Damage-Mitigating Control of Power Systems for Structural Durability and High Performance

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A major goal in the control of complex power systems such as advanced aircraft, spacecraft, and electric power plants is to achieve high performance with increased reliability, availability, component durability, and maintainability. The state of the art in the synthesis of control systems for complex mechanical processes focuses on improving performance and diagnostic capabilities under constraints that do not often adequately represent the dynamic properties of the materials. The reason is that the traditional design is based upon the assumption of conventional materials with invariant characteristics. In view of high performance requirements and availability of improved materials, the lack of appropriate knowledge about the properties of these materials will lead to either less than achievable performance due to overly conservative design, or overstraining of the structure leading to unexpected failures and drastic reduction of the service life. The key idea of the research work reported in this paper is that a significant improvement in service life can be achieved by a small reduction in the system dynamic performance. The concept of damage mitigation is introduced and, to this effect, a continuous-time model of fatigue damage dynamics is formulated. Finally, the results of simulation experiments are presented for transient operations of a reusable rocket engine.

1 Introduction

A major goal in the control of complex mechanical systems such as advanced aircraft, spacecraft, and electric power plants is to achieve high performance with increased reliability, availability, component durability, and maintainability (Lorenzo and Merrill, 1991; Ray et al., 1994a, 1994b). The specific requirements are:

- Extension of the service life of the controlled plant;
- Increase of the mean time between major maintenance actions;
- Reduction of risk in the design of integrated control-structure-materials systems.

Therefore, control systems need to be synthesized by taking performance, mission objectives, service life, and maintenance and operational costs into consideration. The state of the art in the synthesis of decision and control systems for complex power systems focuses on improving dynamic performance and diagnostic capabilities under the constraints that often do not adequately represent the material properties of the critical plant components. The reason is that the traditional design is based upon the assumption of conventional materials with invariant characteristics. In view of high performance requirements and availability of improved materials that may have significantly different damage characteristics relative to conventional materials, the lack of appropriate knowledge about the properties of these materials will lead to either of the following:

- Less than achievable performance due to overly conservative design; or
- Overstraining of the structure leading to unexpected failures and drastic reduction of the useful life span.

As the science and technology of materials continue to evolve, the design methodologies for damage-mitigating control must have the capability of easily incorporating an appropriate representation of material properties. This requires augmentation of the system-theoretic techniques for synthesis of decision and control laws with governing equations and inequality constraints that would model the mechanical properties of the materials for the purpose of damage representation and failure prognosis. The major challenge in this research is to characterize the damage generation process, and then utilize this information in a mathematical form for synthesizing algorithms of robust control, diagnostics, and risk assessment in complex mechanical systems.

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2 The Damage-Mitigating Control System

The goal of the damage-mitigating control system (DCS) is to optimize the plant dynamic performance while simultane-

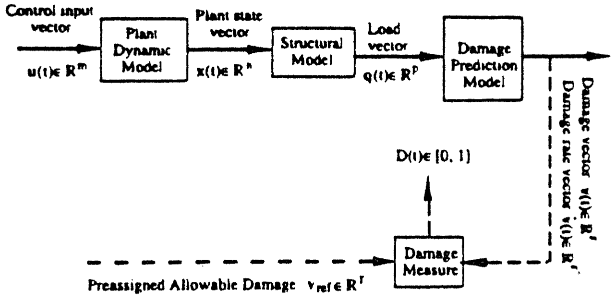

Fig. 1 A conceptual view of the damage-mitigating control system

ously maintaining the accumulated damage and the damage rate of the critical plant component(s) within prescribed limits. Figure 1 shows a conceptual view of the DCS, in which the plant model is a finite-dimensional state-space representation of the power system dynamics. The plant states are inputs to the structural model, which, in turn, generates the necessary information for the damage model. The output of the structural model is the *load vector*, which may consist of (time-dependent) stress, strain, temperature, wear, level of corrosion in gaseous and aqueous environments, and other physicochemical process variables at the critical point(s) of the structure. The damage model is a continuous-time (instead of being a cycle-based) representation of life prediction such that it can be incorporated within the DCS model. The objective is to mitigate the damage rate and accumulated damage at the critical point(s) of the structure, which may be subjected to time-dependent load. The damage state vector $\upsilon(t)$ could indicate the level of microcracking, macroscopic crack length, wear, creep, density of slip bands, etc., at one or more critical points, and its time derivative $\dot{\upsilon}(t)$ indicates how the instantaneous load is affecting the structural components. The overall damage $D(t)$ is a scalar measure of the combined damage at the critical point(s) resulting from (possibly) different sources (e.g., fatigue, creep, corrosion, or wear) relative to the preassigned allowable level υ_{ref} of the damage vector. Although $D(t)$ may not directly enter the feedback or feedforward control of the plant, it can provide useful information for decision-making such as damage prognosis and risk analysis (Ray and Wu, 1994a).

The governing equations of plant dynamic and damage prediction models in Fig. 1 are constituted by nonlinear (and possibly time-varying) differential equations which must satisfy the local Lipschitz condition (Vidyasagar, 1992) within the domain of the plant operating range. The structural model in Fig. 1 consists of solutions of structural dynamic (typically finite element) equations representing the (mechanical and thermal) load conditions. A general structure of the plant and damage dynamics and their constraints is represented in the deterministic setting as follows:

Task period: Starting time t_0 to final time t_f

Plant dynamics:

$$\frac{dx}{dt} = f(x(t), u(t), t); \quad x(t_0) = x_0 \tag{1}$$

Damage dynamics:

$$\frac{d\upsilon}{dt} = h(\upsilon(t), q(x, t)): \quad \upsilon(t_0) = \upsilon_0; \quad h > 0 \quad \forall t \tag{2}$$

Damage measure:

$$D(t) = \xi(\upsilon(t), \upsilon_{\text{ref}}) \text{ and } D(t) \in [0, 1] \tag{3}$$

Damage rate tolerance:

$$0 \le h(\upsilon(t), q(x, t)) < \beta(t) \quad \forall t \in [t_0, t_f] \tag{4}$$

Accumulated damage tolerance:

$$[\upsilon(t_f) - \upsilon(t_0)] < \Gamma \tag{5}$$

where

$x \in \mathbf{R}^n$ is the plant state vector;
$u \in \mathbf{R}^m$ is the control input vector;
$\upsilon \in \mathbf{R}^r$ is the damage state vector;
$\upsilon_{\text{ref}} \in \mathbf{R}^r$ is the preassigned limit for the damage state vector;
$\beta(t) \in \mathbf{R}^r$ and $\Gamma \in \mathbf{R}^r$ are specified tolerances for the damage rate and accumulated damage, respectively;
$q \in \mathbf{R}^p$ is the load vector; and
$D \in [0, 1]$ is a scalar measure of the accumulated damage.

The vector differential Eqs. (1) and (2) become stochastic if the initial plant and damage states, namely, x_0 and υ_0, are random vectors, or if uncertainties in the plant and material characteristics are included in the models as random parameters, or if the plant is excited by discrete events occurring at random instants of time (Sobczyk and Spencer, 1992). The stochastic aspect of damage-mitigating control is a subject of future research, and is not addressed in this paper.

Given an initial condition, the open-loop control policy is obtained via nonlinear programming (Luenburger, 1984) by minimizing a specified cost functional under the prescribed constraints of damage rate and accumulated damage. The objective is to minimize a specified cost functional J (which includes plant state, damage rate, and control input vectors) without violating the prescribed upper bounds of the damage rate and the accumulated damage. The cost functional J is to be chosen in an appropriate form representing a trade-off between the system performance and the damage. The optimization problem is then formulated as follows:

$$\text{Minimize:} \quad J = \sum_{k=0}^{N-1} J_k(\tilde{x}_k, \dot{\upsilon}_k, \tilde{u}_k) \tag{6}$$

$$\text{Subject to:} \quad 0 \le h(\upsilon_k, q(x_k), k) < \beta(k) \text{ and}$$
$$(\upsilon_N - \upsilon_0) < \Gamma \text{ for } k = 1, 2, 3, \ldots, N \tag{7}$$

where $\tilde{x}_k = x_k - x_{ss}$ and $\tilde{u}_k = u_k - u_{ss}$ are deviations of the plant state vector and the control input vector from the respective final steady-state values of x_{ss} and u_{ss}; and $\beta(t) \in \mathbf{R}^r$ and $\Gamma \in \mathbf{R}^r$ are specified tolerances for the damage rate and accumulated damage, respectively.

The open-loop control law was synthesized by minimizing the cost functional in Eq. (6) under: (i) the inequality constraints in Eq. (7); and (ii) the condition that, starting from the initial conditions $x(t_0)$ and $\upsilon(t_0)$, the state trajectory must satisfy the plant dynamic model in Eq. (1). The design variables to be identified are the control inputs, u_k, $k = 0, 1, 2, \ldots, N - 1$, and the goal is to search for an optimal control sequence $\{u_k\}$.

3 Modeling of Damage Dynamics

Damage of mechanical structures is usually a result of fatigue, creep, corrosion, and their combinations (Suresh, 1991). The prime focus in this research is representation of fatigue damage in the continuous-time setting. As discussed earlier, a time-dependent model of damage dynamics, having the structure of Eq. (2), is necessary for analysis and synthesis of DCS. From this perspective, a dynamic model of fatigue damage has been formulated in the continuous-time setting. Although this damage model has a deterministic structure, it can be recast in the stochastic setting to include the effects of both unmodeled dynamics and parametric uncertainties.

Because of the wide ranges in mechanical properties of materials, extensive varieties of experiments have been conducted for fatigue analysis, and many models have been

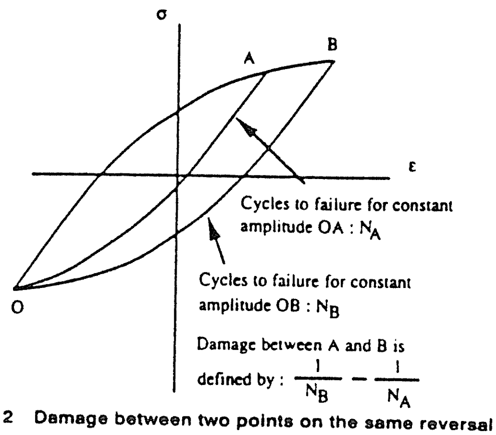

Fig. 2 Damage between two points on the same reversal

proposed for fatigue life prediction in aerospace and ground vehicles (Newman, 1981; Tucker and Bussa, 1977). Each of these models expresses the damage dynamics by an equation with the number of cycles N as the independent variable. In contrast, the damage dynamics in Eq. (2) are expressed as a vector differential equation with respect to time, t, as the independent variable. The advantages of this approach are that it allows the damage model to be incorporated within the constrained optimization problem and that the damage accumulated between any two instants of time can be derived even if the stress-strain hysteresis loop is not closed. This concept is applicable to different models of damage dynamics such as those resulting from cyclic strain or crack propagation. To this end, we propose to model the continuous-time dynamics of fatigue damage based on the following two approaches:

- *Cyclic Strain Life*: In this approach, the local stress-strain behavior is analyzed at certain critical points where failure is likely to occur. The local strain may be directly measured from a strain gage, or computed via finite element analysis. The local stress is estimated from the cyclic stress-strain curve. A cycle-based approach is then used to estimate the fatigue damage from the strain-life curves at different levels of stress and strain in the load history. We propose to determine the damage accumulation within a cycle using the classical Palmgren-Miner rule and subsequently modify it via the damage curve approach of Bolotin (1989).
- *Linear Elastic Failure Mechanics* (*LEFM*): The LEFM approach is built upon the concept of a physical measure of damage in terms of the crack length and the size of the plastic zone at the crack tip (Wheeler, 1972). The accumulated damage is computed by integrating the crack growth rate over the number of cycles. This is based on the crack growth rate equation being approximated by an exponential function of stress intensity factor range of the component (Bannantine et al., 1990). The component is assumed to fail when the crack reaches the critical length, which, in turn, is determined from the fracture toughness of the component on the basis of experimental data.

3.1 Damage Modeling via Cyclic Strain Life. The cyclic strain-life approach recognizes that the fatigue life is primarily controlled by the local strain at the critical point(s) of the component. The first goal is to model linear damage accumulation in the continuous-time setting. Referring to Fig. 2, let point O be the starting (reference) point of a reversal, let A and B be two consecutive points on the same rising reversal, and let N_A and N_B represent the total number of cycles to failure with constant load amplitudes, $OA/2$ and $OB/2$, respectively. Then, the half-cycle increment of linear damage δ between points A and B is defined as:

$$\Delta\delta_{A,B} = \frac{1}{N_B} - \frac{1}{N_A} \tag{8}$$

In Eq. (8), it is assumed that the damage occurs only on the rising reversal, i.e., if the stress is monotonically increasing, and no damage occurs during unloading, i.e., if the stress is monotonically decreasing. This assumption is consistent with the physical phenomena observed in the fatigue crack propagation process. Given that $\Delta\sigma$ is the stress increment between point A and point B, the average damage rate with respect to this stress change is equal to $\Delta\delta/\Delta\sigma$. Let Δt be the time interval from A to B, the average rate of linear damage δ in terms of the stress σ can be transformed into the time domain by $\Delta\delta/\Delta t = (\Delta\delta/\Delta\sigma) \times (\Delta\sigma/\Delta t)$. Making Δt infinitesimally small, the instantaneous damage rate becomes $d\delta/dt = (d\delta/d\sigma) \times (d\sigma/dt)$ where the instantaneous stress rate $d\sigma/dt$ can be generated from direct measurements of strain rate or the finite element analysis, and $d\delta/d\sigma$ is derived from the existing cycle-based formulae (Bannantine et al., 1990). The strain-life and the cyclic stress-strain curves are used to evaluate $d\delta/d\sigma$. Replacing the number of cycles to failure, N_f, by $1/\Delta\delta$ where $\Delta\delta$ is the increment in linear damage during one cycle, the strain life relationship (Dowling, 1983) in terms of the elastic damage and plastic damage modes is written as:

$$\frac{|\epsilon - \epsilon_r|}{2} = \frac{\sigma_f' - \sigma_m}{E}\left(\frac{\delta}{2}\right)^{-b} + \epsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}\left(\frac{\delta}{2}\right)^{-c} \tag{9}$$

where b, c, σ_f', and ϵ_f' are material constants; σ_m is the mean stress; ϵ_r is the total strain corresponding to the reference stress σ_r at the starting point of a given reversal as determined from the rainflow cycle counting method; and $|\epsilon - \epsilon_r|/2$ is the strain amplitude between the current point and the reference point. This equation does not provide a closed-form solution for the predicted damage δ. The general approach to solve this problem is to separate Eq. (9) into two different modes. The first term on the right side is a modification of the Basquin equation and corresponds to the so-called elastic damage δ_e. The second term on the right side of Eq. (9) is a modification of the Coffin-Manson equation and corresponds to the so-called plastic damage δ_p. These two damages, δ_e and δ_p, can be derived separately in an explicit form.

$$\frac{\Delta\epsilon_e}{2} = \frac{\sigma_f' - \sigma_m}{E}\left(\frac{\Delta\delta_e}{2}\right)^{-b} \quad \text{(modified Basquin equation)} \tag{10a}$$

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f'\left(1 - \frac{\sigma_m}{\sigma_f'}\right)^{\frac{c}{b}}\left(\frac{\Delta\delta_p}{2}\right)^{-c} \quad \text{(modified Coffin-Manson equation)} \tag{10b}$$

where the elastic and plastic strain amplitudes, $\Delta\epsilon_e/2$ and $\Delta\epsilon_p/2$, are related to the state of stress following the cyclic stress-strain characteristics:

$$\frac{\Delta\epsilon_e}{2} = \frac{|\sigma - \sigma_r|}{2E} \tag{11a}$$

$$\frac{\Delta\epsilon_p}{2} = \left(\frac{|\sigma - \sigma_r|}{2K'}\right)^{\frac{1}{n'}} \tag{11b}$$

where n' is the cyclic strain hardening exponent, K' is the cyclic strength coefficient, and σ_r is the reference stress obtained from the cycle counting under spectral loading. From Eqs. (10a), (10b), (11a), and (11b), the closed-form solutions for $\Delta\delta_e$ and $\Delta\delta_p$ can be obtained in terms of stress instead of strain as given below:

$$\Delta \delta_e = 2 \times \left(\frac{|\sigma - \sigma_r|}{2(\sigma_f' - \sigma_m)} \right)^{-\frac{1}{b}} \qquad (12a)$$

$$\Delta \delta_p = 2 \times \left(\frac{1}{\epsilon_f'} \times \left(\frac{|\sigma - \sigma_r|}{2K'} \right)^{\frac{1}{n'}} \times \left(1 - \frac{\sigma_m}{\sigma_f'} \right)^{-\frac{c}{b}} \right)^{-\frac{1}{c}} \qquad (12b)$$

Step changes in the reference stress σ_r can occur only at isolated points in the load spectrum. Since the damage increment is zero at any isolated point, the damage accumulation can be evaluated at all points, excluding these isolated points, which constitute a set of zero Lebesgue measure (Royden, 1988). Exclusion of the points of step changes in σ_r does not cause any error in the computation of damage, and $d\sigma_r/dt$ can be set to zero because σ_r is piecewise constant. Further, since it is assumed that no damage occurs during unloading, the damage rate can be made equal to zero if $\sigma < \sigma_r$. When the stress is increasing, i.e., $\sigma > \sigma_r$, the elastic damage rate $d\delta_e/dt$ and the plastic damage rate $d\delta_p/dt$ are computed from Eqs. (12a) and (12b) in terms of the instantaneous stress rate $d\sigma/dt$ as:

If $\sigma \geq \sigma_r$, then

$$\frac{d\delta_e}{dt} = 2 \times \frac{d}{d\sigma} \left(\left(\frac{\sigma - \sigma_r}{2(\sigma_f' - \sigma_m)} \right)^{-\frac{1}{b}} \right) \times \frac{d\sigma}{dt}, \text{ and} \qquad (13a)$$

$$\frac{d\delta_p}{dt} = 2 \times \frac{d}{d\sigma} \left(\left(\frac{1}{\epsilon_f'} \times \left(\frac{\sigma - \sigma_r}{2K'} \right)^{\frac{1}{n'}} \times \left(1 - \frac{\sigma_m}{\sigma_f'} \right)^{-\frac{c}{b}} \right)^{-\frac{1}{c}} \right) \times \frac{d\sigma}{dt}; \qquad (13b)$$

else $d\delta_e/dt = 0$ and $d\delta_p/dt = 0$.

The damage rate $d\delta/dt$ is then obtained as the weighted average of the elastic and plastic damage rates as:

$$\frac{d\delta}{dt} = w\frac{d\delta_e}{dt} + (1 - w)\frac{d\delta_p}{dt} \qquad (14)$$

where the weighting function, w, is selected on the basis of the elastic and plastic strain amplitudes in Eqs. (11a) and (11b) as:

$$w = \frac{\Delta\epsilon_e}{\Delta\epsilon_e + \Delta\epsilon_p} \text{ and } 1 - w = \frac{\Delta\epsilon_p}{\Delta\epsilon_e + \Delta\epsilon_p} \qquad (15)$$

Equations (13) to (15) are then used to obtain the damage rate at any instant of time. The damage increment between two consecutive points t_k and t_{k-1} on the same reversal can be calculated by integrating the damage differential $d\delta$. This continuous-time damage model has been verified using the experimental data generated in the SAE cumulative fatigue test program (Newman, 1981) for Man-Ten steel. A comparison of the model results and experimental data is reported by Ray and Wu (1994b) and Wu (1993).

The continuous-time damage model in Eqs. (13) to (15) is derived on the basis of linear damage accumulation following the Palmgren–Miner rule. Although this concept of linear damage accumulation has been widely used due to its simplicity in computation, the cumulative damage behavior is actually nonlinear (Bolotin, 1989; Suresh, 1991). Experimental results show that, for variable amplitude loading, the accumulated damage is dependent on the order in which the load cycles are applied. This phenomenon is known as the sequence effect, which is further explained in the next section. Since the structural components of complex mechanical systems are subjected to loads of varying amplitude, the linear rule of damage accumulation, which is commonly used for fatigue life assessment, could lead to erroneous results due to this sequence effect. Therefore, a nonlinear damage rule needs to be established for accurate prediction of the damage rate and damage accumulation in the critical components.

3.2 Modeling of Nonlinear Cumulative Damage Using the Damage Curve Approach. The concept of a nonlinear damage curve to represent the damage was first conceived by Marco and Starkey (1954). No mathematical representation of damage curve was proposed at that time because the physical process of damage accumulation was not adequately understood. Manson and Halford (1981) proposed the double linear rule primarily based on the damage curve approach for treating cumulative fatigue damage. In their paper, an effort was made to represent the damage curve mathematically and approximate it by two piecewise line segments. The total fatigue life was then divided into two phases so that the linear damage rule can be applied in each phase of the life. A concept similar to the damage curve approach was proposed by Bolotin (1989) with a mathematical representation, which does not necessarily assess the damage on the basis of cycles and is more appropriate for modeling in the continuous-time setting. Bolotin used the following analytical relationship between D and δ:

$$D = (\delta)^{\gamma} \qquad (16)$$

In Eq. (12), the γ parameter describes the nonlinearity of the damage curve and is usually assumed to be dependent solely on the stress amplitude level (Manson and Halford, 1981). High-strength materials usually yield very large values of γ especially under high-cycle fatigue. At the initial stage of the fatigue life, a large γ creates a very small damage, which could lead to numerical and computational problems. This necessitates formulation of a computationally practical method of damage prediction. It follows from a crack propagation model such as the Paris model (Paris and Erdogan, 1963) that the crack growth rate is dependent not only on the stress amplitude but also on the current crack length. Recognizing the fact that the crack itself is an index of accumulated damage, it is reasonable to assume the γ parameter to be dependent on both stress amplitude and the current level of damage accumulation, i.e., $\gamma = \gamma(\sigma_a, D)$. Therefore, we propose the following modification of Eq. (16):

$$D = (\delta)^{\gamma(\sigma_a, D)} \qquad (17)$$

where D and δ are the current states of nonlinear and linear damage accumulation, respectively. Although Eq. (13) has an implicit structure, it can be solved via a recursive relationship.

Ray and Wu (1994a) have developed a nonlinear damage model in the continuous-time setting. In this model, the accumulated damage at any point within a reversal can be obtained by solving the following nonlinear equations:

$$\gamma = \gamma(\sigma_a, D) \text{ and } D = (\delta + \Delta\delta)^{\gamma} \qquad (18)$$

where δ is the linear damage at the reference point of a reversal and $\Delta\delta$ is the linear damage increment for the stress amplitude σ_a relative to the reference point. A noniterative numerical approach that operates on a set of discrete points in the load history has been developed for solving the set of implicit Eqs. (18).

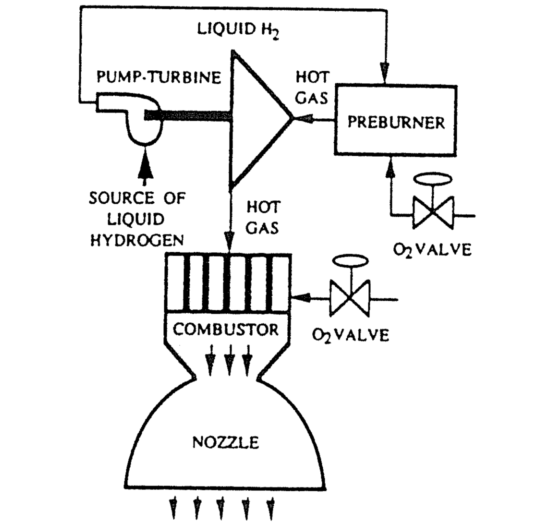

Fig. 3 Schematic diagram of a bipropellant rocket engine

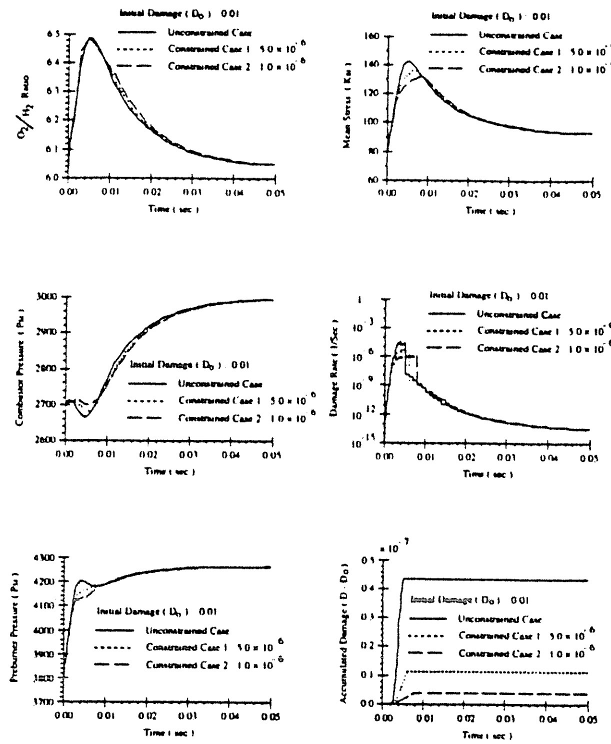

Fig. 4 Upthrust transients of a bipropellant rocket engine

3.3 The γ Parameter Fitting for the Nonlinear Cumulative Damage Model. One major task in this approach is to identify a mathematical representation for the γ parameter as a function of the stress amplitude and the current damage state. It requires knowledge of the physical process of damage accumulation, which may be obtained from either experimental data or a combination of experimental data and analysis, and an appropriate definition of damage. The γ parameters are different, in general, for different materials and follow different structures of the governing equations. Furthermore, because the mechanisms attributed to the damage accumulation at various stages of fatigue life are different, no single approach apparently provides a sufficiently accurate prediction of damage throughout the fatigue life of a component. It is difficult, if not impossible, to construct a single structure for representation of γ. An alternative approach is to evaluate γ by interpolation. Once the damage is appropriately defined, and an analytical method for damage prediction is selected, the information needed for the nonlinear damage model can be generated via experimentation or analysis for a set of constant stress amplitudes. The values of γ are then computed by Eq. (17) for a given stress amplitude and the damage data ranging from the initial damage state D_0 to the failure condition at $D = 1$. The generated data, γ versus D, are then plotted for various amplitudes of stresses. These curves can be either fitted by nonlinear equations, if possible, or as linear piecewise representations. Since it is not practical to perform experiments at infinitesimally small increments of stress amplitude, γ can be only experimentally evaluated at selected discrete levels of stress amplitude. The values of γ for other stress amplitudes can then be interpolated. Since the characteristics of γ may strongly depend on the type of the material, availability of pertinent experimental data for the correct material is essential for the damage-mitigating control synthesis. The γ parameter has been fitted based on the experimental data (Swain et al., 1990) for AISI 4340 steel. Details are reported by Ray and Wu (1994b).

4 Simulation Results and Discussion

The damage mitigation concept is verified by simulation experiments for open loop control of a reusable rocket propulsion engine such as one described by Sutton (1992) and Duyar et al. (1991). The plant model under control is a simplified representation of the dynamic characteristics of a bipropellant rocket engine as shown in Fig. 3. The preburner serves as a gas generator for driving the liquid hydrogen (LH_2)-fuel turbopump. In this model, oxidant is separately supplied to the preburner and the main combustor chamber. Standard lumped parameter methods have been used to model the nonlinear plant dynamics in state-space form where the plant state vector consists of eight state variables, namely turbine shaft speed, pump (LH_2-fuel) mass flow rate, preburner gas pressure, preburner gas density, combustor gas pressure, combustor gas density, and the positions of the two oxidant flow valves.

The critical plant output variables are combustor gas pressure and the oxidant/fuel (O_2/H_2) ratio, and the control inputs are commands for manipulation of the two oxidant valves. The structural model (Ray and Wu, 1994b) represents the cyclic mechanical stresses at the root of a typical turbine blade that is presumed to be a critical component. The blade is represented by a three-node beam model with six degrees of freedom at each node while the first node is kept fixed. The load on the blade is assumed to consist of two components. The first component is due to the (time-dependent) drive torque, which is derived as an output of the plant model. The second component is a dynamic term, which represents the oscillatory load on the blade as it passes each stator. It is the second component that causes high cycle fatigue at the root of the blade while the first component is largely responsible for the mean stress. The fatigue damage model formulated in Sections 2 and 3 was used to generate the results in Fig. 4.

The purpose of this simulation study is to examine the dynamic performance and damage of the nominal plant when the oxidant valves are manipulated to vary the engine thrust according to the open loop control policy described in Section 2. To demonstrate the broad concepts of fatigue damage mitigation, the nominal plant model was used in these simulation experiments with exact initial conditions and no disturbances and noise. However, if these conditions are not met, additional feedback control will be necessary because the open loop control alone would be inadequate. Following the

Table 1 The constraints for the simulation experiment

Time	Contrained case 1	Constrained case 2
0 ms to 2 ms	$\beta = 1.0 \times 10^{-6}s^{-1}$	$\beta = 0.2 \times 10^{-6}s^{-1}$
2 ms to 3 ms	$\beta = 2.5 \times 10^{-6}s^{-1}$	$\beta = 0.5 \times 10^{-6}s^{-1}$
3 ms to 50 ms	$\beta = 5.0 \times 10^{-6}s^{-1}$	$\beta = 1.0 \times 10^{-6}s^{-1}$

structure in Eq. (6), the cost functional J for nonlinear programming was selected as:

$$J = \sum_{k=0}^{N-1} \left[\tilde{x}_k^T Q \tilde{x}_k + \dot{\upsilon}_k^T S \dot{\upsilon}_k + \tilde{u}_k^T R \tilde{u}_k + W(O_2/H_2)_k^2 \right] \quad (15)$$

where the deviations, $\tilde{x}_k$ and $\tilde{u}_k$, in the plant state vector and the control input vector are defined in Eq. (6); and the diagonal matrices Q, R, and W, and the scalar S serve as relative weights on the individual variables. Since the rocket engine performance is very sensitive to the oxidant/fuel (O_2/H_2) ratio, it was brought into the cost functional in Eq. (15) to prevent any large deviations from the desired value of 6.02 through the transients. In the future research, if the main combustion chamber is considered as one of the critical components for damage mitigation, then the explicit dependence of the cost functional on the O_2/H_2 ratio may not be necessary. The simulation results in the plates of Fig. 4 were obtained under the following conditions:

Chamber pressure weight $Q_{55} = 12$;
and all other weights $Q_{ii} = 1$, $i \neq 5$,
Control input weight $R = I$ where I is the identity matrix,
Damage rate weight $S = 0$; and O_2/H_2 ratio weight $W = 10$.

The rocket engine model is initiated from an initial equilibrium condition at 2700 psi chamber pressure and O_2/H_2 ratio of 6.02. From this condition the optimization procedure steers the plant to a new equilibrium position at 3000 psi and the same O_2/H_2 ratio of 6.02 in 50 milliseconds. The control commands to the two oxidant valves are updated at every one millisecond. That is, N is equal to 50 in Eq. (15). The performance cost to be minimized is based on the deviations from the final equilibrium condition at 3000 psi. Simulation experiments were conducted with no constraints on the accumulated damage. Assuming that the blade is already used or minor manufacturing defects are present, the initial damage of the turbine blade is set to $D_0 = 0.01$. Two constrained cases along with the unconstrained case are simulated to examine the effects of damage rate constraints on the dynamic responses of the system. These constraints are listed in Table 1.

The plates on the left side of Fig. 4 exhibit the effects of the varying oxygen inlet flow into the preburner and the main combustor on the resulting transients of the process variables, namely, O_2/H_2 ratio, and the pressure in the preburner and main combustion chamber, for the given simulation condition. For a given level of initial damage, pressure dynamics tend to be slower as the constraint is made more severe. The combustor pressure is seen to rise monotonically in all cases after a small dip at about 2.5 ms while the preburner pressure keeps on increasing. These plots are largely similar except for the transients from 1 ms to 10 ms. Virtually all fatigue damage accumulation in this transient operation takes place during this short interval, and this results in a peak overshoot in the mean stress. This sharp increase in stress, displayed in the top plate on the right side of Fig. 4, is the cause of enhanced damage in the turbine blades. Since the turbine blades are the critical components for damage analysis in this study, the pressure ratio across the turbine that directly influences the torque is very important. For a given preburner pressure, a reduction in the combustor pressure causes an increase in the turbine torque. It is the turbine torque and speed that set the cyclic stress and fatigue damage on the turbine blades. Therefore, the dip in the combustor pressure at about 2.5 ms is largely responsible for the peak mean stress. Both the mean stress and stress amplitude are the basic inputs to the damage model.

The two plates on the bottom right side of Fig. 4 compare the damage rate and the accumulated damage for the given simulation condition. For the unconstrained case, the peak stress causes the largest overshoot in the damage rate, which is plotted on a logarithmic scale. The accumulated damage, plotted on a linear scale, is seen to be significantly influenced by the constraints. This suggests that, for reusable rocket engines, the constraints need to be appropriately specified. The damage rate is dependent on the two sequences of control commands, and the oxidant flows into the preburner and combustor are changed in response.

The important observation in these simulation experiments is the substantial reduction in the accumulated damage, which will extend the service life of the turbopump. The accumulated damage in the unconstrained case is seen to be about four to twelve times that of the constrained case. This is a clear message that the consideration of damage in the control of transients to which a rocket engine is exposed can have a considerable impact on the life of critical components (in this case, the turbine blades). It is noted that there is practically insignificant penalty in the response times of the chamber pressure, i.e., the engine thrust, between the unconstrained and the constrained cases. If one is willing to pay a small price in response time, much larger gains on damage accumulation can be achieved. Additional simulation results, not presented in this paper but reported by Wu (1993) and Ray et al. (1994b), show the effects of nonlinear damage accumulation, which are generated under three different initial values of the accumulated damage, namely, $D_0 = 0.001$, 0.01, and 0.1 while the damage rate constraint is set identical to that of the constrained Case 1 in Table 1. The damage rate and damage accumulation are found to be strongly dependent on the initial damage D_0. This suggests that, for reusable rocket engines, the constraints must be specified based on the knowledge of the initial damage D_0, which must be accurately assessed during the previous flight. However, if D_0 cannot be accurately assessed, then it might be safe to generate the control command sequences on the assumption of a conservative, i.e., larger, value of the initial damage at the expense of the engine performance.

The graphs in Fig. 4 exemplify the effects of upthrust transients for a short period of 50 ms. Complete operations of a rocket engine during a single flight include many such upthrust transients, and the steady-state operation may last for several hundreds of seconds. Although the damage rate during steady state is much smaller than that during a transient operation, the total damage accumulation during steady state may not be relatively insignificant. Therefore, during one flight of a (reusable) rocket engine, the cumulative effects of the transient and steady-state operations need to be considered for estimation of total accumulated damage.

The simulation experiments, described above, only consider a single point of critical stress, namely, the turbine blades. In this case, the damage vector is one dimensional. Simultaneous control of damage at several other critical areas in the rocket engine, such as the nozzle lining, shall render the damage vector to be multidimensional. The optimization problem is then to generate control sequences that will not only make a trade-off between the performance and damage but also strike a balance between potentially conflicting requirements of damage mitigation at these critical points.

5 Summary and Conclusions

The theme of the research in damage-mitigating control of power systems as reported in this paper is summarized as follows:

- For control of complex power systems, on-line damage prediction and damage mitigation are carried out based on the available sensory and operational information such that the plant can be inexpensively maintained, and safely and efficiently steered under diverse operating conditions.
- High performance is achieved without overstraining the mechanical structures such that the functional life of critical components is increased resulting in enhanced safety, operational reliability, and availability.

Extended life coupled with enhanced safety and high performance will have a significant economic impact in diverse industrial applications. Furthermore, as the science and technology of materials evolve, the damage characteristics of the structural components can be incorporated within the framework of the proposed damage-mitigating control system.

This ongoing research in damage-mitigating control addresses interdisciplinary work in the fields of active control technology and structural integrity, as applied to fatigue life in particular. Efficacy of the proposed damage mitigation concept is evaluated for life extension of the turbine blades of a bipropellant rocket engine via simulation experiments.

Applications of damage-mitigating control of power systems include a wide range of engineering applications such as reusable rocket engines for space propulsion, rotating and fixed wing aircraft, electric power generation plants, automotive and truck engine/transmission systems, and large rolling mills. In each of these systems, damage-mitigating control can enhance safety and productivity accompanied by reduced life cycle cost. A continuous-time damage model will allow timely warnings of these failures, and the resulting decision and control actions will not only avoid an early shutdown but also improve maintainability. Such incidents could be alleviated by a relatively small reduction in plant performance if the control system is structured with the capability of predicting these impending failures and thereby exercising authority over the lower level control modules. A more complex application of the damage mitigation concept is the startup and scheduled shutdown of rocket engines and power plants, and take-off and landing of aircraft, in which the damage information can be utilized for real-time plant control either in the fully automated mode or with human operator(s) in the loop. Another example is the control of automotive engine and transmission systems to reduce stresses in the drive-train components without comprising driving performance and comfort. The result will be a combination of extended life and reduced drive-train mass with associated savings in the fuel consumption.

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References

Bannantine, J. A., Comer, J. J., and Handrock, J. L., 1990, *Fundamentals of Metal Fatigue Analysis*, Prentice Hall, Englewood Cliffs, NJ.

Bolotin, V. V., 1989, *Prediction of Service Life for Machines and Structures*, ASME, New York.

Dowling, N. E., 1983, "Fatigue Life Prediction for Complex Load Versus Time Histories," ASME *Journal of Engineering Materials and Technology*, Vol. 105, pp. 206–214.

Duyar, A., Eldem, V., Merrill, W. C., and Guo, T-H., 1991, "State Space Representation of the Open-Loop Dynamics of the Space Shuttle Main Engine," ASME *Journal of Dynamic Systems, Measurement, and Control*, Vol. 113, pp. 684–690.

Lorenzo, C. F., and Merrill, W. C., 1991, "Life Extending Control: A Concept Paper," *Proc. American Control Conference*, Boston, MA, June, pp. 1080–1095.

Luenberger, D. G., 1984, *Linear and Nonlinear Programming*, Addison-Wesley, Menlo Park, CA.

Manson, S. S., and Halford, G. R., 1981, "Practical Implementation of the Double Linear Damage Rule and Damage Curve Approach for Treating Cumulative Fatigue Damage," *Int. Journal of Fracture*, Vol. 17, No. 2, pp. 169–192.

Marco, S. M., and Starkey, W. L., 1954, "A Concept of Fatigue Damage," *Trans. ASME*, Vol. 76, No. 4, pp. 627–632.

Newman, J. C., Jr., 1981, "A Crack Closure Model for Predicting Fatigue Crack Growth Under Aircraft Spectrum Loading," *Methods and Models for Predicting Fatigue Crack Growth Under Random Loading*, ASTM STP 748, pp. 53–84.

Paris, P. C., and Erdogan, F., 1963, "A Critical Analysis of Crack Propagation Laws," ASME *Journal of Basic Engineering*, Vol. D85, pp. 528–534.

Ray, A., Wu, M-K., Carpino, M., and Lorenzo, C. F., 1994a, "Damage-Mitigating Control of Mechanical Systems: Part I—Conceptual Development and Model Formulation," ASME *Journal of Dynamic Systems, Measurement, and Control*, Vol. 116, No. 3, pp. 437–447.

Ray, A., Wu, M-K., Carpino, M., and Lorenzo, C. F., 1994b, "Damage-Mitigating Control of Mechanical Systems: Part II—Formulation of an Optimal Policy and Simulation," ASME *Journal of Dynamic Systems, Measurement, and Control*, Vol. 116, No. 3, pp. 448–456.

Ray, A., and Wu, M-K., 1994a, "Fatigue Damage Control of Mechanical Systems," *SMART Materials and Structures*, Vol. 3, No. 1, pp 47–58.

Ray, A., and Wu, M-K., 1994b, "Damage-Mitigating Control of Space Propulsion Systems for High Performance and Extended Life," NASA Contractor Report 194470.

Royden, H. L., 1988, *Real Analysis*, 3rd ed., Macmillan Publishing Company, New York.

Sobczyk, K., and Spencer, B. F., Jr., 1992, *Random Fatigue: Data to Theory*, Academic Press, Boston, MA.

Suresh, S., 1991, *Fatigue of Materials*, Cambridge University Press, Cambridge, United Kingdom.

Sutton, G. P., 1992, *Rocket Propulsion Elements*, Wiley Interscience, New York.

Swain, M. H., Everett, R. A., Newman, J. C., Jr., and Phillips, E. P., 1990, "The Growth of Short Cracks in 4340 Steel and Aluminum–Lithium 2090," AGARD Report, No. 767, pp. 7.1–7.30.

Tucker, L., and Bussa, S., 1977, "The SAE Cumulative Fatigue Damage Test Program," *Fatigue Under Complex Loading: Analyses and Experiments*, Vol. AE-6, pp. 1–54.

Vidyasagar, M., 1992, *Nonlinear Systems Analysis*, 2nd ed., Prentice-Hall, Englewood Cliffs, NJ.

Wheeler, O. E., 1972, "Spectrum Loading and Crack Growth," ASME *Journal of Basic Engineering*, Vol. 94, pp. 181–186.

Wu, M-K., 1993, "Damage-Mitigating Control of Mechanical Systems," Ph.D. Dissertation in Mechanical Engineering, The Pennsylvania State University, University Park, PA.