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Technical Note

Robust Multivariable Control of Rotorcraft in Forward Flight: Impact of Bandwidth on Fatigue Life

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This paper presents the analysis and synthesis of a multivariable control systems for a UH-60 Black Hawk helicopter in forward flight by taking the fatigue life of critical components into consideration. In particular, this paper investigates the impact of the control system bandwidth on both the handling qualities of the helicopter and the fatigue life of a rotor component control horn. The results of non-linear model simulations of on-axis doublets and one maneuver reveals that fatigue life is very sensitive to the control system bandwidth and the level of initial damage. For example, a 45° right turn at 140 knots may cause a 148% increase in fatigue damage in the control horn for only a 1.3 radian per second increase in control system bandwidth.

Nomenclature

u, v, w, p, q, r, θ, ϕ, ψ	Fuselage body states
$\dot{h}$	Climb rate
$W_d(s)$, $W_c(s)$, $W_i(s)$, $W_p(s)$	Weighting functions
M(s)	Transfer function of the idea helicopter model
$G_0(s)$	Nominal helicopter plant transfer function
A(s)	Actuator transfer function
K(s)	Controller transfer function
γ	Exponent
$\sigma(t)$	Stress at critical location
$\dot{D}(t)$, D(t), D_L	Accumulated damage, damage rate, linear accumulated damage

Introduction

Future rotorcraft are required to be light weight, cost effective, and fly more aggressively. Therefore emphasis must be placed on improving the handling qualities performance, structural durability, and operating costs of the rotorcraft. In addition, the impact of the control system design on fatigue life and cost also needs to be assessed. For example, the ADS-33 Handling Qualities Specification states the minimum achievable bandwidth requirement for each axis. The maximum allowable bandwidth is more abstract and is not reflected in any specification. However, it is widely accepted that the structural durability of critical components (and hence the operating cost) represents a maximum bandwidth limit. Typically control system designers focus on achieving and exceeding the minimum handling qualities bandwidth requirements with not much consideration given to the fatigue life of critical rotor components. This investigation shows that the fatigue life of rotorcraft components is sensitive to small changes in the controller bandwidth. Due to space limitation only one airspeed and one maneuver are discussed in this paper. Nevertheless, it demonstrates the basic understanding of how the control system bandwidth affects the overall handling qualities and fatigue life of a rotorcraft.

Rozak and Ray, (Ref. 1) reported the H_∞ design of a multivariable control system along with the simulation results based on a nonlinear model which consists of several multivariable 3-axis rate command (RC) controllers. The controller bandwidths range from the minimum ADS-33 requirement to the maximum achievable limit and did not violate the requirements of robust stability. Each controller was integrated into the nonlinear simulation model which in turn was commanded to fly doublets as well as a 45° right turn at 0.9 V_h (140 knots) to represent the following physical phenomenon: As airspeed increases from hover to full forward flight both the steady and vibratory loads on the control horn greatly increase. In addition, a majority of fatigue life testing is performed at this airspeed. The mechanical load vector on the control horn was then converted into localized stresses using a photo-elastic model. Given the localized stresses, the damage accumulation and rate (for that maneuver) was then calculated by the continuous-time damage model. Controller performance is evaluated using the structured singular value, (μ-analysis), and the ADS-33 handling qualities specification. From the various simulation runs at 140 knots, this investigation shows that an increase of 1.3 radians per second in control system bandwidth may cause as much as a 148% increase in fatigue damage.

Rotorcraft Model Description

The rotorcraft model used in this investigation is shown in Fig. 1. The model contains four sub-models; (1) Non-linear helicopter plant; (2) Structural model; (3) Damage model; and (4) Robust controller model. The rationale for adopting the control as a critical component is that it has the shortest fatigue life amongst the rotor components on this aircraft. A brief description of each of the sub-models is presented below.

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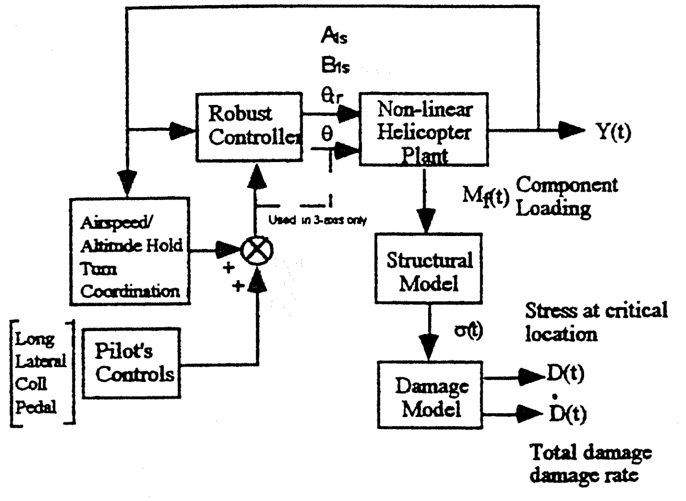
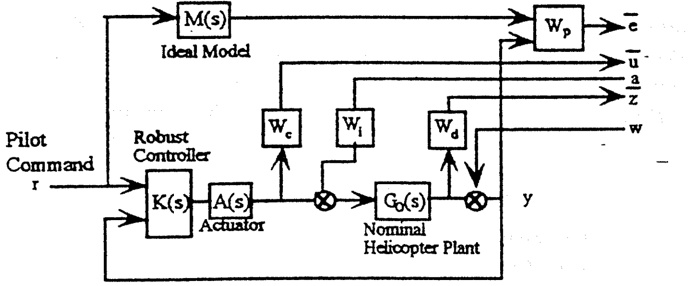


Fig. 1a. Model Structure.



$$\begin{array}{c} \text{Output} \\ \begin{bmatrix} z \\ u \\ e \\ r \\ y \end{bmatrix} \end{array} = \begin{array}{c} \text{P-Block} \\ \left[\begin{array}{ccc|c} 0 & W_d G_o & 0 & W_d G_o A \\ 0 & 0 & 0 & W_c A \\ W_p & W_p G_o & W_p M & W_p G_o A \\ \hline 0 & 0 & I & 0 \\ I & G_o & 0 & G_o A \end{array}\right] \end{array} * \begin{array}{c} \text{Input} \\ \begin{bmatrix} w \\ a \\ r \\ u \end{bmatrix} \end{array}$$

Fig. 1b. Robust Controller Structure.

Non-linear helicopter model. An accurate model of the rotorcraft flight dynamics as well as the loads imparted on the control horn of the Black Hawk is required. A non-linear model of the Black Hawk helicopter was created in FLIGHTLAB using a similar structure as GEN HEL. The model was validated throughout the entire flight regime using both frequency and time domain data gathered by Sikorsky Aircraft from flight test programs. A flexible blade model was used to provide a more accurate representation of the rotor system dynamics as well as the loads imparted onto the control horn. The flexible blade model uses a variety of 3-dimensional translational and rotational kinematic elements distributed over each blade segment, and it uses beam theory and linear superposition of the mode shapes to calculate the elastic deformation along the blade. It was validated using R-DYNE and K-TRAN simulations for all the mode shapes present in the blade model. Since a steady airflow model is used in the FLIGHTLAB simulation, the higher blade frequency modes (4P, 5P, & 6P) are not excited. (As future work, the next step would be to incorporate an unsteady airflow model into FLIGHTLAB.)

Structural model. Every component in the rotor head experiences a variety of both steady and vibratory loads due to the aerodynamic forces and cyclic control inputs imparted onto the flexible blades. One of the places that fatigue occurs on the control horn is the radius undercut at the base of the arm extending to the pitch control pushrod. Structural models of the control horn are very difficult to create because of its complex shape. Therefore, a photo-elastic model of the control horn was fabricated and tested for Sikorsky Aircraft. The report identifies a maximum local stress of 6550 psi at the radius cut-out per 1000 pounds of force applied to the pitch control pushrod attachment location. The control horn is made of high strength aluminum. Aluminum 7075-T6 is the closest to the actual control horn material and was used in this study.

Damage model. To evaluate the amount of control horn fatigue damage occurring from a flight maneuver, a continuous-time damage model developed by Ray et al (Ref. 2) was used in this investigation. The continuous-time damage model is based on the Cyclic Strain-life as well as the Linear Elastic Fracture Mechanics approaches. The damage model based on the cyclic strain-life approach looks at the damage increment as defined by the difference between fatigue lives of different amplitude cycles. It is assumed that damage occurs only when stress is increasing. The non-linear damage is expressed as follows:

$$D = (D_L)^{\gamma(\sigma_a,\, D)} \tag{1}$$

Where D is the non-linear damage, D_L is the linear damage cycle defined by (n/N), and the exponent γ is a function of the stress amplitude and the current level of accumulated damage as outlined by Ray et al. The γ-parameter is unique for different types of materials. It represents the relationship between linear and non-linear damage at various levels of stress amplitude and damage. The procedure to obtain γ for 7075-T6 aluminum is the same as outlined in Wu (Ref. 3) for 4340 steel. It requires that both linear and non-linear damage be known for different values of constant stress amplitudes.

The γ-parameters were computed for each stress amplitude at different values of damage. Figure 2 shows the relationship between γ and non-linear damage (D). As can be seen in this figure, the γ-parameter is strongly dependent on the current level of accumulated damage. The apparent cross-over of the γ-parameter curves is due to the initial damage values selected to match the fatigue life predicted from the strain-life approach. For values of stress amplitude and damage between two consecutive data points, the γ-parameter is linearly interpolated from the logarithmic scale shown in Fig. 2. The γ-parameter is extrapolated for values beyond these curves. It should be noted that for each γ-parameter curve, as the level of initial damage is decreased the corresponding minimum

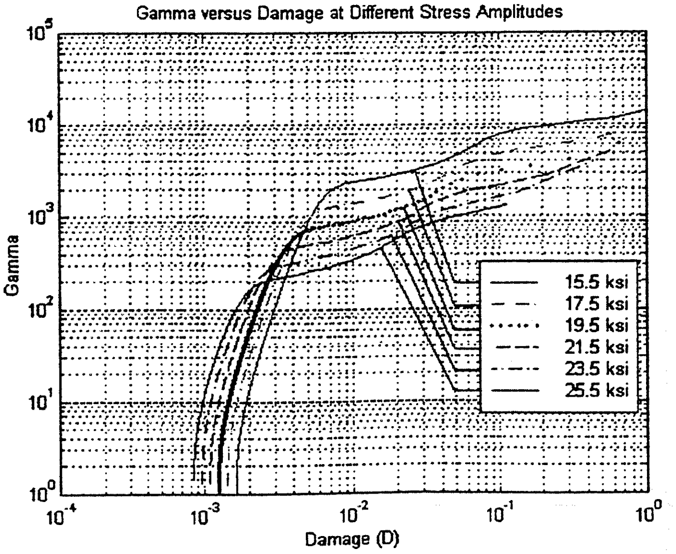


Fig. 2. γ-parameter versus Damage at Different Stress Amplitudes for 7075-T6 Aluminum.

value of γ must be greater than the previous curve. If this is not done the damage model will produce a negative damage increment which is not physically possible. It should be noted that the degree of non-linearity increases as the stress amplitude decreases for high cycle fatigue. Therefore, accumulated damage is much smaller in the initial stages of fatigue life as compared to the linear accumulated damage. Likewise, when the component is near failure the accumulated damage is much faster than the linear accumulated damage.

Robust Controller Design. To apply robust control theory requires a linear-time-invariant-finite-dimensional representation of the aircraft dynamics. Since helicopter dynamics are best described as a function of airspeed, the non-linear model was trimmed at 140 knots forward airspeed, level flight, and linearized. The resultant linear model contains a maximum of 85-states when using the flexible blade model. Further reduction to a 26-state linear model is derived by setting the unnecessary state derivatives to zero and algebraically setting the unwanted states as a function of the desired state space vector. The 26-state linear model consists of 9-states that describe the rigid body dynamics of the helicopter fuselage, 12-states that describe rotor flapping and lagging motions of the non-rotating frame, 3-states representing the main rotor down-wash, and the last two states represent the tail servo rate and position of the Black Hawk stabilator. These last two states are critical in defining plant uncertainty especially in the forward flight regime, as discussed in Ref. 1. The robust controller structure is based on a 3-axis rate command (RC) control system. Figure 1b illustrates the robust controller structure as well as the P-block (input/output) representation that was used in this investigation (Refer to Ref. 1 for [illegible]tion criteria). The structure uses an ideal model, M, which represents the desired closed loop plant dynamics. The weighting function W_p seeks to minimize the error between the actual plant and the ideal model. W_c is used to limit each actuators travel, rate, and acceleration. The position and rate limits were chosen to be 100% and 100%/sec. W_i tries to minimize external input disturbance (i.e. wind gust). The weighting function W_d represents the plant uncertainty due to neglected helicopter states and airspeed variations. The neglected states are due to reducing the 85 state-space linear plant down to 26 states. The uncertainty derived from the flexible blade model is much higher than that of the rigid blade model (above 1 rad/sec) especially in yaw rate. The control structure was recast into the LFT format (Ref. 4) so that H_∞ synthesis and μ-analysis could be accomplished. The robust stability μ-value increased to between 0.85 and 0.93 (depending on desired controller bandwidth) using a flexible blade model instead of a rigid blade model.

Control Law Design Goals

The goal of all new aircraft control system designs is to achieve "level 1 ADS-33C handling qualities performance. The ADS-33C specification is broken up into hover/low speed and forward flight regimes and depicts the various requirements to achieve level 1, 2, or 3 handling quality ratings, (Level 1 being the most desirable). The Black Hawk is a utility aircraft and falls under the "Moderate Maneuvering" MTE category. There are primarily five ADS-33 requirements that the controller needs to adhere:

ADS-33 Requirement	Section	Requirement
1. Bandwidth: Pitch	3.4.1.1	1 rad/sec
Roll & Yaw Axis	3.4.5.1, 3.4.7.1	2 rad/sec
2. Fully Attend Operations (Pole Placement)	3.4.1.2.1	ζ=0.35 (except @ low frequencies)
3. Inter-axis Coupling	3.4.4.1, 3.4.4.2	Less than 0.25
4. Attitude Quickness	3.4.5.2	
5. Collective Climb Rate Response	3.4.4.1.1	

The last objective is to fabricate a controller that provides nominal and robust performance. Robust stability is the major area of concern because it illustrates the amount of plant variation that controller can handle and maintain stability. Each controller is evaluated using μ-analysis. The goal is to try to achieve μ values that are less than 1.

Controller Bandwidth Variation Simulation Results

The dynamic loads and consequent fatigue life for the control horn is a function of control system bandwidth. Therefore several rate command (RC) 3-axis robust controllers were created at different bandwidths ranging from the minimum allowable (as determined by ADS-33C level 1 criteria) to the maximum possible due to the uncertainty present in the system. The controller bandwidth was set by adjusting the ideal model and the performance weighting (W_p) between the allowable range. Therefore, the ideal model and the Wp crossover frequencies are the same. The only exception is in the pitch axis. To meet the ADS-33 performance criteria for the collective axis, the performance weighting function for the pitch axis can not be less than 8 radians per second (Note: The ideal model ranges from 2 to 9 rad/sec). Figure 3 illustrates the changes in bandwidth as a function of the ideal model and the performance weighting function. The μ-analysis of the minimum bandwidth controller yielded nominal performance of 1.98, robust stability of 0.85, and robust performance of 2.29, while the maximum bandwidth controller resulted in 2.68, 0.93, and 2.83 respectively. As the performance weighting function Wp was increased towards the cross-over frequency of W_d, the μ values increased as expected since the solution space is smaller. Regardless of bandwidth, these controllers demonstrate the necessary condition for robust stability (μ_s<1).

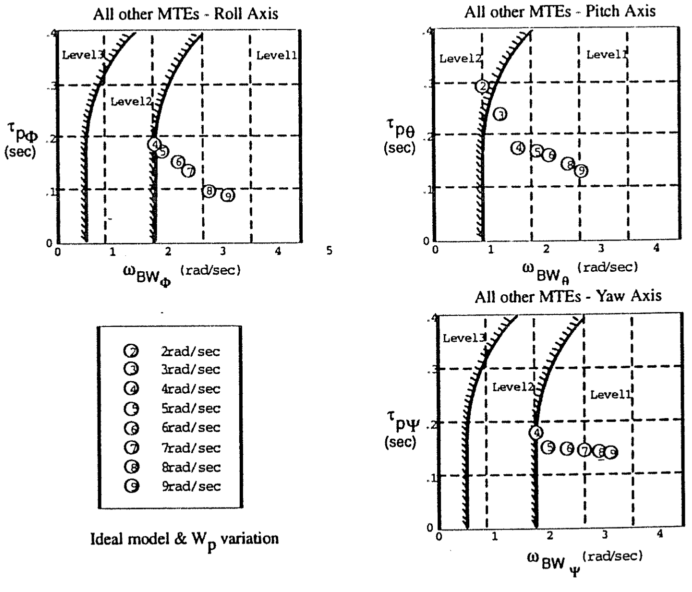


Fig. 3. Summary of Aircraft ADS-33 Bandwidth Response due to Robust Controller Bandwidth Variations.

Simulation Results of the Integrated Model at 140 Knots

Each of the robust controllers fabricated at different bandwidths were integrated into the non-linear FLIGHTLAB model (using the flexible blade model). Flight simulation of each controller was performed by placing a moderate amplitude doublet into each pilot channel, as well as an aggressive 45° right turn maneuver for 18 seconds. Initial damage of 0.002 and 0.010 were both investigated because each value represents a different location on the non-linear damage curve (Ref. 5). The integrated model of the control system with the damage model is shown in Fig. 1. Figure 4

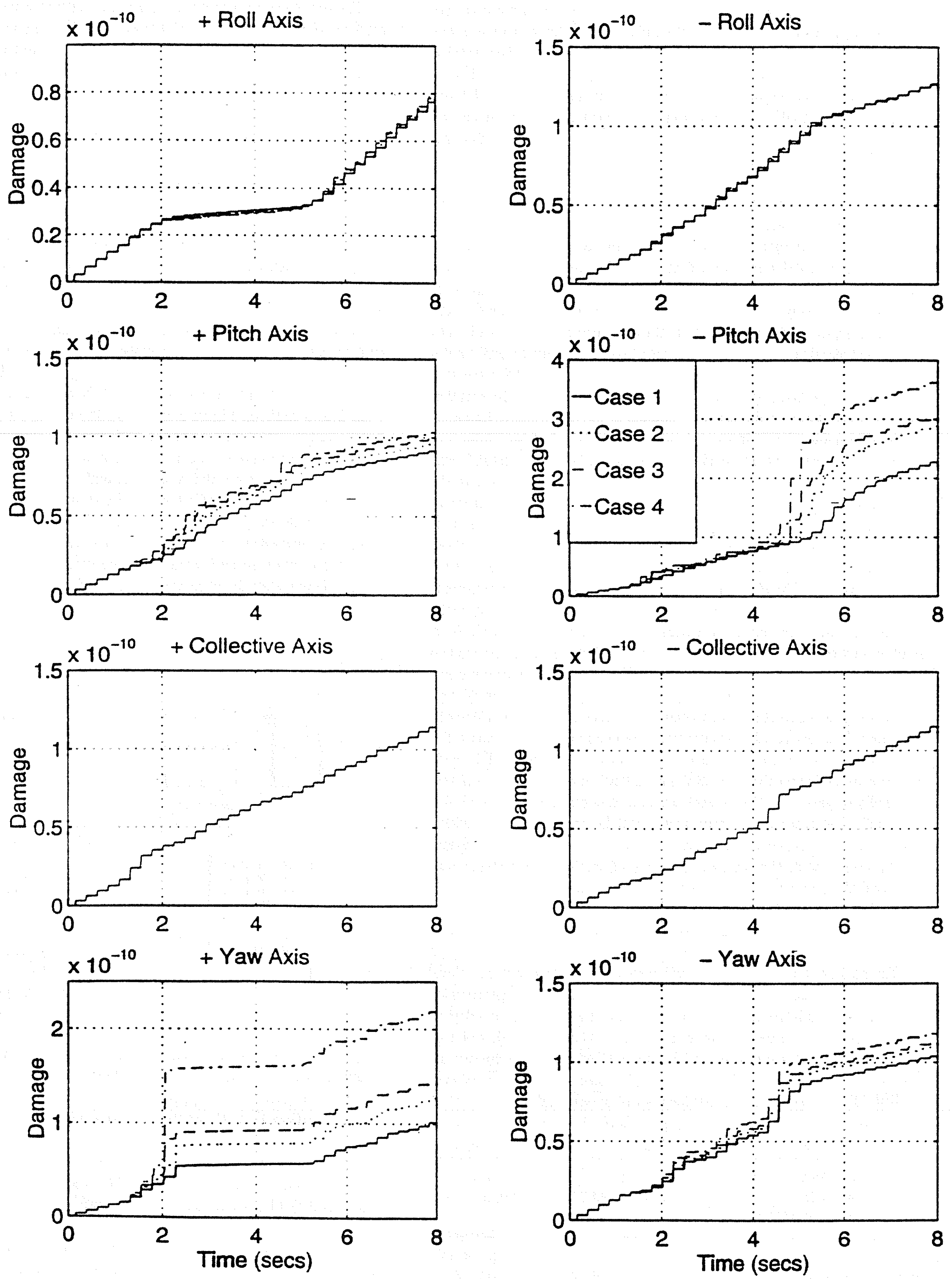
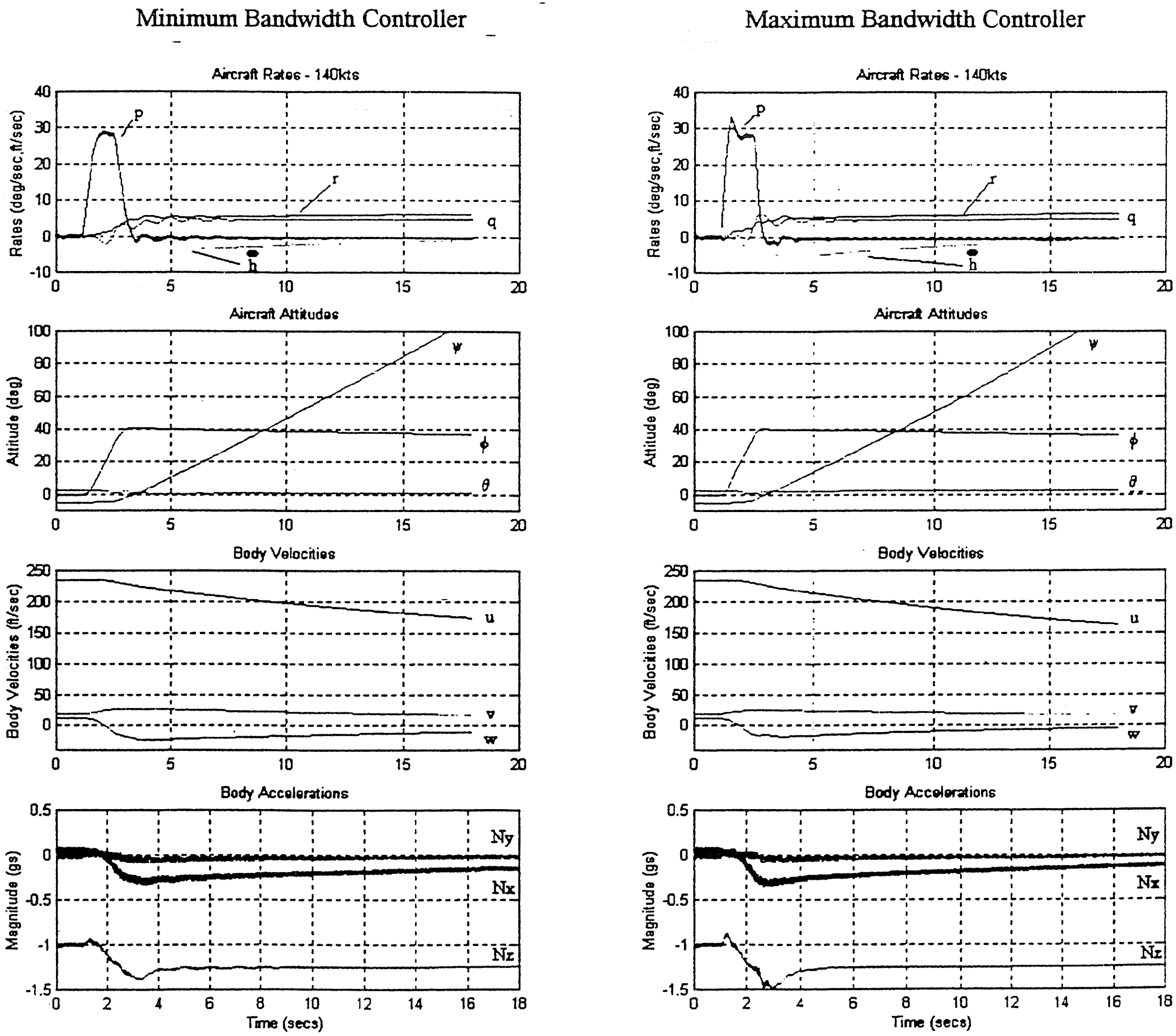


Fig. 4. Change in Accumulated Damage to Positive (+) and Negative (–) On-Axis Doublets – Di = 0.002. Case 1: min. bandwidth; Case 4: max. bandwidth controller.

illustrates the changes in the accumulated damage as a result of placing positive and negative on-axis doublet commands into the integrated FLIGHTLAB model. Case 1 represents the results obtained from using the minimum bandwidth controller (2 radians per second for roll and yaw axes and 1.0 in the pitch axis.) whereas Case 4 presents the maximum bandwidth controller (3.11 & 3.24 radians per second for roll and yaw axes and 2.75 in the pitch axis). The initial damage is set to 0.002 in the damage model (slow growth region). There is almost no change in the roll and collective axes due to the increased controller bandwidth. However, there is a change in both the pitch and yaw axes. This can be explained by the higher forces required to overcome the large moment of inertia in these axes as compared to the roll axis. The simulation was repeated with initial damage in the damage model set to 0.010 which is near component failure. Similar results were obtained, with the exception that the accumulated damage was less sensitive to changes in controller bandwidth. Since the aircraft is not symmetric in forward flight, the non-linear FLIGHTLAB simulation was repeated with on-axis negative doublet commands with initial damage set to 0.010 and 0.002. For the negative doublet, the pitch and yaw axes are, again, sensitive to changes in control system bandwidth. However, the pitch axis is especially sensitive to negative doublets and the yaw axis to positive doublets.

Next, the non-linear FLIGHTLAB simulation was commanded to fly a 45° right turn at 140 knots. This was accomplished by placing at 30 degree per second rate command into the roll axis for 1.5 seconds and activating the turn coordination algorithm. Figure 5 shows the time domain response of the aircraft using the minimum and maximum bandwidth robust controllers. The airspeed hold algorithm is not active for this turn as explained in Ref. 1. Notice there is only a slight improvement in the aircraft response with the maximum bandwidth control system. Figure 6 shows the stresses experienced by the control horn during the turn as well as the change in the accumulated damage during the maneuver as a function of controller bandwidth and initial damage (Di=0.01 & 0.002). At the maximum stress level, the change in stress amplitude is about 4.5 ksi between the minimum and maximum bandwidth control systems. Notice that the accumulated damage is more sensitive to changes in control system bandwidth when the initial damage is in the slow growth area of the non-linear damage curve. Using Figures 3, 5, and 6, there can be a 148% increase in fatigue damage for only an average 1.3 radian per second increase in control system bandwidth. This is for initial damage set to a low value of 0.002. For a higher level of initial damage (0.010), there is a 30% increase in fatigue damage for the same change in control system bandwidth. However, at this level of initial damage, the component is very near the end of


Minimum Bandwidth Controller

Maximum Bandwidth Controller

Fig. 5. FLIGHTLAB Response to a 45° Right Turn Maneuver – 140 knots – Minimum and Maximum Bandwidth Controllers.

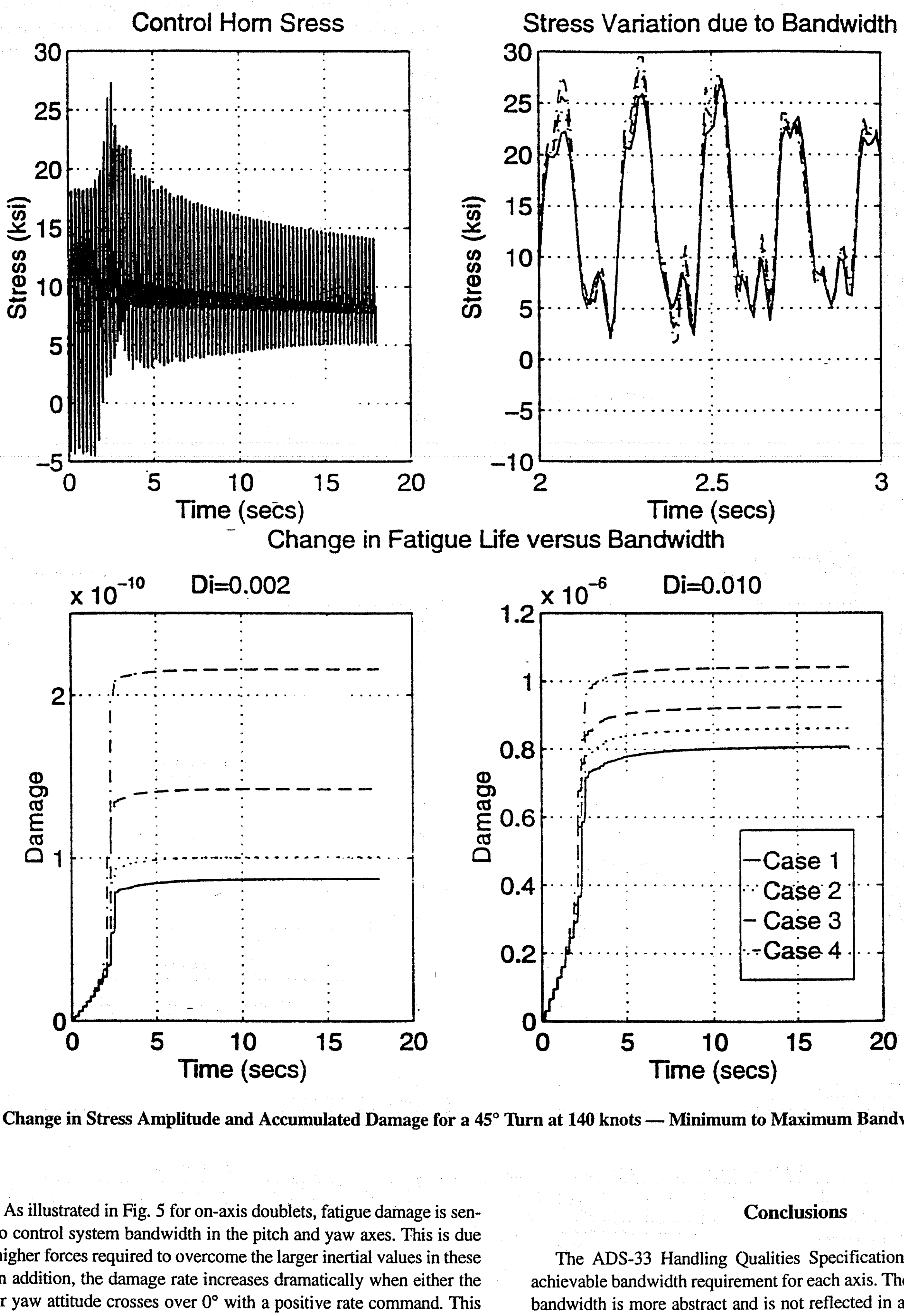


Fig. 6. Change in Stress Amplitude and Accumulated Damage for a 45° Turn at 140 knots — Minimum to Maximum Bandwidth Controllers.

its life. As illustrated in Fig. 5 for on-axis doublets, fatigue damage is sensitive to control system bandwidth in the pitch and yaw axes. This is due to the higher forces required to overcome the larger inertial values in these axes. In addition, the damage rate increases dramatically when either the pitch or yaw attitude crosses over 0° with a positive rate command. This is because the aircraft is not symmetric in forward flight. The non-linear FLIGHTLAB simulations of on-axis doublets and one maneuver reveals that fatigue damage is very sensitive to the control system bandwidth and the level of initial damage.

Conclusions

The ADS-33 Handling Qualities Specification states the minimum achievable bandwidth requirement for each axis. The maximum allowable bandwidth is more abstract and is not reflected in any specification. This investigation shows that the fatigue life of rotorcraft components is sensitive to small changes in the controller bandwidth, especially in the pitch and yaw axes. Therefore control systems designs should take special consideration not to arbitrarily exceed the minimum handling qualities bandwidth requirements without looking at the fatigue life characteristics. This investigation presented an interdisciplinary methodology (by using robust control synthesis and a continuous-time fatigue model) to evaluate the impact that control system bandwidth has upon fatigue life of critical components. This research illustrates that there is a need for an interdisciplinary approach to system design that will optimize the control system bandwidth, fatigue life, weight and cost of the component.

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