Book Reviews

Mathematical Methods in Robust Control of Linear Stochastic Systems (Mathematical Concepts and Methods in Science and Engineering Series)—Yasile Dragan, Toadar Morozan, and Adrian-Mihail Stoica (New York: Springer Science + Business Media, 2006). *Reviewed by Asok Ray*

I. Background Information

Although the concept of linear control systems emerged in the deterministic setting with the work of Nyquist, Bode, and other scientists, *linear stochastic control* as a discipline formed its nucleus in the early 1960s following the seminal work of Wiener, Kalman, and other contributors in the United States and Europe. The decade of the sixties saw a rapid growth in conference and journal publications on stochastic optimal control. Textbooks (e.g, those written by Bryson and Ho [1] and Kwakernaak and Sivan [2]) in this field started appearing almost a decade later. At that point, the word "*robustness*" was not used in the control literature. Following the December 1971 issue of the IEEE Transactions on Automatic Control [3], many graduate students (including the reviewer himself) and even experienced researchers in academia started believing that linear quadratic Gaussian (LQG) was the panacea for linear stochastic control problems, ignoring issues of sensitivity to parametric and nonparametric uncertainties that had already been raised by several distinguished scientists, namely, Kwakernaak, MacFarlane, and Rosenbrock. In this context, Horowitz and Shaked [4] raised a lot of controversy by criticizing the usage of the state–space analysis instead of the classical frequency domain. By the end of the decade of 1970s, it became clear that the sole usage of LQG does not provide solutions to the general class of linear stochastic control problems (e.g., see the short note of Doyle [5] on lack of robustness of LQG).

The theory of robust linear control was firmly established in the deterministic setting through the seminal work of Zames [6] and major contributions from Doyle *et al.* [7]. Nevertheless, other researchers (for example, Sofonov and Bernstein) also contributed to unification of the concepts of robust linear optimal control and linear optimal estimation in output feedback control problems. This task has been achieved by combining the two Riccati equations that are generated as solutions of two-point-boundary-value problems in optimal control and estimation in the respective mutually adjoint spaces.

In the present decade, the scientific and engineering community in the control field is working toward developing a unified theory for control of nonlinear, nonstationary, and multitime-scale spatiotemporal stochastic processes. The resulting control laws must address the issues of stability and performance robustness due to both parametric and nonparametric uncertainties; obviously, these uncertainties (e.g., white noise and Markovian jump perturbations) are not guaranteed to be additive within the structures of stochastic differential and difference equations. These problems pose significant mathematical and systems-theoretic challenges. In the reviewer's opinion, there are two (and perhaps more) general ways to address these issues. One way of solving optimal stochastic control problems (see, for example, Chattopadhyay and Ray [8] and Ray [9]) is the ensemble approach that involves symbolic dynamics [10], automata theory [11], and information theory [12]. The physics community has already adopted the ensemble approach in the discipline of statistical mechanics [13], [14] and also for stability and performance analysis of complex dynamical systems under quasistationary equilibria [15], [16]. The second way is the traditional approach through solution of stochastic differential and difference equations based on Itô Stochastic Calculus [17]; in essence, the problems in this approach are posed in the setting of Itô integrals for nonadditive uncertainties, instead of Wiener integrals for additive white Gaussian noise. It is the second way that the monograph under review has taken to address robust control problems in the linear stochastic setting with nonadditive white noise and Markovian jumps.

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II. Review of the Book

In this monograph, the authors present both general and specialized theories for different aspects of robust control of linear stochastic systems. Apart from valuable background information on mathematical and systems-theoretic concepts, the monograph presents new theoretical results based on stability theory in both deterministic and stochastic settings. In particular, it addresses the robust stabilization problem in stochastic systems that are subjected to both white noise and Markovian jump perturbations. The theory is developed in terms of coupled Riccati equations and inequality constraints by making use of existing concepts from Game Theory.

The monograph consists of seven chapters. Equipped with the preliminary mathematical concepts and fundamental results of linear and nonlinear stochastic differential equations in *Chapter 1*, the authors proceed in *Chapter 2* to more specialized topics on stability. Starting with exponential stability in the deterministic setting on finite-dimensional Hilbert spaces, the necessary and sufficient conditions are established in the stochastic (i.e., mean square) setting in terms of Lyapunov functionals. *Chapter 3* deals with structural properties of linear stochastic systems (e.g., controllability and stabilizability and their dual concepts of observability and detectability) with two types of uncertainties—white noise and Markovian jump perturbations; these two types of uncertainties become the focal point of theoretical work in subsequent chapters.

Chapter 4 begins with stochastic control and filtering problems, analogous to the settings of Kolmogorov backward and forward diffusion, respectively. Specifically, this chapter introduces differential and algebraic generalized Riccati equations that arise in control and estimation problems of linear stochastic systems. Special emphasis is laid on rigorous derivation of necessary and sufficient conditions for maximal, minimal, and stabilizing solutions of the Riccati equations. The chapter ends with an iterative procedure for computation of the maximal solution of Riccati equations. The remaining three chapters provide information that is not apparently available in textbooks and other monographs.

Chapter 5 addresses the linear quadratic problem for optimal control on the infinite horizon for stochastic systems with both white noise and Markovian jump perturbations. General solutions are obtained for both state-multiplicative and control-multiplicative white noise under the class of nonanticipative stochastic controls. The optimal control law is derived in terms of coupled generalized Riccati equations. The chapter is concluded with an application example of the optimal tracking problem. The focal points in this chapter are as follows.

1) Formulation and solution of linear quadratic problems for control and estimation in the infinite horizon.
2) Derivation of stochastic H_2 control laws.

Chapter 6 investigates robust properties of linear stochastic systems with various classes of uncertainties. This chapter augments the known results (e.g., bounded real lemma and small gain theorem) on robustness properties from the deterministic setting to the stochastic setting. New results have been derived not only for stochastic systems with white noise and Markovian jump perturbations individually, but also for a general class of systems involving both types of uncertainties. The focal points in this chapter are as follows.

1) Construction of input–output operators.
2) Formulation of a stochastic version of the bounded real lemma.
3) Derivation of robust stability with respect to linear structured uncertainties.

Chapter 7 is devoted to robust stabilization of linear stochastic systems that are subjected to both multiplicative white noise and Markovian jump perturbations with respect to parametric uncertainties. In line with the established methods of robust stabilization, a disturbance attenuation problem (DAP), called γ-attenuation by the authors, is presented as an extension of the standard H_∞ control problem. Necessary and sufficient conditions are established for existence of stabilizing γ-attenuating controllers in terms of coupled game-theoretic Riccati equations and inequalities. Special attention is paid to attenuation of exogenous perturbations at a specified level. The solution of the general attenuation problem is presented in the setting of linear matrix inequalities (LMI) to provide necessary and sufficient conditions for solvability. The focal points in this chapter are as follows.

1) Formulation and solution of the DAP for full-state access and output measurements.
2) Robust stabilization of linear stochastic systems with Markovian jump perturbations.
3) Formulation of an H_∞-type filtering problem for signals corrupted with multiplicative white noise.

III. Specific Comments

The monograph presents a focused treatment of the mathematics of robust control in linear stochastic systems with few examples. It addresses both established and new theories in the field of robust control, which are supported by adequate proofs and supporting statements. The contents of the monograph are addressed to the research community in applied mathematics and systems theory as well as to mathematically oriented researchers in other fields such as biological sciences, economics, and social sciences, who might be interested in systems-theoretic applications. From this perspective, a researcher who has successfully completed two first-level graduate courses in real analysis [18], a graduate course in linear robust control [2], [19], and graduate courses in stochastic approximation [20] and stochastic control [21] should be able to follow the details presented in the monograph.

Although the authors of this monograph modestly say, "This book is not intended to be a textbook nor a guide for control engineers," it contains valuable information for instructors of graduate courses in control and estimation, who may choose to include certain parts of this monograph in the course contents. Similarly, doctoral students in engineering may benefit not only from the new theories presented in the monograph, but also by following the rigorous mathematical style of presentation.

The monograph is intended to be self-contained although the authors have achieved this goal only from the perspective of proving the theorems. As such, a striking deficiency in the monograph is its lack of meaningful application examples and discussion of potential numerical problems that one may encounter while solving real-life problems. The few numerical examples given in this monograph do not convey the message of how complexities in physical processes could hinder robust control analysis and synthesis. Specifically, the monograph does not address robust identification [22] and how and what kind of uncertainties enter the plant model. The readers would significantly benefit if the monograph's contents are augmented with the following information.

1) Examples showing how white noise and Markovian jumps enter into the models of physical plants during robust identification of the plant model for the purpose of control system analysis and synthesis.
2) A single example of a plant (e.g., longitudinal-motion rigid-body dynamics of an aircraft) to be followed through in Chapters 5–7.
3) Numerical examples (of preferably low-order systems) to exemplify the sources of potential problems and the methods for circumvention of these problems.
4) Provision of a glossary for all acronyms (e.g., DAP for *disturbance attenuation problem*) used in the monograph.

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