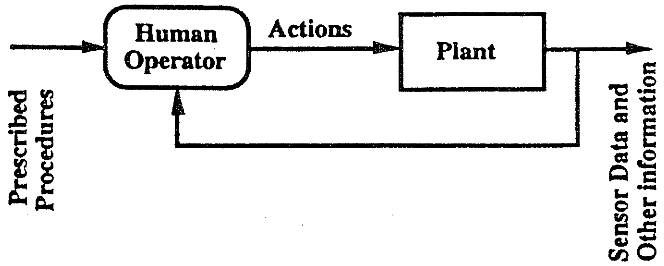

Fig. 2. Plant operations under human operator control.

plants,[4] aircraft,[5] and robotic systems,[6] the application of FF/FBC to wide-range robust control of nuclear reactors apparently has not been reported in the open literature. Recently, Suzuki, Shimazaki, and Shinohara[7] reported synthesis of an H_∞-based robust control system for a boiling water reactor. This control system is based solely on linear feedback principles with specified bounds on stability and performance, and therefore, its robustness can be guaranteed only for narrow-range operations of the reactor. In contrast, the control system reported in this paper takes advantage of FFC for wide-range operations, and FBC is exercised to overcome possible deviations of the plant response from the prescribed nominal trajectory. Furthermore, the μ-synthesis approach adopted in this paper allows quantification of the uncertainties resulting from unmodeled dynamics, modeling inaccuracies, and linearization of the nonlinear model. The specific advantage of the H_∞-based μ approach is that it allows a systemic evaluation of the robust performance measure of the synthesized control system. Details of the μ approach are reported in the literature.[8-10]

The objective here is to synthesize an automatic control strategy that would be similar to the sequence of actions taken by a human operator. The basic control actions exercised by a human operator follow the prescribed procedures while the necessary adjustments are made based on the on-line sensor data and the plant operating conditions. From the perspectives of plant operations and control, the human operator's actions can be viewed as a combination of both FFC and FBC having the structure shown in Fig. 2. The prescribed procedures followed by the human operator are analogous to the FFC input that brings the plant from the initial operating point to the final desired operating point under nominal conditions. The actions of the human operator, who makes the necessary corrections to the prescribed input according to experience, and the on-line sensor data are analogous to the feedback controller.

In Fig. 1, U^{ff} is the FFC input sequence, and Y^{ff} is the corresponding output sequence. The optimized nominal performance is achieved by synthesizing the FFC input U^{ff} by nonlinear programming,[11] which has the following two advantages: (a) the input sequence is optimal for the nonlinear control process and (b) the constraints of process operations and control can be easily specified. The state-of-the-art structured singular value μ approach[9] is adopted to synthesize FBC, whose role is to provide good robustness for both stability and performance. This integrated structure of FF/FBC thus enhances robustness of stability and performance, which assure safety and operational efficiency over a wide operating range. The efficacy of the proposed control concept is examined by comparing it with the previous work by using an observer-based linear quadratic regulator[1] (OBLQR). The OBLQR approach makes use of the state feedback concept to modify the demand signal for an embedded classical output feedback controller and belongs to the class of FBC.

II. CONCEPT OF THE CONTROL SYSTEM

The plant under consideration is a pressurized water reactor (PWR) under normal operations and is modeled with six delayed neutron groups and temperature feedback based on the lumped fuel and coolant temperature as follows[1]:

$$\frac{dn_r}{dt} = \frac{\rho - \beta}{\Lambda} n_r + \sum_{i=1}^{G} \frac{\beta_i}{\Lambda} c_{ri} , \tag{1}$$

where

$$\beta = \sum_{i=1}^{G} \beta_i$$

and G is the number of delayed neutron groups,

$$\frac{dc_{ri}}{dt} = \lambda_i n_r - \lambda_i c_{ri} , \quad i = 1,2,\ldots,G , \tag{2}$$

$$\frac{dT_f}{dt} = \frac{1}{\mu_f}[f_f P_{a0} n_r(t) - \Omega(T_f - T_c)] , \quad (3)$$

$$\frac{dT_l}{dt} = \frac{1}{\mu_c}[(1 - f_f) P_{a0} n_r(t) + \Omega(T_f - T_c) - M(T_l - T_e)] , \quad (4)$$

$$\rho = \rho_r + \alpha_f(T_f - T_{f0}) + \alpha_c(T_c - T_{c0}) , \quad (5)$$

and

$$\frac{d\rho_r}{dt} = g_r Z_r . \quad (6)$$

The variables in the foregoing equations are defined in the Nomenclature on p. 184. This plant model contains $(4 + G)$ state variables for G delayed groups, namely, relative power n_r, relative precursor density c_{ri}, average fuel temperature T_f, average coolant temperature leaving the reactor T_l, and control reactivity ρ_r. The plant output is the relative power n_r. The control input is the control rod speed Z_r. The plant dynamics in Eqs. (1) through (6) are expressed in the standard state-space representation as follows:

$$\text{plant dynamics: } \dot{x} = f(x,u,t) \; ; \quad x(t_0) = x_0 . \quad (7)$$

II.A. Optimized Feedforward Control

The optimized FFC law is formulated in the discrete-time setting as a standard nonlinear programming problem as the sequence $U^{ff} = \{u_0, u_1, \ldots, u_{n-1}\}$, where the subscripts 0 and n indicate the initial time and the final time, respectively. For a given initial condition, a specified objective functional is minimized under the prescribed constraints. The sequences $Y^{ff} = \{y_1, y_2, \ldots, y_n\}$ and $X^{ff} = \{x_1, x_2, \ldots, x_n\}$ are the corresponding sequences of output and state variables where the initial state x_0 is specified as a known condition. The final state x_n can be made arbitrarily close to the desired final value by the appropriate selection of the state weights in the cost functional. The discrete-time constrained optimization problem is summarized as follows:

minimize the system performance cost functional:

$$J(x,u) = \sum_{k=1}^{n-1} (\tilde{x}_k^T Q_k \tilde{x}_k + \tilde{u}_k^T R_k \tilde{u}_k) \quad (8)$$

plant dynamic constraints:

$$x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} f(x,u,t)\, dt \quad (9)$$

plant input constraints:

$$u_k - u_k^{ub} \leq 0 \quad \text{and} \quad u_k^{lb} - u_k \leq 0 \quad (10)$$

plant output constraints:

$$g_k(x_k,u_k) \leq \beta_k , \quad (11)$$

where

$\tilde{x}_k = x_k - x_{ss}$, $\tilde{u}_k = u_k - u_{ss}$ = deviations of the plant state vector and the control input vector from the respective final steady-state values of x_{ss} and u_{ss}

matrices Q_k, R_k = relative weights of the state and the input variables

$\{u_k^{ub}\}$ = sequence of the upper bound of the plant input

$\{u_k^{lb}\}$ = sequence of the lower bound of the plant input

β_k = upper bound of the plant output $g_k(x_k,u_k)$ at time k.

II.B. Robust Feedback Control

Nonlinear programming generates an open-loop FFC policy to achieve the optimal trajectory under the specified constraints of the plant input and output. However, because of plant modeling uncertainties (including unmodeled dynamics), sensor noise, and disturbances, the actual plant response will deviate from that of the modeled system when the plant is excited by the sequence of open-loop control commands. Therefore, a closed-loop control system is necessary to compensate for these deviations, and a dynamic output feedback controller may serve this purpose. If the deviations from the nominal trajectory are not large, the gain matrices of the dynamic output feedback controller could be synthesized based on a linearized model of the plant. However, since the plant is required to be operated over a wide range (for example, scheduled shutdown of the plant from full power), then linearization would be carried out at several operating points, and the closed-loop control would be made piecewise linear by adopting the concept of gain scheduling. Alternatively, in our approach, the dynamic output FBC law is formulated by using the structured singular value μ technique[9,12,13] of robust multi-input/multi-output control synthesis, which relies on approximation of the plant dynamics by a single linear time-invariant model. Details of the μ-analysis and the μ-synthesis approaches are reported in the literature.[8-10]

III. DEVELOPMENT OF THE CONTROL SYSTEM

The objective of the control system is to regulate the reactor power in the 100 to 25% range with a good temperature response. To this effect, a computationally efficient nominal plant model for FFC was determined to be a two-delayed-neutron-group representation[14] $(G = 2)$ with zero-lifetime approximation[15] for prompt neutrons. Since the prompt neutron lifetime is very short ($\sim 10^{-5}$ s), the singular perturbation approach was adopted to approximate the fast dynamics of the prompt neutron by an algebraic relation describing the instantaneous response, which is called the zero-lifetime approximation. Equation (1) is accordingly modified as follows:

$$n_r = \frac{1}{\beta - \rho} \sum_{i=1}^{G} \beta_i c_{ri} \ . \tag{12}$$

The structure of the previously reported OBLQR controller reported in Ref. 1 is shown in Fig. 3. A one-delayed-neutron-group nonlinear observer has been used in the OBLQR to generate the state estimate, and the quadratic performance objective is identified as

$$J = \int (0.01T_f^2 + 0.1T_l^2 + 1000Z_r^2)\, dt \ , \tag{13}$$

where average fuel temperature T_f and average coolant temperature T_l leaving the reactor are the plant state variables and the control rod speed Z_r is the plant input. In this paper, the cost functional of the optimized FFC is chosen structurally similar to that in Eq. (13). The objective functional for FFC penalizes relative power n_r and average fuel temperature T_f and is independent of control effort Z_r, which is already constrained as follows:

$$\text{minimize } J = \sum_{k=1}^{n} [400(n_r^k)^2 + 0.01(T_f^k)^2] \tag{14}$$

and

$$\text{subject to } Z_r - 0.2 \le 0 \ ; \quad -Z_r - 0.2 \le 0 \ ;$$
$$\dot{x} - f(x,u,t) = 0 \ , \tag{15}$$

where $\dot{x} = f(x,u,t)$ is the reactor model as defined in Eq. (7) along with the modification in Eq. (12). For this example, the constraints in Eq. (15) are specified such that the rod speed Z_r does not exceed 20% of the total length per second. The time step was chosen to be 1 s. The objective functional in Eq. (14) is different from that for OBLQR in Eq. (13), which directly penalizes average coolant temperature T_l. The FFC requires a relatively small penalty on n_r to prevent power oscillations. In contrast to OBLQR, which does not incorporate any constraints, FFC does accommodate the known physical constraints of actual plant operations. These constraints, when included in OBLQR for implementation, would render it suboptimal with possible loss of performance. The FFC approach thus maintains optimality with respect to the performance objective, specified constraints, and the nominal nonlinear plant model. Furthermore, although not demonstrated in this example, FFC can accommodate time-varying performance objectives for tracking problems and specifications of hard constraints on states such as reactor temperature.

The FBC synthesis is formulated as a robust performance problem with multiplicative plant uncertainty at the plant input described as follows:

1. Since the two-delayed-neutron-group model with zero-lifetime approximation is used as the nominal plant model to generate the optimized FFC input sequence, the discrepancy between this model and the regular six-group model is defined as a source of plant modeling uncertainty. The rationale for model order reduction is to mitigate the computational load of the optimization process via nonlinear programming while still achieving desirable plant response.

2. The full-order nonlinear plant model is linearized at the middle of the reactor power range (62.5% in this case) to serve as a nominal model for synthesizing FBC. The variations between this nominal plant model and the models linearized at the two extreme

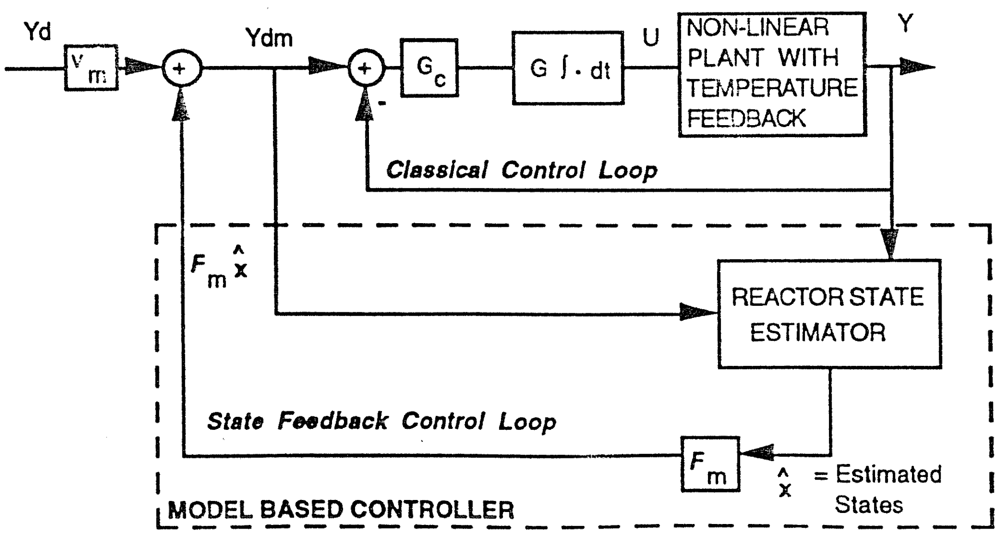

Fig. 3. The OBLQR control system.

power levels (100 and 25%) are realized as another source of plant modeling uncertainty.

Based on the low-order nominal model at the 62.5% power level and the high-order model at the extreme points of 100 and 25% power levels, the frequency-dependent error bounds were obtained as shown in Fig. 4. The resulting uncertainty weighting function $W_{del}(s)$ is modeled as the following fourth-order transfer function from these two bounds:

$$W_{del}(s) = \frac{0.668s^4 + 4.7272s^3 + 3s^2 + 0.41s + 0.01}{s^4 + 7.02s^3 + 5.71s^2 + 1.21s + 0.06} \quad . \tag{16}$$

The performance criterion is represented by the plant output sensitivity function. The frequency-domain performance requirement is specified as the steady-state tracking error being <1% at frequencies of 10^4 rad/s or less. To this effect, the performance weighting function $W_p(s)$ is set as

$$W_p(s) = \frac{(s + 1.0)}{(3s + 0.01)} \quad . \tag{17}$$

The structured singular value μ of the synthesized control system is shown in Fig. 5. Since the desired goal of $\mu < 1$ is achieved, this control system is guaranteed to have robust performance (and also robust stability)

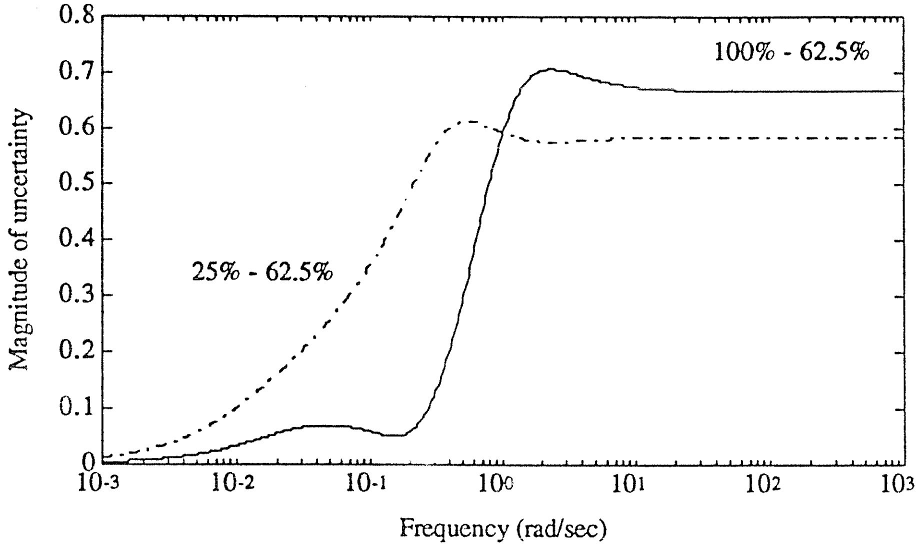

Fig. 4. Magnitude plots of the uncertainty bounds.

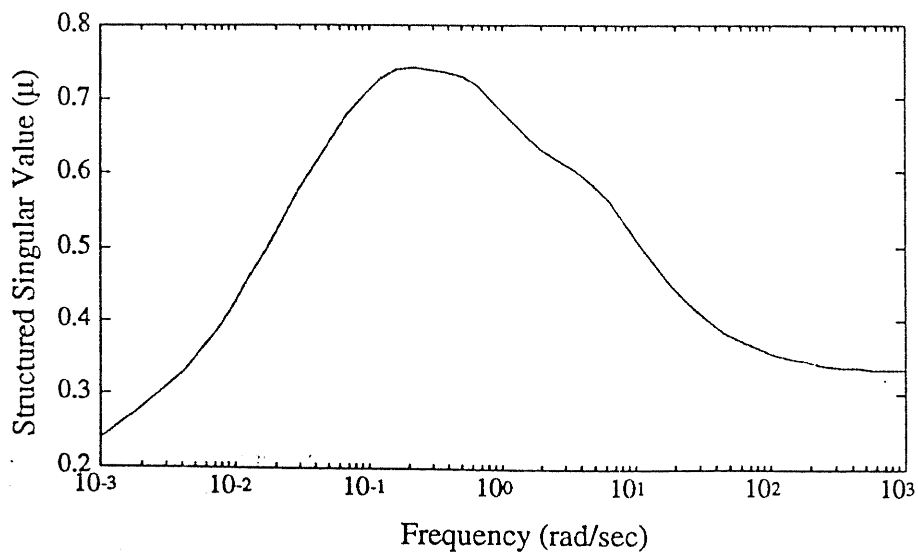

Fig. 5. The structured singular value plot μ for robust performance.

for the defined uncertainties by virtue of the main-loop theorem.[9]

IV. RESULTS AND DISCUSSION

In the development of OBLQR to achieve desirable performance and robustness to uncertainties, the locations of two dominant eigenvalues were used as an indicator of stability and performance robustness of the closed-loop control system.[1] Figure 6 presents the dominant eigenvalues of the OBLQR system when the nonlinear reactor model consisting of six delayed neutron groups is linearized at different operating points ranging from 10 to 120% power. The results under similar conditions are presented in Fig. 7 for the FF/FBC system developed in this paper. A comparison of Figs. 6 and 7 shows that the dominant eigenvalues of the closed-loop system under FF/FBC is less sensitive to variations in the reactor power than those under OBLQR. Therefore, FF/FBC is superior to OBLQR in view of stability and performance robustness. Furthermore, the FF/FBC control system is guaranteed to have robust performance over the full operating range of 25 to 100% power in the presence of prescribed uncertainties. It is also important that the FF/FBC controller was synthesized in a single step without any trial and error when the uncertainty and performance weights were specified as in Eqs. (16) and (17), respectively. In contrast, the OBLQR controller was derived via an iterative process of performance objective specification, linear quadratic regulator design, eigenvalue analysis over the power range, and simulation verification.

The simulation results of OBLQR and FF/FBC are now compared in the time domain to demonstrate the improvements achieved under FF/FBC. The system responses under both controllers are shown in Figs. 8, 9, and 10 for a load change from 100 to 25% power. A rapid power reduction may be desired to quickly lower reactor fuel temperature within the safety limits in response to off-normal operations. Safety systems, independent of automated control systems, are incorporated in nuclear plants to shut down the power generation; however, their activation can severely stress the plant, reduce the plant service life, and require a prolonged

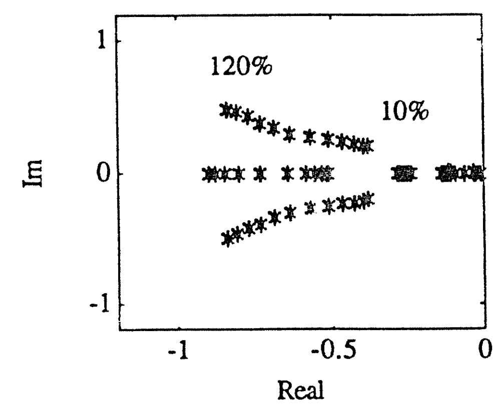

Fig. 6. Dominant eigenvalue sensitivity analysis of OBLQR.

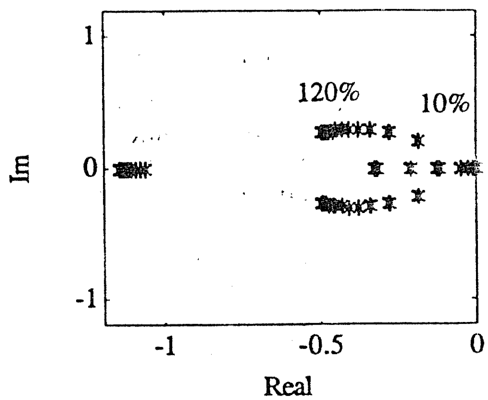

Fig. 7. Dominant eigenvalue sensitivity analysis of FF/FBC.

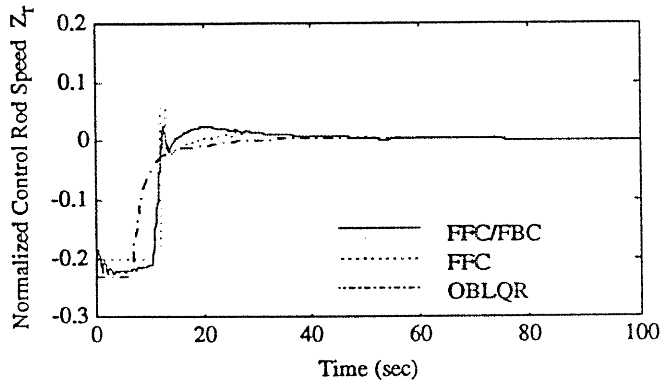

Fig. 8. Normalized control rod speed response.

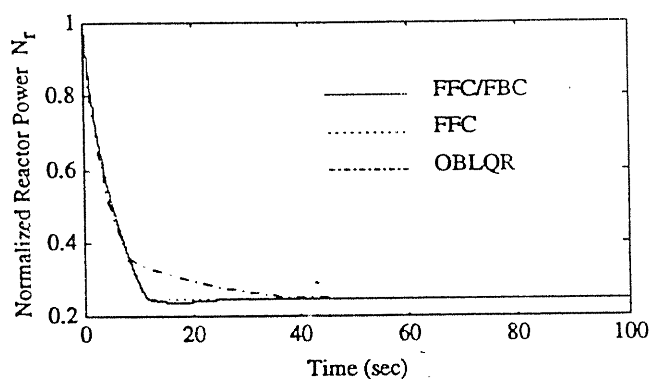

Fig. 9. Normalized reactor power response.

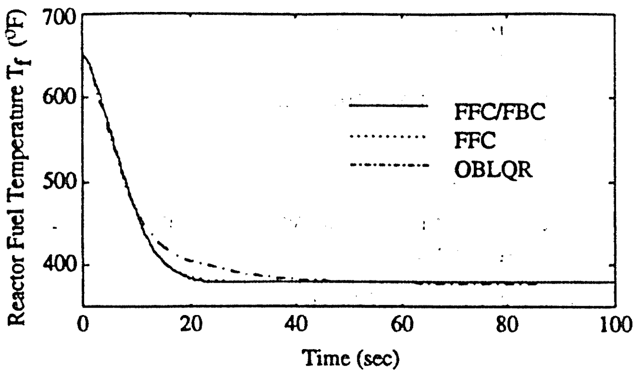

Fig. 10. Reactor fuel temperature response.

outage to restart the reactor from the hot standby condition. An efficient automated system to rapidly maneuver the plant to a safe intermediate power level in a controlled manner is thus desirable. Furthermore, the reactor fuel temperature serves as a good indicator of the structured integrity of the reactor and is thus penalized in the performance objective functionals of both OBLQR and FFC in Eqs. (12) and (14), respectively. The time domain simulation results are discussed as follows:

1. The FFC rod speed U^{ff} generated by nonlinear programming assures that it is never greater than the prespecified constraint. Since the constraints can be precisely specified, it is not necessary to include the control input in the objective functional. However, depending on the uncertainties, this constraint may be violated under FBC. In this simulation demonstration, the plant input constraints $\{u_k^{ub}\}$ and $\{u_k^{lb}\}$ in Eq. (10) were conservatively selected to avoid the constraints.

2. The OBLQR design procedure does not accommodate any constraints. However, the simulation results were generated with a control rod speed constraint. The control rod speed under OBLQR is constrained in the simulation experiment as seen in Fig. 8 to be compatible with the real-world environment where the control variables do have physical limits. An important advantage of the proposed FF/FBC control is that not only can physical constraints be accommodated in the design but also specified constraints on safety critical process variables can be taken into account in the controller design phase instead of solely relying on the plant safety systems.

3. As shown in Fig. 9, the optimized FFC system reached 99% of the desired power level in <15 s, which is much less than the time of 40 s taken by the OBLQR controller while acting on the nominal plant model. The rationale for this significantly superior performance of FFC is that OBLQR is a linear optimal controller based on the plant model linearized at 100% power level. The FFC system is synthesized via nonlinear optimization over the full operating range and hence has a much better performance.

4. The FBC compensates for the uncertainties to maintain the actual plant trajectory close to the nominal optimal trajectory. As seen in Fig. 9, the FF/FBC system reached 99% of the desired power level in <15 s, which is much less than the time of 40 s taken by the OBLQR controller while acting on the actual plant model. Figure 10 shows the significantly superior performance of FF/FBC in rapidly bringing the reactor fuel temperature down to the desired value.

The major advantages of the FF/FBC system synthesized via nonlinear programming and μ are summarized as follows:

1. The closed-loop control system under FF/FBC is expected to have a significantly superior performance compared with that of a control system that is solely based on feedback. This is possible because the feedforward controller takes the role of coarse control while the feedback controller maintains the system close to the optimal trajectory.

2. Nonlinear constraints in the time domain setting can be conveniently specified in the nonlinear programming part of the FF/FBC synthesis. This approach significantly reduces the number of iterations in the process of controller design and simulation verification when compared with the OBLQR design.

3. The μ synthesis guarantees the robust stability and robust performance of FF/FBC under specified bounds of allowable uncertainties. No ad hoc testing of the closed-loop control system is necessary.

Extension of this FF/FBC concept to the wide-range multivariable control problem of a commercial scale PWR nuclear power plant is under progress.

V. IMPLEMENTATION ISSUES FOR THE PROPOSED CONTROL STRATEGY

Implementation of the FF/FBC system should be achieved by a set of FFC input and output sequences for different operating ranges generated off-line and stored in the database of the control computer. The FBC alone should be sufficient for plant maneuvering within a relatively small operating range, say ~5% of full power. Under these circumstances, the FFC input U^{ff}, in the FF/FBC system (see Fig. 1) is set to zero, and the reference signal replaces the feedforward output sequence Y^{ff}. For wide-range operations, however, the full authority of FF/FBC should be exercised. Since only a finite number of operating ranges can be stored in the database of the control computer, a given desired operating range may not be identical to any one of the stored FFC sequences. A multistep control strategy is suggested to overcome this problem as follows:

Step 1: Select an appropriate FFC sequence whose initial and final states are close to those of the desired operating range.

Step 2: Bring the plant from the current state to the initial state of the selected FFC sequence via FBC.

Step 3: Use FF/FBC to steer the plant to the final state of the selected FFC sequence.

Step 4: Bring the plant to the desired final state via FBC.

The foregoing four steps can be conveniently executed on-line in the control computer. The FF/FBC is applicable to almost all operating conditions of practical interest.

Since the feedback controller is synthesized via the μ approach, the control law may be of high order relative to the plant model. This calls for an order reduction of the feedback controller from the viewpoint of real-time operations. After an appropriate order reduction, the FBC law should be tested via μ analysis to ensure robustness of the closed-loop system.

Implementation of the FF/FBC concept at the Pennsylvania State University TRIGA reactor is in progress. In this effort, the control law is implemented in the discrete-time setting on a commercial-grade Bailey NETWORK 90 microprocessor-based controller. Although the capabilities of FF/FBC are not fully exploited in planned experiments at the present time, preliminary results have shown excellent performance while the reactor was operated over a modest range. The results of this ongoing experimental research will be reported in a forthcoming publication.

NOMENCLATURE

c_{ri} = relative precursor density (state variables)

$\mathbf{D}$ = space of complex diagonal matrices of appropriate dimension

f_f = fraction of reactor power deposited in fuel

G = number of the delayed neutron groups

G_c = classical control gain

g_r = control rod worth

M = mass flow rate times heat capacity of the water (MW/°C)

n_r = relative power (state variable or plant output)

P_{a0} = reactor power at rated condition (MW)

Q = space of unitary matrices of appropriate dimension

T_c = average coolant temperature = $(T_e + T_l)/2$ (°C)

T_{c0} = initial coolant temperature (°C)

T_e = temperature of the water entering the reactor (°C)

T_f = average reactor fuel temperature (state variable) (°C)

T_{f0} = initial fuel temperature (°C)

T_l = temperature of the water leaving the reactor (state variable) (°C)

Z_r = control rod speed (plant input)

Greek

α_c = coolant temperature reactivity coefficient

α_f = fuel temperature reactivity coefficient

β = total delayed neutron fraction

β_i = fraction of fission neutrons that come from delayed group i

Δ = uncertainty structure

Λ = effective prompt neutron lifetime (s)

λ_i = radioactive decay constant of precursor group i (s)

μ = structured singular value

μ_f = total heat capacity of the fuel and structural material

μ_c = total heat capacity of the reactor coolant

Ω = heat transfer coefficient between fuel and coolant (MW/°C)

ρ = reactivity

ρ_r = control reactivity (state variable)

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